A Bracelet of Numbers

by Don Allen, Ottawa, Canada

The numbers that we deal with in this simple investigation are the ten digits, 0 to 9. Choose one such number. Choose a second, which need not be different. Obtain a new number by adding the first two numbers, and recording only the ones digit of the sum. Thus, for 8, 9, we have 8, 9, 7, ..., the start of an unending sequence. Obtain a "next" number by similarly combining the final two. The sequence becomes 8, 9, 7, 6, 3, 9, 2, 1...Continue. Predict what may happen, and investigate to see if you are right.

Try, at leisure, a triple of starting digits instead of a pair. How, otherwise, might the procedure be modified to produce different, but interesting, results.

Hailstone Numbers

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Known now for over 60 years, an attractive "number" investigation involves the so-called hailstone numbers - you'll also see references to the Collatz conjecture and to the "3*x* + 1" problem. Choose any positive integer. The number that you choose necessarily is even or is odd. If it is even, divide it by 2. If it is odd multiply it by 3, then add 1. You obtain in this way a new number. Repeat the process with the number that you obtain. Continue to a sequence of positive whole numbers. Stop only if you reach 1. Numbers that take you to 1 are hailstone numbers.

An example: You choose the starting number, 15. Your sequence is: 15, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1. Your sequence rises to a height of 160, and does reach 1 - in 17 steps; 15, therefore, is a hailstone number. Try 100 (25 steps). Try 31 (106 steps). Try 27.

Which numbers are hailstone numbers? Are all numbers hailstone numbers? What might happen if you were to change the rules?

Kaprekar's Constant

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This number activity has more general possibilities, but to introduce it we focus on its four-digit case. First, write down a four-figure number (not all digits the same), say 3529. Now, rearrange the digits to form the largest number that you can (9532, I suggest) and the smallest such number (2359). Subtract these two numbers (large minus small), and get a new number, 7173. Repeat the process: 7731- 1377 = 6354. Repeat again: 6543 - 3456 = 3087. And again: 8730-0378 = 8352. And again: 8532 - 2358 = 6174. Now, observe: 7641 -1467 = 6174… again, and again! For the amateur mathematician who discovered it, D. R. Kaprekar (he lived in India), this number, 6174, is now known as Kaprekar's constant. Four-figure numbers "lead to" 6174. Why? Show that 3256 leads to 6174 in two steps, 9017 leads to 6174 in seven steps. Would there be a Kaprekar constant for three-figure numbers? ...for five-figure numbers?