The Buffon Needle Problem

by Paul Goodey, University of Oklahoma,
Former Mu Alpha Theta National Secretary-Treasurer

The Buffon Needle Problem was described in 1777 by George-Louis Leclerc, Comte de Buffon [Essai d' arithmetique morale, in volume 44 of the *Supplement à l'Historie Naturelle*]. It leads to an experimental method of approximating π. The results of various Buffon needle experiments have appeared in the literature. In many ways the most incredible (that is, least credible) of these is due to M. Lazzarini. [Un' applicazione del calcolo della probabilità alla ricerca sperimentale di un valore approssimato di π, *Periodico di Matemtica* **4** (1901), 140-143].

This article is intended to provide readers with an analysis sufficient for them to draw their own conclusions as to the veracity of Lazzarini's experiments. Much more detailed and interesting information can be found in the papers of N. T. Gridgeman [Geometric probability and the number π, *Scripta Math.*, **25** (1960), 183-195] and L. Badger [Lazzarini's lucky approximation of π, *Math. Magazine* **67** (1994), 83-91], on which this article is based.

A good starting point for our story is in the works of Archimedes (possibly the most famous of all male streakers) who showed that 22/7 is a good approximation of π -- this is accurate to 2 decimal places. In increasing order of magnitude of their denominator, the next best rational approximations to π are

$\frac{333}{106}$	accurate to 3 decimal places	Athoniszoon, 1583
$\frac{355}{113}$	accurate to 6 decimal places	Tsu Chung-Chin, 480
$\frac{52163}{16604}$	accurate to 5 decimal places	Found by computer
$\frac{103993}{33102}$	accurate to 8 decimal places	Lambert, 1767

A very interesting (and opinionated) history of π can be found in the book *A History of* π (St. Martin's Press, New York, 1974) by Petr Beckmann.

The Buffon Needle Problem envisages the experimenter dropping a needle of length l onto a set of parallel lines distance d apart ($d > l$). The problem asks what is the probability with which the needle will hit a line. In fact, Buffon himself gave the correct answer, $2l/d\pi$. Consequently, if one carries out this experiment the numbers N of experiments and H of hits can be observed. The quantities l and d can, of course, be measured. According to the experiment, the probability of a hit is *H*/*N* and according to the theory, it is $2l/d\pi$. Setting these two equal yields an equation for π , that is,

$$\pi = \frac{2lN}{dH}$$

Lazzarini reported that he carried out this experiment with *l*/*d* = 0.83. He dropped the needle 3408 times and observed 1808 hits. No doubt you are already asking yourself why would anyone set out to do 3408 experiments and then stop! Well, if you know that the number of experiments was 3408 then you know that the value of π is going to be

$$\pi = \left(\frac{2l}{d}\right)\left(\frac{3408}{H}\right)$$

Had you read something of the history of π you would know that 355/113 is an excellent approximation and you might further note the advantage of carrying out a number of experiments that shares a factor with 355:

$$\pi = \left(\frac{2l}{d}\right)\left(\frac{2^4\times3\times71}{H}\right) = \left(\frac{2l}{d}\right)\left(\frac{2^4\times3\times355}{5H}\right)$$

By now you see the opportunity to have 355 in the numerator. Unfortunately, you now have a 5 in the denominator and this is not a factor of the number 113, which you would be delights to see down there. However, all is not lost. After all, you have to report of the length, *l*, of the needle. If you're lucky it might turn out to be 5. Naturally the distance between the lines has to exceed this, why not 6?

$$\pi = \left(\frac{2\times5}{6}\right)\left(\frac{2^4\times3\times355}{5H}\right) = \left(\frac{2^4\times355}{H}\right)$$

If only you could observe $2^4\times113$ hits! The prosecution rests its case.