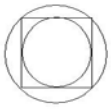
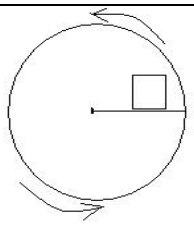
2007 – 2008 Log1 Contest Round 1
Theta Geometry

Name: _____

	4 points each	
1	The perimeter of a regular pentagon is 60. What is the side length?	
2	Which of the following regular polygons <u>cannot</u> be used to tessellate (cover with no overlaps or gaps) the plane: Triangle, Square, Pentagon, Hexagon, Octagon.	
3	You receive a present wrapped in a rectangular box with a volume of 60 cubic feet and side lengths that are three consecutive integers. What was the minimum amount of wrapping paper (in square feet) used to wrap the present?	
4	The tires of a bicycle have a radius of 1 ft. On the ride from home to school, the wheels rotate 500 times. How far (in feet) is school from home?	
5	A circle is inscribed in a square and that square is inscribed in a larger circle. What is the ratio of the big circle's area to the little one? 	

	5 points each	
6	In triangle ABC, $m\angle A = m\angle B - 10^\circ$ and $m\angle B = m\angle C - 13^\circ$. Find the measure of $\angle C$.	
7	A square pyramid (in other words, a pyramid with a square base and equilateral triangle sides) has a volume of 2/3. What is its height?	
8	A "hectogon" is a regular polygon in which each interior angle measures exactly 176.4° How many sides does a hectogon have?	
9	I take a spherical container full of milk and pour it into a cylindrical container of equal diameter. The milk fills the cylinder right to the top without spilling. What is the height of the cylinder in terms of its radius?	
10	I'd like to have a perfect ice cream cone. The perfect ice cream cone would have a base diameter exactly equal to the diameter d of a ball of ice cream. Besides the half of the top ice cream ball that's in the cone, there should be room in the cone for another ice cream ball of the same size to be stuffed inside! In terms of the diameter d of the spherical ice cream balls, what is the height of the perfect cone?	

	6 points each	
11	Guido Anchovy is exceptionally lazy at cutting pizza. He likes to make as many slices as he can with as few straight cuts possible. He doesn't care if the pieces are even close to the same shape or size. Determine how many cuts Guido will make if he has to serve at least 20 people.	
12	How many cans of 3" diameter will fit in a 10.5" X 16" box?	
13	A dartboard is a series of concentric circles alternating between red and black. Each ring is 1.5" wide and there are 5 rings around a black center 2" wide. If I throw a dart at the board in a way that is considered random (but it is guaranteed to hit the board), find the probability that it will land in a red ring.	

14	A Rubik's cube 4" on a side is placed on a record player such that one corner is 4" from the record player's center and one face of the cube lines up perfectly on a radius of the record player. When the record player spins, what volume does the cube sweep out in the air? 	
15	Two goats are tethered by ropes, each 20 feet long, to trees $20\sqrt{2}$ feet apart. To the nearest square foot, what is the area of the grass both goats can graze in together?	

2007 – 2008 Log1 Contest Round 1
Alpha Geometry

Name: _____

4 points each		
1	The perimeter of a regular pentagon is 60. What is the side length?	
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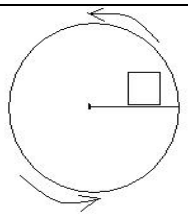
3	You receive a present wrapped in a rectangular box with a volume of 60 cubic feet and side lengths that are three consecutive integers. What was the minimum amount of wrapping paper (in square feet) used to wrap the present?	
4	The tires of a bicycle have a radius of 1 ft. On the ride from home to school, the wheels rotate 500 times. How far (in feet) is school from home?	
5	I put 27 sugar cubes together to form a larger cube, then I lick the outside faces of the large cube, except for the bottom. How many more sugar cubes are there that have two faces licked than those that are completely unlicked?	

5 points each		
6	In triangle ABC, $m\angle A = m\angle B - 10^\circ$ and $m\angle B = m\angle C - 13^\circ$. Find the measure of $\angle C$.	
7	A square pyramid (in other words, a pyramid with a square base and equilateral triangle sides) has a volume of 2/3. What is its height?	
8	A "hectogon" is a regular polygon in which each interior angle measures exactly 176.4° How many sides does a hectogon have?	
9	I take a spherical container full of milk and pour it into a cylindrical container of equal diameter. The milk fills the cylinder right to the top without spilling. What is the height of the cylinder in terms of its radius?	
10	The volume of a torus (a donut shape) is given by the equation $V=2\pi^2 Rr^2$, where R is the distance from the center of the tube to the center of the torus, and r is the radius of the tube. A donut-hole is a sphere with a diameter equal to the diameter of the hole in the torus that it was removed from. If a particular donut has R=1.2" and r=0.6", how many donut-holes (a whole number) must I eat to consume a volume of dough equivalent or greater than one donut?	

6 points each		
11	Guido Anchovy is exceptionally lazy at cutting pizza. He likes to make as many slices as he can with as few straight cuts possible. He doesn't care if the pieces are even close to the same shape or size. Determine how many cuts Guido will make if he has to serve at least 20 people.	
12	How many cans of 3" diameter will fit in a 10.5" X 16" box?	
13	A dartboard is a series of concentric circles alternating between red and black. Each ring is 1.5" wide and there are 5 rings around a black center 2" wide. If I throw a dart at the board in a way that is considered random (but it is guaranteed to hit the board), find the probability that it will land in a red ring.	

14	A Rubik's cube 4" on a side is placed on a record player such that one corner is 4" from the record player's center and one face of the cube lines up perfectly on a radius of the record player. When the record player spins, what volume does the cube sweep out in the air? 	
15	Find the area of a convex hexagon with vertices (-1,2), (5,7), (4,-1), (0,-2), (8,3) and (1,7).	

2007 – 2008 Log1 Contest Round 1
Mu Geometry

Name: _____

4 points each		
1	The perimeter of a regular pentagon is 60. What is the side length?	
2	Which of the following regular polygons <u>cannot</u> be used to tessellate (cover with no overlaps or gaps) the plane: Triangle, Square, Pentagon, Hexagon, Octagon.	
3	You receive a present wrapped in a rectangular box with a volume of 60 cubic feet and side lengths that are three consecutive integers. What was the minimum amount of wrapping paper (in square feet) used to wrap the present?	
4	I ride an old-fashioned bicycle down the block. The radius of the front tire is 2.5 feet, but the back tire is only 1 foot in diameter! If the distance of the trip is 500π feet, how many more times will my back tire rotate than the front tire?	
5	I put 27 sugar cubes together to form a larger cube, then I lick the outside faces of the large cube, except for the bottom. How many more sugar cubes are there that have two faces licked than those that are completely unlicked?	

5 points each		
6	In triangle ABC, $m\angle A = m\angle B - 10^\circ$ and $m\angle B = m\angle C - 13^\circ$. Find the measure of $\angle C$.	
7	A square pyramid (in other words, a pyramid with a square base and equilateral triangle sides) has a volume of 2/3. What is its height?	
8	A "hectogon" is a regular polygon in which each interior angle measures exactly 176.4° How many sides does a hectogon have?	
9	A ferris wheel is 200 feet in diameter, turns at 1 revolution per minute and the lowest point while riding is 2 feet above the platform. You happen to glance at your watch as you start at the bottom. You pass over the top, but then the ferris wheel gets stuck! It's been 50 seconds since you last looked at your watch, so how high above the platform are you now?	
10	The volume of a torus (a donut shape) is given by the equation $V=2\pi^2 Rr^2$, where R is the distance from the center of the tube to the center of the torus, and r is the radius of the tube. A donut-hole is a sphere with a diameter equal to the diameter of the hole in the torus that it was removed from. If a particular donut has R=1.2" and r=0.6", how many donut-holes (a whole number) must I eat to consume a volume of dough equivalent or greater than one donut?	

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11	Guido Anchovy is exceptionally lazy at cutting pizza. He likes to make as many slices as he can with as few straight cuts possible. He doesn't care if the pieces are even close to the same shape or size. Determine how many cuts Guido will make if he has to serve at least 20 people.	
12	How many cans of 3" diameter will fit in a 10.5" X 16" box?	

13	A dartboard is a series of concentric circles alternating between red and black. Each ring is 1.5" wide and there are 5 rings around a black center 2" wide. If I throw a dart at the board in a way that is considered random (but it is guaranteed to hit the board), find the probability that it will land in a red ring.	
14	Two ants are tethered, each by a gossamer thread of the same length, to opposite corners (at ends of a long diagonal) of a cube and can travel anywhere on the faces of the cube that their tethers permit. The ants can meet at several (finite) unique points on the cube, but otherwise have no overlap in their regions of movement. If the side of the cube has a length of 1, how long are the gossamer threads?	
15	Find the area of a convex hexagon with vertices (-1,2), (5,7), (4,-1), (0,-2), (8,3) and (1,7).	

2007 – 2008 Log1 Contest Round 1
Geometry Answers

Theta Answers	
1	12
2	Pentagon and Octagon (must have both)
3	94
4	1000π
5	2:1 or 2
6	72°
7	1
8	100
9	4/3 r
10	3d
11	6
12	18
13	171/289
14	256π
15	228

Alpha Answers	
1	12
2	Pentagon and Octagon (must have both)
3	94
4	1000π
5	10
6	72°
7	1
8	100
9	4/3 r
10	10
11	6
12	18
13	171/289
14	256π
15	54

Mu Answers	
1	12
2	Pentagon and Octagon (must have both)
3	94
4	400
5	10
6	72°
7	1
8	100
9	52
10	10
11	6
12	18
13	171/289
14	$\frac{\sqrt{5}}{2}$
15	54

2007 – 2008 Log1 Contest Round 1
Geometry Solutions

Th	Al	Mu	Solution
1	1	1	A pentagon has 5 sides; 60 divided by 5 is 12.
2	2	2	The measure of an interior angle must divide 360; only triangle(60), squares(90) and hexagons(120) will work.
3	3	3	The box's sides are 3, 4, and 5. The surface area is: 2 *((3*4)+(3*5)+(4*5)) = 94
4	4		The distance is the product of the circumference of the wheel, 2π, and the number of rotations, 500.
		4	The back tire rotates 500 times, while the front tire rotates 100 times.
5			If the small circle's diameter is x, then the diagonal of the square and also the diameter of the larger circle is $x\sqrt{2}$. The ratio of areas, $(x/2)^2:(x\sqrt{2}/2)^2$ simplifies to 2:1.
	5	5	12 cubes have two faces licked, and 2 cubes are unlicked: the center of the middle layer and the bottom layer. So 12-2=10
6	6	6	Substitute values for $\angle A$ and $\angle B$, then solve for $\angle C$: $(m\angle C - 13) - 10 + m\angle C - 13 + m\angle C = 180°$. $3^* m\angle C = 216°$, so $m\angle C = 72°$.
7	7	7	If L is the length of any side, then $V=\sqrt{2} / 6 * L^3$, which makes $L=\sqrt{2}$. Plug this value into the equation $V=1/3 * L^2 h$ (where h is the height) and you find that $h=1$.
8	8	8	Each exterior angle of his polygon will measure 3.6 degrees. Dividing this into 360 means there are 100 sides. Actually, the greek-derived name "hectogon" is not usually used.
9	9		The sphere and cylinder have identical volume, so $V=\pi r^2 h=4/3\pi r^3$.
		9	At 5/6 of a complete revolution, the point you are stuck at forms a 30-60-90 right triangle where the hypotenuse is the radius and the legs are your gondola's horizontal and vertical distance from the ferris wheel's center. The vertical distance below the center is half the hypotenuse, 100/2=50. The center itself is 102 feet above the platform, so you are 52 feet above it. Don't jump!
10			Volume of the cone, $(1/3)\pi r^2 h$ = 1.5 ice cream balls, $(3/2)*(4/3)\pi r^3$. This simplifies to 6 r = h, or $3d = h$.
	10	10	The radius of a donut-hole is $R-r$, and its volume is $4/3\pi(R-r)^3$. The general equation for the donut to donut-hole ratio is $(2\pi^2 Rr^2) / (4/3\pi(R-r)^3)$. For this particular donut (with R =1.2" and r =0.6" thus $2r = R$ and $R-r = r$), one must eat 3π donut-holes. Round up to 10.
11	11	11	The number of pieces of pizza, starting with 1 (the whole pizza), increases according to an additive series where the difference between consecutive terms is always 1 greater than the difference of the preceding pair. 1,(+1=)2,(+2=)4,(+3=)7,(+4=)11,(+5=)16,(+6=)22...
12	12	12	The total amount of room taken up by 18 cans, if arranged in a offset "honeycomb" manner, will be 10.5" X ≈15.99". 3 cans, plus half a can's width from the offset row makes 3.5*3=10.5. The horizontal distance between can centers in offset rows is the $\sqrt{3}$ leg of a 30-60-90 triangle. 5 offset widths plus the two half-can widths on the edges makes $5*3(\sqrt{3})/2 + 2*1.5 \approx 15.99$.
13	13	13	The dartboard's radius is 8.5 and its total area is $289\pi/4$. Now add up the area of the red rings: beginning with the outer ring, the areas are $93\pi/4$, $57\pi/4$ and $21\pi/4$. The answer is 171/289.

<table>
<tr><td>14</td><td>14</td><td></td><td>The furthest edge of the Rubik's cube is a distance of $4\sqrt{5}$ from the record player's center. This distance is determined by finding the hypotenuse of the triangle where the other two legs are 1) the edge of the cube on the outside edge of the record player, 4, and 2) the edge parallel to the record player's radius + its distance from the center, 8. $4\sqrt{5}$ is the outer diameter of the swept volume, which is ring shaped, with a constant height of 4. The inner diameter is 4. The volume of outer shape minus the inner hole will give us the swept volume: $4\pi*((4\sqrt{5})^2 - 4^2) = 256\pi$</td></tr>
<tr><td></td><td></td><td>14</td><td>Unfold two adjacent faces of the cube as though coplanar. For the ants to have no area of overlap, they can only meet at one point, which is the midpoint between the two corners. The location of this point is 1 unit in one direction and $\frac{1}{2}$ unit in the other, making the distance = $\sqrt{(1^2 + 0.5^2)}$, which reduces to $(\sqrt{5})/2$. The ants can only meet in the middle of 6 of the cubes edges.</td></tr>
<tr><td>15</td><td></td><td></td><td>The distance between the trees is the diagonal of a square of side length 20. The area of overlap can be calculated be taking twice the area of the region obtained by subtracting the 45-45-90 triangle from the quarter-circle sector within the square: $2 (20^2\pi/4 - 20^2/2) \approx 228$.</td></tr>
<tr><td></td><td>15</td><td>15</td><td>Use the shoelace method (remember to re-order the points) or draw a rectangle around the graphed hexagon and subtract off the areas of the surrounding shapes.
<table>
<tr><td></td><td>-1</td><td>2</td><td></td></tr>
<tr><td>2</td><td>1</td><td>7</td><td>-7</td></tr>
<tr><td>35</td><td>5</td><td>7</td><td>7</td></tr>
<tr><td>56</td><td>8</td><td>3</td><td>15</td></tr>
<tr><td>12</td><td>4</td><td>-1</td><td>-8</td></tr>
<tr><td>0</td><td>0</td><td>-2</td><td>-8</td></tr>
<tr><td>2</td><td>-1</td><td>2</td><td>0</td></tr>
<tr><td>107</td><td></td><td></td><td>-1</td></tr>
</table>
(1/2)(107 - -1)=54</td></tr>
</table>