2007 – 2008 Log1 Contest Round 1
Theta Logs and Exponents

Name: ___________________

4 points each		
1	Which of the following is smaller, 5^{25} or 10^{20}?	
2	Solve for x, $4^{16} = 16^{x}$.	
3	Solve $\log_{10} 1000$, assuming that the digits are in base 4 and not base 10.	
4	$12^{a}8^{b}27^{c} = 216$. Solve a+b+c .	
5	If you express $\sqrt{65000}$ in the form $a\sqrt{b}$, where a is the largest possible number such that a and b are integers, what is the value of $a+b$?	

5 points each		
6	For problems 6 and 7, use the following information. Benford's Law states that for certain random selections of numbers, the leading, leftmost digit, d, is not uniformly distributed, but occurs with probability proportional to: $\log_{b}\left(\frac{d+1}{d}\right)$, where b is the base. In base 16, what is the probability that the leading digit is not 1?	
7	In base 10, what is the probability that the leading digit is less than 5? Leave answer in log terms.	
8	What is the product of the roots of $4x^3 - 13x + 6 = 0$?	
9	What is the sum of the coefficients of the expansion of $(3x - y/2 + 1/2)^3$?	
10	What is the remainder when 2^{15} is divided by 7?	

6 points each		
11	$\log_{10}2$ is approximately equal to .3010. To four significant figures, find the value of $\log_{10}25$.	
12	When Nina runs, each 1000 meters (1K) takes 10% longer than the previous one. She runs 1K in 2 minutes and 57 seconds and her time for 5K is 18 minutes. To the nearest minute, how long will it take her to run 10K?	
13	Solve for all real values of x: $e^{2x} - e^{x} = 6$.	
14	What is the positive x-intercept of $y = 1 - \ln(x^2 + 1)$?	
15	What is the value of the constant term of the expansion of $\left(2x^2 + \frac{1}{x}\right)^6$?	

2007 – 2008 Log1 Contest Round 1
Alpha Logs and Exponents

Name: ____________________

	4 points each	
1	Which of the following is smaller, 5^{25} or 10^{20}?	
2	Solve for x, $4^{16} = 16^x$.	
3	Solve $\log_{10} 1000$, assuming that the digits are in base 4 and not base 10.	
4	$12^a 8^b 27^c = 216$. Solve a+b+c .	
5	A = $\begin{bmatrix} \ln(2) & -1 \\ \ln(4) & 3 \end{bmatrix}$ The determinate of matrix A can be expressed in the form ln(x). What is the value of x?	

	5 points each	
6	For problems 6 and 7, use the following information. Benford's Law states that for certain random selections of numbers, the leading, leftmost digit, d, is not uniformly distributed, but occurs with probability proportional to: $\log_b\left(\frac{d+1}{d}\right)$, where b is the base. In base 16, what is the probability that the leading digit is not 1?	
7	In base 10, what is the probability that the leading digit is less than 5? Leave answer in log terms.	
8	What is the product of the roots of $4x^3 - 13x + 6 = 0$?	
9	What is the sum of the coefficients of the expansion of $(3x - y/2 + 1/2)^3$?	
10	What is the remainder when $555^{555^{555}}$ is divided by 7?	

	6 points each	
11	$\log_{10}2$ is approximately equal to .3010. To four significant figures, find the value of $\log_{10}25$.	
12	When Nina runs, each 1000 meters (1K) takes 10% longer than the previous one. She runs 1K in 2 minutes and 57 seconds and her time for 5K is 18 minutes. To the nearest minute, how long will it take her to run 10K?	
13	Solve for all real values of x: $e^{2x} - e^x = 6$.	
14	What is the positive x-intercept of $y = 1 - \ln(x^2 + 1)$?	
15	Simplify $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5$.	

2007 – 2008 Log1 Contest Round 1
Mu Logs and Exponents

Name: ____________________

	4 points each	
1	Which of the following is smaller, 5^{25} or 10^{20}?	
2	Solve for x, $4^{16} = 16^x$.	
3	Solve $\log_{10} 1000$, assuming that the digits are in base 4 and not base 10.	
4	$\lim_{n \to \infty} \frac{4^{n+1}}{2^{2n} + 1} = ?$	
5	A = $\begin{bmatrix} \ln(2) & -1 \\ \ln(4) & 3 \end{bmatrix}$ The determinate of matrix A can be expressed in the form ln(x). What is the value of x?	

	5 points each	
6	For problems 6 and 7, use the following information. Benford's Law states that for certain random selections of numbers, the leading, leftmost digit, d, is not uniformly distributed, but occurs with probability proportional to: $\log_b\left(\frac{d+1}{d}\right)$, where b is the base. In base 16, what is the probability that the leading digit is not 1?	
7	In base 10, what is the probability that the leading digit is less than 5? Leave answer in log terms.	
8	What is the product of the roots of $4x^3 - 13x + 6 = 0$?	
9	f(x) = x ln(x) -3x, where $x \in (0, \infty)$. Find the minimum value of f(x).	
10	What is the remainder when $555^{555^{555}}$ is divided by 7?	

	6 points each	
11	$\log_{10}2$ is approximately equal to .3010. To four significant figures, find the value of $\log_{10}25$.	
12	When Nina runs, each 1000 meters (1K) takes 10% longer than the previous one. She runs 1K in 2 minutes and 57 seconds and her time for 5K is 18 minutes. To the nearest minute, how long will it take her to run 10K?	
13	Solve for all real values of x: $e^{2x} - e^x = 6$.	
14	$y = 2^{x+5}$. Solve for y' (the derivative of y with respect to x). Answer in terms of y.	
15	Simplify $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5$.	

2007 – 2008 Log1 Contest Round 1
Logs and Exponents Answers

	Theta Answers	Alpha Answers	Mu Answers
1	5^{25}	5^{25}	5^{25}
2	8	8	8
3	3	3	3
4	2	2	4
5	76	32	32
6	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$
7	$\log_{10}(5)$	$\log_{10}(5)$	$\log_{10}(5)$
8	$-\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$
9	27	27	$-e^2$
10	1	1	1
11	1.398	1.398	1.398
12	47 [minutes]	47 [minutes]	47 [minutes]
13	ln(3)	ln(3)	ln(3)
14	$\sqrt{e-1}$	$\sqrt{e-1}$	$\ln(2)y$
15	60	$-\frac{\sqrt{3}}{2}+\frac{i}{2}$	$-\frac{\sqrt{3}}{2}+\frac{i}{2}$

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Logs and Exponents Solutions

Th	Al	Mu	Solution
1	1	1	Quick comparison, $5^{25} = 5^5 * 5^{20}$ $10^{20} = 2^{20} * 5^{20}$ $5^5 < 2^{20}$, therefore 5^{25} is smaller.
2	2	2	Rewrite in like terms : $4^{16} = 16^x$ $(2^2)^{16} = (2^4)^x$ $2^{32} = 2^{4x}$ x = 8
3	3	3	Whether the base is 4, 10, or any integer, $1000 = 10^3$, so the answer will be 3.
4	4		The simplest solution may be the trial and error method. 216 is 6^3, 8 is 2^3, and 27 is 3^3, so one solution is (0, 1, 1) which sums to 2. A more full proof method would be to write all number in prime factorizations and solve in a system of equations. $12^A 8^b 27^c = 216 =$ $(2^2 \cdot 3)^a (2^3)^b (3^3)^c = 2^3 3^3$ 2a + 3b = 3 a + 3c = 3 Combine terms: 3a + 3b +3c = 6 a + b + c = 2
		4	$\frac{4^{n+1}}{2^{2n}+1} = \frac{4 \cdot 4^n}{4^n+1}$. As this approaches infinity, the one in the denominator drops off, and you are left with 4.
5			65000 contains a factor of 100, which reduces b to 650. This is also divisible by 25, which reduces b further to 26, a number that is divisible by no more squares. $\sqrt{100 \cdot 25} = 50$. 50 +26 = 76.
	5	5	The determinate of matrix A is 3ln(2) - (-1ln(4)) or ln(8) +ln(4) = ln(32).
6	6	6	Find the probability that the leading digit is a 1: $\log_{16}(1 + 1) - \log_{16}(1) = \log_{16}(2) = \frac{1}{4}$. The probability that the leading digit is not 1 is $1 - \frac{1}{4}$ or $\frac{3}{4}$.
7	7	7	P = $\log_{10}\left(\frac{2}{1}\right) + \log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right)$ Combine terms and all the terms except the denominator of the first entry and the numerator of the last will cancel, leaving $\log_{10}(5)$.
8	8	8	The product of the roots of polynomial $y = ax^n + bx^{n-1} + ... + c$ is equal to -c/a if n is odd (c/a if n is even), or in this case -7/2.
9	9		One shortcut method is to realize that the variable x, y, and z will not change the sum of the coefficients and can be substituted by the number 1. Therefore the sum of the coefficients is equal to $(3 - 1/2 + 1/2)^3$ or $3^3 = 27$.
		9	Take the derivative of the function using the product rule to get f'(x) = ln(x) + 1 – 3. Solve for when f'(x) is equal to zero. 0 = ln(x) -2 2 = ln(x) $x = e^2$. $f''(e^2)=1/e^2 > 0$ so it is indeed a minimum. If you plug e^2 back into f(x), $f(e^2) = -e^2$.
10			2^1 has a remainder of 2 when divided by 7. 2^2 has a remainder of 4, 2^3 a remainder of 1; 2^4 a remainder of 2, etc. The remainder repeats every multiple of 3. Since 15 is a multiple of 3, 2^{15} will have a remainder of 1.

Th	Al	Mu	Solution
	10	10	555/7 = 79 remainder 2. We can rewrite the problem in the following way: $(7\cdot 79+2)^{555^{555}}$. Since the terms containing a factor of 7 will always be divisible by 7, the only term we need to care about is the power of 2. $2^1 \bmod 7 \equiv 2$ $2^2 \bmod 7 \equiv 4$ $2^3 \bmod 7 \equiv 1$ $2^4 \bmod 7 \equiv 2(7+1) \bmod 7 \equiv 2$ The division repeats every multiple of 3. Since 555 is a multiple of 3, 555 raised to any positive integer must also be divisible by 3. Therefore $2^{555^{555}} \bmod 7 \equiv 1$.
11	11	11	In order to find $\log_{10}25$, you must find $\log_{10}5$. $\log_{10}2$ is given and 1 - $\log_{10}2$ = $\log_{10}5$, or in decimal approximation, .699. Since $\log_{10}25$ = 2 $\log_{10}5$, 2*.699, or 1.398 is the answer.
12	12	12	The information about her 1K time is unnecessary. Nina's 5K is equal to K + (1.1)K + $(1.1)^2$K + $(1.1)^3$K + $(1.1)^4$K. The next 5K will be equal to $(1.1)^5$ multiplied by her 5K time. Finding $(1.1)^5$ is somewhat time consuming, but the powers of 11 are easier than some. $(1.1)^2$ = 1.21, $(1.1)^3$ = 1.331, $(1.1)^4$ = 1.4641, $(1.1)^5$ = 1.61501. 1.61 * 18 will give the time of the second 5K, equal to about 29. 29 +18 or 47 minutes gives her total 10K time.
13	13	13	Solve using substitution: e^{2x} -e^x – 6 = 0 can be rewritten as A^2 – A – 6 = 0, where A = e^x. Factoring , 3 and -2 are the roots. However, -2 = e^x has no real solutions. ln(3) is the only real solution.
14	14		Set equal to zero and solve for x: 0 = 1 – ln(x^2 +1) 1 = ln(x^2 +1) e = (x^2 +1) e – 1 = x^2 Since we are only looking for the positive value $x=\sqrt{e-1}$.
		14	Rewrite in terms of e (Be careful of the constant term): $y=2^{x+5}$ = $2^5 2^x$ $\ln(y)=\ln(2^5)+x\ln(2)$ $y=e^{\ln(2^5)+x\ln(2)}$ $y'=\ln(2)e^{\ln(2^5)+x\ln(2)}$ $y'=\ln(2)y$ or differentiate implicitly to get y'/y = ln(2)
15			$\left(2x^2+\frac{1}{x}\right)^6$ is equal to $\left(\frac{1}{x}\right)^6\left(2x^3+1\right)^6$ and the constant term occurs at $\binom{6}{2}\left(\frac{1}{x}\right)^6(2x^3)^2(1)^4$ or 15*4 = 60
	15	15	Either solve directly, or use Euler's formula, $e^{\theta i}=\cos(\theta)+i\sin(\theta)$ and realize that $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5$ is equal to $\left(e^{\frac{\pi i}{6}}\right)^5$ = $\left(e^{\frac{5\pi i}{6}}\right)$. Plugging back into Euler's formula to get $\cos(\frac{5\pi}{6})+i\sin(\frac{5\pi}{6})$ or $\left(-\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$.