2009 – 2010 Log1 Contest Round 1
Theta Logs and Exponents

Name: _____

	4 points each	
1	A population of lions quadruples every month. If the initial population of the lions was 50, what is the population after 2 months?	
2	Simplify and express as a fraction: $(27^{-2/3}) + (\log_2 8)$.	
3	My great-great-grandfather is $\frac{3}{4}$ Norwegian Buffalo Stomper. Assuming that no one else in my family has any Norwegian Buffalo Stomper heritage, what fraction Norwegian Buffalo Stomper am I?	
4	Evaluate: log(log10000000000)	
5	Let $a = \log 5$ and $b = \log 3$. Write log16.2 in terms of a and b.	

	5 points each	
6	Evaluate: $\log_{16} 36 - \log_{16} 9$	
7	Solve for x: $3^{\log 5} = x^{\log 3}$.	
8	The First National Bank of Pythagoras was offering a special compounding rate for all new customers. Suppose Elliot has $1000 and invests in an account that pays 50% interest compounded annually. How much money does he have at the end of 2 years?	
9	r and s are rational numbers and $6^r 9^s = 648$. What is s?	
10	The number of aardvarks varies jointly with the $\log_2$ of the number of bears and the $\log_3$ of the number of cats. When there are 16 bears and 3 cats, there are 16 aardvarks. How many bears are there if there are 27 cats and 12 aardvarks?	

	6 points each	
11	Solve: $\log_2(64!) - \log_2(63!)$	
12	If $(i^{46})(i^{101}) + 3i^{39} - i^{2035} = a + bi$, where $a, b \in R$ and $i = \sqrt{-1}$, what is $a + b$?	
13	Evaluate: $\ln(\ln e^{4e^4})$	
14	Evaluate b if $(3\log_a b)(\log_7 a) = 9$.	
15	Solve for x: $\frac{1}{\log_4 x} + \frac{1}{\log_8 x} + \frac{1}{\log_{16} x} = 3$	

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Alpha Logs and Exponents

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2	Simplify and express as a fraction: $(27^{-2/3}) + (\log_2 8)$.	
3	My great-great-grandfather is $\frac{3}{4}$ Norwegian Buffalo Stomper. Assuming that no one else in my family has any Norwegian Buffalo Stomper heritage, what fraction Norwegian Buffalo Stomper am I?	
4	Solve for x: $\log_8(\log_2(\log_3(\log_2 x))) = 0$	
5	Let $a = \log 5$ and $b = \log 3$. Write $\log 16.2$ in terms of a and b.	

	5 points each	
6	Evaluate: $\log_{16} 36 - \log_{16} 9$	
7	Solve for x: $x^{x^{x^{x^{x\ldots}}}} = 3$	
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13	Evaluate: $\ln(\ln e^{4e^4})$	
14	Evaluate b if $(3\log_a b)(\log_7 a) = 9$.	
15	Find the sum of the distinct values of x, if: $\frac{e^{2x^3}(e^{-3x^2})}{e^{17x}(e^{12})} = 1$	

2009 – 2010Log1 Contest Round 1
Mu Logs and Exponents

Name: _____

	4 points each	
1	A population of lions quadruples every month. If the initial population of the lions was 50, what is the population after 2 months?	
2	Simplify and express as a fraction: $(27^{-2/3}) + (\log_2 8)$.	
3	My great-great-grandfather is $\frac{3}{4}$ Norwegian Buffalo Stomper. Assuming that no one else in my family has any Norwegian Buffalo Stomper heritage, what fraction Norwegian Buffalo Stomper am I?	
4	Solve for x: $\log_8(\log_2(\log_3(\log_2 x))) = 0$	
5	Let $a = \log 5$ and $b = \log 3$. Write $\log 16.2$ in terms of a and b.	

	5 points each	
6	Evaluate: $\log_{16} 36 - \log_{16} 9$	
7	Solve for x: $x^{x^{x^{x^{x\ldots}}}} = 3$	
8	The First National Bank of Pythagoras was offering a special compounding rate for all new customers. Suppose Elliot has $1000 and invests in an account that pays 50% interest compounded annually. How much money does he have at the end of 2 years?	
9	Evaluate: $\frac{d}{dx}(x \log_2 x)$	
10	The number of aardvarks varies jointly with the $\log_2$ of the number of bears and the $\log_3$ of the number of cats. When there are 16 bears and 3 cats, there are 16 aardvarks. How many bears are there if there are 27 cats and 12 aardvarks?	

	6 points each	
11	Solve: $\log_2(64!) - \log_2(63!)$	
12	If $(i^{46})(i^{101}) + 3i^{39} - i^{2035} = a + bi$, where $a, b \in R$ and $i = \sqrt{-1}$, what is $a + b$?	
13	Evaluate: $\ln(\ln e^{4e^4})$	
14	Find the sum of the distinct values of x, if: $\frac{e^{2x^3}(e^{-3x^2})}{e^{17x}(e^{12})} = 1$	
15	If $3^{x_1} = 6$, $6^{x_2} = 9$, $9^{x_3} = 12 \ldots 726^{x_{242}} = 729$, then what is $x_1 x_2 x_3 x_4 \ldots x_{242}$?	

2009 – 2010Log1 Contest Round 1
Logs and Exponents Answers

Theta Answers	
1	800
2	$\frac{28}{9}$
3	$\frac{3}{64}$
4	1
5	$4b - a$
6	$\frac{1}{2}$
7	x= 5
8	$2250
9	s = 1/2
10	2
11	6
12	-3
13	$4 + \ln 4$
14	343
15	8

Alpha Answers	
1	800
2	$\frac{28}{9}$
3	$\frac{3}{64}$
4	512
5	$4b - a$
6	$\frac{1}{2}$
7	$3^{\frac{1}{3}}$ or $\sqrt[3]{3}$
8	$2250
9	s = 1/2
10	2
11	6
12	-3
13	$4 + \ln 4$
14	343
15	$\frac{3}{2}$

Mu Answers	
1	800
2	$\frac{28}{9}$
3	$\frac{3}{64}$
4	512
5	$4b - a$
6	$\frac{1}{2}$
7	$3^{\frac{1}{3}}$ or $\sqrt[3]{3}$
8	$2250
9	$\log_2 x + \frac{1}{\ln 2}$
10	2
11	6
12	-3
13	$4 + \ln 4$
14	$\frac{3}{2}$
15	6

2009 – 2010Log1 Contest Round 1
Logs and Exponents Solutions

Mu	Al	Th	Solution
1	1	1	Population is equal to $50(4^t)$ So to find the population after 2 months just substitute 2 for t to get 800.
2	2	2	$27^{-2/3}$ can be simplified to: $(3^3)^{-2/3} = (3)^{-2} = \frac{1}{9}$ $\log_2 8 = 3$ To get the answer simply add the two results: $\frac{1}{9} + 3 = \frac{28}{9}$
3	3	3	Great Great Grandpa $= \frac{3}{4}$ Great Grandpa $= \frac{3}{8}$ Grandpa $= \frac{3}{16}$ Parent $= \frac{3}{32}$ Me $= \frac{3}{64}$
4	4		$\log_8(\log_2(\log_3(\log_2 x))) = 0$ $\log_2(\log_3(\log_2 x)) = 8^0 = 1$ $\log_3(\log_2 x) = 2^1 = 2$ $\log_2 x = 3^2 = 9$ $x = 2^9 = 512$
		4	The 1 is followed by ten 0's, thus it can be rewritten as 10^{10}. $\log(\log 10^{10}) = \log 10 = 1$
5	5	5	$16.2 = \frac{81}{5} = \frac{3^4}{5}$ $\log(\frac{3^4}{5}) = \log 3^4 - \log 5$ $= 4\log 3 - \log 5$ $= 4b - a$
6	6	6	Since the quotient of two logs is the difference of the logs: $\log_{16} 36 - \log_{16} 9 = \log_{16}(\frac{36}{9})$ $= \log_{16} 4$ $= \frac{1}{2}$
7	7		$x^{x^{x^{x^{\cdots}}}} = 3$ $x^3 = 3$ $x = 3^{\frac{1}{3}}, \sqrt[3]{3}$

		7	$3^{\log 5} = x^{\log 3}$ $\log(3^{\log 5}) = \log(x^{\log 3})$ $(\log 5)(\log 3) = (\log 3)(\log x)$ $x = 5$
8	8	8	To determine the solution you would use the following equation: $A = P(1 + \frac{r}{n})^{nt}$ Substitute the appropriate values to get: $A = P(1 + \frac{.5}{1})^2 = 1000(1.5)^2 = 1000(2.25) = 2250$
	9	9	$6^r 9^s = 2^r 3^r 3^{2s} = 2^r 3^{r+2s} = 648 = 2^3 3^4$ r=3, s=1/2.
9			$\frac{d}{dx}(x \log_2 x) = \log_2 x + x \frac{d}{dx}\left(\frac{\ln x}{\ln 2}\right) = \log_2 x + \frac{1}{\ln 2}$
10	10	10	a = $k \log_2$ (b) $\log_3$(c). 16 = $k \log_2$ (16) $\log_3$ (3) = 4k. Therefore k = 4. Then, 12 = 4 $\log_2$ (b) $\log_3$ (27) = 4(3) $\log_2$ (b). Therefore $\log_2$(b) = 1 and b=2
11	11	11	$\frac{64!}{63!} = 64 = 2^6$ Thus answer is 6
12	12	12	$i^{46} = -1$ $i^{101} = i$ $3i^{39} = 3(-i)$ $i^{2035} = -i$ $\therefore (-1)(i) + (-3i) - (-i) = -3i + 0$ $b = -3, a = 0$ $a + b = (-3) + (0) = -3$
13	13	13	Starting in the parentheses: $\ln e^{4e^4} = 4e^4$ Because the log of a product is the sum of the logs: $\ln(4e^4) = \ln 4 + \ln e^4$ $= 4 + \ln 4$
	14	14	$(3\log_a b)(\log_7 a) = 9$ By change of base: $\frac{\log b^3}{\log a} \cdot \frac{\log a}{\log 7} = 9$ $\frac{\log b^3}{\log 7} = 9$ $b^3 = 7^9$ $b = 7^3 = 343$

		15	$\frac{1}{\log_4 x}+\frac{1}{\log_8 x}+\frac{1}{\log_{16} x}=3$ By changing the bases: $\frac{\log 4+\log 8+\log 16}{\log x}=3$ Because the sum of the logs is the log of the products: $\log(4(8)(16))=3\log x$ $\log 512=\log x^3$ $x^3=512$ $x=8$
14	15		$\frac{e^{2x^3}(e^{-3x^2})}{e^{17x}(e^{12})}=1$ $=\frac{e^{2x^3-3x^2}}{e^{17x+12}}=1$ $2x^3-3x^2=17x+12$ $2x^3-3x^2-17x-12=0$ Sum of roots $=-\frac{b}{a}$ $=\frac{3}{2}$
15			$3^{x_1}=6$ $\quad 6^{x_2}=9$ $\log 3^{x_1}=\log 6$ $\quad \log 6^{x_2}=\log 9$ $x_1\log 3=\log 6$ $\quad x_2\log 6=\log 9$ $x_1=\frac{\log 6}{\log 3}$ $\quad x_2=\frac{\log 9}{\log 6}$ $9^{x_3}=12$ $\quad 12^{x_4}=15$ $\log 9^{x_3}=\log 12$ $\quad \log 12^{x_4}=\log 15$ $x_3\log 9=\log 12$ $\quad x_4\log 12=\log 15$ $x_3=\frac{\log 12}{\log 9}$ $\quad x_4=\frac{\log 15}{\log 12}$ From this we can see that by multiplying out all of the terms, everything will cancel out except for log3 (the denominator) and log729 (the numerator). Solving the resulting fraction: $\frac{\log 729}{\log 3}=\log_3 729$ $=6$