2010 – 2011 Log1 Contest Round 3
Theta Individual

Name: _____

	4 points each	
1	Evaluate: $4 - 6\left(3 - \frac{4}{3}\right) + 2 \cdot 4^2$	
2	What is the sum of the multiples of 3 between 1 and 100?	
3	What is the probability of getting exactly 2 heads and 2 tails on 4 flips of a fair coin?	
4	What is the area of a regular hexagon with side length equal to 2?	
5	What is the y-intercept of the parabola: $y = 3x^2 - 8x + 5$?	

	5 points each	
6	Three boys, Abe, Bob and Charlie and three girls Alice, Barb and Carol go to the movies. In how many ways can they be ordered in a row if there must be a boy on each end?	
7	What is the units digit of the number $(6+1)(6^2+1)(6^3+1)\cdots(6^{2011}+1)$?	
8	In a regular hexagon ABCDEF, diagonals AE and DF are drawn and intersect at point G. What is the measure of angle $\widehat{DGA}$ in degrees?	
9	A test has 30 multiple-choice questions and scoring is 5 points for a correct answer, 1 point for a blank and 0 points for a wrong answer. Some scores are possible, for example 0(all wrong) and 150 (all right) while others such as 149 are impossible. How many scores from 0 to 150 are impossible?	
10	Suppose P(x) is a polynomial with integer coefficients which has -3 and 5 as two of its roots. Which of the following are possible values for P(8): 3, 5, 20, 54, 60 and/or 66?	

	6 points each	
11	A circle is drawn inside quadrilateral ABCD such that it is tangent to all four sides. Side AB = 8 and CD = 11, what is the perimeter of the quadrilateral?	
12	Bob cashed a check at his bank. He didn't notice that the teller switched the number of dollars and cents when he was paid. Later, a friend gave him 35 cents for a newspaper. Bob was surprised to find that he now had, in total, only half the value of his check. What was the value of his check?	
13	If the expression $(a+b+c)^3$ is expanded and simplified, how many terms will it have?	
14	Consider the sequence, $1!, 1!+2!, 1!+2!+3!, \ldots, 1!+2!+3!+\cdots 20!$, where n! is n factorial. How many perfect squares are in this sequence?	
15	How many solutions are there to the equation: $\log_x(19x - 30) = 3$?	

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Alpha Individual

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4 points each		
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3	What is the probability of getting exactly 2 heads and 2 tails on 4 flips of a fair coin?	
4	A triangle has sides of lengths: 8, 15 and 17. What is the cosine of the angle opposite the side with length 17?	
5	What is the y-intercept of the parabola: $y = 3x^2 - 8x + 5$?	

5 points each		
6	Three boys, Abe, Bob and Charlie and three girls Alice, Barb and Carol go to the movies. In how many ways can they be ordered in a row if there must be a boy on each end?	
7	What is the units digit of the number $(6 + 1)(6^2 + 1)(6^3 + 1) \cdots (6^{2011} + 1)$?	
8	In a regular hexagon ABCDEF, diagonals AE and DF are drawn and intersect at point G. What is the measure of angle $\widehat{DGA}$ in degrees?	
9	What is $(\sin^2 75°)(\cos^2 75°)$?	
10	Suppose P(x) is a polynomial with integer coefficients which has -3 and 5 as two of its roots. Which of the following are possible values for P(8): 3, 5, 20, 54, 60 and/or 66?	

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13	If the expression $(a + b + c)^3$ is expanded and simplified, how many terms will it have?	
14	Two points are given in polar coordinates as $\left(2, \frac{\pi}{12}\right)$ and $\left(3, \frac{5\pi}{12}\right)$; what is the distance between them?	
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Mu Individual

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3	What is the probability of getting exactly 2 heads and 2 tails on 4 flips of a fair coin?	
4	A triangle has sides of lengths: 8, 15 and 17. What is the cosine of the angle opposite the side with length 17?	
5	The speed of a car at time t is given by: $s(t) = -4t^2 + 8t + 30$. What is the maximum speed attained?	

5 points each		
6	Three boys, Abe, Bob and Charlie and three girls Alice, Barb and Carol go to the movies. In how many ways can they be ordered in a row if there must be a boy on each end?	
7	What is the units digit of the number $(6 + 1)(6^2 + 1)(6^3 + 1) \cdots (6^{2011} + 1)$?	
8	In a regular hexagon ABCDEF, diagonals AE and DF are drawn and intersect at point G. What is the measure of angle $\widehat{DGA}$ in degrees?	
9	What is $(\sin^2 75°)(\cos^2 75°)$?	
10	What is the $\lim_{x \to 0} x \cot x$?	

6 points each		
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13	If the expression $(a + b + c)^3$ is expanded and simplified, how many terms will it have?	
14	Two points are given in polar coordinates as $\left(2, \frac{\pi}{12}\right)$ and $\left(3, \frac{5\pi}{12}\right)$; what is the distance between them?	
15	Use a linear differential approximation to estimate $\sqrt{10}$ near the value of $\sqrt{9} = 3$. Express your answer as an improper fraction.	

2010 – 2011 Log1 Contest Round 3
Theta Individual

Name: _____

4 points each		
1	Evaluate: $4 - 6\left(3 - \frac{4}{3}\right) + 2 \cdot 4^2$	26
2	What is the sum of the multiples of 3 between 1 and 100?	1683
3	What is the probability of getting exactly 2 heads and 2 tails on 4 flips of a fair coin?	$\frac{3}{8}$
4	What is the area of a regular hexagon with side length equal to 2?	$6\sqrt{3}$
5	What is the y-intercept of the parabola: $y = 3x^2 - 8x + 5$?	5 or (0,5)

5 points each		
6	Three boys, Abe, Bob and Charlie and three girls Alice, Barb and Carol go to the movies. In how many ways can they be ordered in a row if there must be a boy on each end?	144
7	What is the units digit of the number $(6+1)(6^2+1)(6^3+1)\cdots(6^{2011}+1)$?	3
8	In a regular hexagon ABCDEF, diagonals AE and DF are drawn and intersect at point G. What is the measure of angle $\widehat{DGA}$ in degrees?	120°
9	A test has 30 multiple-choice questions and scoring is 5 points for a correct answer, 1 point for a blank and 0 points for a wrong answer. Some scores are possible, for example 0(all wrong) and 150 (all right) while others such as 149 are impossible. How many scores from 0 to 150 are impossible?	6
10	Suppose P(x) is a polynomial with integer coefficients which has -3 and 5 as two of its roots. Which of the following are possible values for P(8): 3, 5, 20, 54, 60 and/or 66?	66

6 points each		
11	A circle is drawn inside quadrilateral ABCD such that it is tangent to all four sides. Side AB = 8 and CD = 11, what is the perimeter of the quadrilateral?	38
12	Bob cashed a check at his bank. He didn't notice that the teller switched the number of dollars and cents when he was paid. Later, a friend gave him 35 cents for a newspaper. Bob was surprised to find that he now had, in total, only half the value of his check. What was the value of his check?	$86.42
13	If the expression $(a+b+c)^3$ is expanded and simplified, how many terms will it have?	10
14	Consider the sequence, 1!, 1! + 2!, 1! + 2! + 3!, … , 1! + 2! + 3! + ⋯ 20!, where n! is n factorial. How many perfect squares are in this sequence?	2
15	How many solutions are there to the equation: $\log_x(19x - 30) = 3$?	2

2010 – 2011 Log1 Contest Round 3
Alpha Individual

Name: _____

4 points each		
1	Evaluate: $4 - 6\left(3 - \frac{4}{3}\right) + 2 \cdot 4^2$	26
2	What is the sum of the multiples of 3 between 1 and 100?	1683
3	What is the probability of getting exactly 2 heads and 2 tails on 4 flips of a fair coin?	$\frac{3}{8}$
4	A triangle has sides of lengths: 8, 15 and 17. What is the cosine of the angle opposite the side with length 17?	0
5	What is the y-intercept of the parabola: $y = 3x^2 - 8x + 5$?	5 or (0,5)

5 points each		
6	Three boys, Abe, Bob and Charlie and three girls Alice, Barb and Carol go to the movies. In how many ways can they be ordered in a row if there must be a boy on each end?	144
7	What is the units digit of the number $(6 + 1)(6^2 + 1)(6^3 + 1) \cdots (6^{2011} + 1)$?	3
8	In a regular hexagon ABCDEF, diagonals AE and DF are drawn and intersect at point G. What is the measure of angle $\widehat{DGA}$ in degrees?	120°
9	What is $(\sin^2 75°)(\cos^2 75°)$?	1/16
10	Suppose P(x) is a polynomial with integer coefficients which has -3 and 5 as two of its roots. Which of the following are possible values for P(8): 3, 5, 20, 54, 60 and/or 66?	66

6 points each		
11	A circle is drawn inside quadrilateral ABCD such that it is tangent to all four sides. Side AB = 8 and CD = 11, what is the perimeter of the quadrilateral?	38
12	Bob cashed a check at his bank. He didn't notice that the teller switched the number of dollars and cents when he was paid. Later, a friend gave him 35 cents for a newspaper. Bob was surprised to find that he now had, in total, only half the value of his check. What was the value of his check?	$86.42
13	If the expression $(a + b + c)^3$ is expanded and simplified, how many terms will it have?	10
14	Two points are given in polar coordinates as $\left(2, \frac{\pi}{12}\right)$ and $\left(3, \frac{5\pi}{12}\right)$; what is the distance between them?	$\sqrt{7}$
15	How many solutions are there to the equation: $\log_x(19x - 30) = 3$?	2

2010 – 2011 Log1 Contest Round 3
Mu Individual

Name: _____

4 points each		
1	Evaluate: $4 - 6\left(3 - \frac{4}{3}\right) + 2 \cdot 4^2$	26
2	What is the sum of the multiples of 3 between 1 and 100?	1683
3	What is the probability of getting exactly 2 heads and 2 tails on 4 flips of a fair coin?	$\frac{3}{8}$
4	A triangle has sides of lengths: 8, 15 and 17. What is the cosine of the angle opposite the side with length 17?	0
5	The speed of a car at time t is given by: $s(t) = -4t^2 + 8t + 30$. What is the maximum speed attained?	34

5 points each		
6	Three boys, Abe, Bob and Charlie and three girls Alice, Barb and Carol go to the movies. In how many ways can they be ordered in a row if there must be a boy on each end?	144
7	What is the units digit of the number $(6 + 1)(6^2 + 1)(6^3 + 1) \cdots (6^{2011} + 1)$?	3
8	In a regular hexagon ABCDEF, diagonals AE and DF are drawn and intersect at point G. What is the measure of angle $\widehat{DGA}$ in degrees?	120°
9	What is $(\sin^2 75°)(\cos^2 75°)$?	1/16
10	What is the $\lim_{x \to 0} x \cot x$?	1

6 points each		
11	A circle is drawn inside quadrilateral ABCD such that it is tangent to all four sides. Side AB = 8 and CD = 11, what is the perimeter of the quadrilateral?	38
12	Bob cashed a check at his bank. He didn't notice that the teller switched the number of dollars and cents when he was paid. Later, a friend gave him 35 cents for a newspaper. Bob was surprised to find that he now had, in total, only half the value of his check. What was the value of his check?	$86.42
13	If the expression $(a + b + c)^3$ is expanded and simplified, how many terms will it have?	10
14	Two points are given in polar coordinates as $\left(2, \frac{\pi}{12}\right)$ and $\left(3, \frac{5\pi}{12}\right)$; what is the distance between them?	$\sqrt{7}$
15	Use a linear differential approximation to estimate $\sqrt{10}$ near the value of $\sqrt{9} = 3$. Express your answer as an improper fraction.	19/6

2010 – 2011 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
1	1	1	$4-6\left(3-\frac{4}{3}\right)+2\cdot 4^2=4-(18-8)+2\cdot 16=4-10+32=26$
2	2	2	$3+6+\cdots+99=3(1+2+\cdots 33)=\frac{3(33)(34)}{2}=3(33)(17)=1683$
3	3	3	$4C2\left(\frac{1}{2}\right)^4=\frac{6}{16}=\frac{3}{8}$
4	4		This is a right triangle and 17 is the hypotenuse. Therefore, the angle opposite it is 90 degrees with a cosine of 0.
		4	The hexagon can be divided into 6 equilateral triangles with each side being 2. The area of each triangle is $A_T=\frac{s^2\sqrt{3}}{4}=\sqrt{3}$ and the total area is $6\sqrt{3}$
5			$s'(t)=-8t+8=0$, so t=1. $s(1)=-4+8+30=34$
	5	5	The y-intercept is when x=0. Substituting in the equation: y=5.
6	6	6	There are three boys to choose from for the left end and two for the right end. The four others can be ordered in any way in the middle. $3(2)(4!)=6(24)=144$
7	7	7	Since 6 to any positive integer power ends in 6, the units digit of all the factors is 7. The powers of 7 end in respectively, 7, 9, 3, 1, 7, 9… So the product of 2011 such terms has a units digit of 3.
8	8	8	The measure, in degrees, of each interior angle will be 180-360/6=120.Since triangle EFA is isosceles, the measure of angle AEF is 30 degrees (180-120)/2. The same is true angle EFD. This makes angle EGF and DGA equal to 120 degrees.
9	9		$(\sin^2 75°)(\cos^2 75°)=\left(\frac{2(\sin 75°)(\cos 75°)}{2}\right)^2=\left(\frac{\sin 150°}{2}\right)^2=\left(\frac{\sin 30°}{2}\right)^2=\left(\frac{1}{4}\right)^2=\frac{1}{16}$
		9	All multiples of 5 are clearly possible and if there are four or more questions not correct then the numbers in between are possible. That leaves scores of 139, 143, 144, 147, 148 and 149 as impossible or 6 in all.
10			Writing $\cot x=\cos x/\sin x$ and using L'Hopital's rule: $\lim_{x\to 0} x\cot x=\lim_{x\to 0}\frac{x\cos x}{\sin x}=\lim_{x\to 0}\frac{x(-\sin x)+\cos x}{\cos x}=1$
	10	10	The polynomial can be written as $P(x)=(x+3)(x-5)Q(x)$. Therefore $P(8)=11(3)Q(8)=33Q(8)$, and since Q(8) is an integer, so P(8) must be divisible by 33 and only 66 is possible.
11	11	11	Label the point of tangency along AB, a', along BC, b', along CD, c' and along DA, d'. Since the tangent lines drawn from an external point to a circle are equal, we have: a'B = Bb'; b'C=Cc'; c'D=Dd'; and d'A=Aa'. The perimeter is: $Aa'+a'B+Bb'+b'C+Cc'+c'D+Dd'+d'A=2(Aa'+a'B+Cc'+c'D)=2(AB+CD)=38$

12	12	12	Let d=# of dollars and c=# of cents, then $$100c + d + 35 = \frac{1}{2}(100d + c)$$ $$d = \frac{199c + 70}{98} = 2c + \frac{3c + 70}{98}$$ Since d and c are both integers between 0 and 99. $3c + 70 = 2(98), c = 42$ and $d = 86$.
13	13	13	In each of three factors, an a, b, or c is multiplied. Therefore, we each term will have $a^l b^m c^n$ where $l + m + n = 3$ and each is greater than or equal to 0. One can use combinatorics to get the answer or easily count: (3,0,0), (2,1,0), (2,0,1), (1,2,0), (1,1,1), (1,0,2), (0,3,0), (0,2,1), (0,1,2), (0,0,3).
14	14		Consider the lines between the points and the origin. The angle between these lines is $\frac{\pi}{3}$. Using the Law of Cosines, the distance between the points is: $c^2 = 2^2 + 3^2 - 2(2)(3)\cos\frac{\pi}{3} = 13 - 12\frac{1}{2} = 13 - 6$. So c = $\sqrt{7}$.
		14	The first few terms are: 1, 3, 9, 33, and 153. Since all the factorials above 5! end in 0, all the other terms end in 3. Perfect squares cannot end in 3, so there are only two terms, 1 and 9.
15			The function $f(x) = \sqrt{x}$ with derivative $f'(x) = \frac{1}{2\sqrt{x}}$ is used with $f(10) \approx f(9) + f'(9)(10 - 9) = 3 + \frac{1}{6} = \frac{19}{6}$.
	15	15	Switching to exponential form: $x^3 = 19x - 30$. Factoring this has solutions *x*=-5, 2 and 3. Ruling out x=-5 leaves two solutions.