Theta

Area and Volume

Test #113

Directions:

1. Fill out the top section of the Round 1 Google Form answer sheet and select **Theta- Area and Volume** as the test. Do not abbreviate your school name. Enter an email address that will accept outside emails (some school email addresses do not).

2. Scoring for this test is 5 times the number correct plus the number omitted.

3. TURN OFF ALL CELL PHONES.

4. No calculators may be used on this test.

5. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

6. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

7. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

8. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

9. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

10. Unless a question asks for an approximation or a rounded answer, give the exact answer.

The choice E, or NOTA, should be selected when none of the answer answers given for the question are correct. If you get stuck, try to follow your nose!

1. A circle with area 4π is stretched horizontally to form an ellipse with area 12π. What is the length of the major axis of the ellipse.

   A. $2\sqrt{3}$ B. 6 C. $4\sqrt{3}$ D. 12 E. NOTA

2. A dog is bound to the southwest corner of his house by a 2-foot-long leash. The base of his house is a square with an area of 9 square feet, and there is a one-foot-wide opening at the south side of his house for a door, perfectly centered on the side. What is the total area that the dog can possibly roam?

   A. $\frac{5\pi}{2}$ B. $\frac{7\pi}{2}$ C. $\frac{9\pi}{2}$ D. $\frac{11\pi}{2}$ E. NOTA

3. Billy the bumblebee is trapped in a cube of side length 6 and is tied down to a point at the center of the cube with a leash that has length 9. Interestingly, he can just manage to squeeze through each of the eight corners of the cube. What is the volume of the possible space that Billy can roam?

   A. 216π B. 400π C. 432π D. 468π E. NOTA

4. Find the total aggregate length of all the edges put together, in inches, of an icosahedron with edge length 1 foot.

   A. 20 B. 240 C. 360 D. 1440 E. NOTA

5. Find the area of a triangle with side lengths $4\sqrt{3}$, $\sqrt{61}$, and $\sqrt{13}$

   A. 48 B. $\sqrt{624}$ C. $\sqrt{732}$ D. $\sqrt{347}+\sqrt{156}$ E. NOTA

6. Joanne the baker has baked a delicious pie but ate a 45° slice before she could serve it to her customers. If Joanne bakes perfectly cylindrical pies with height 2 and diameter 16, then what is the surface area of the remaining pie?

   A. $32+140\pi$ B. $16+112\pi$ C. $16+168\pi$ D. 140π E. NOTA

7. Unit circles ω_1, ω_2, and ω_3 are such that each circle's center lies on the circumference of the other two circles. Solve for the perimeter of the union of the areas of the three circles.

A. 1.5π B. 2π C. 3π D. 4π E. NOTA

8. Refer to the configuration in the previous problem. Let A_1, A_2, A_3 be the regions bounded by $\omega_1, \omega_2, \omega_3$ respectively. Find the perimeter of $(A_1 \cap A_2) \cup (A_2 \cap A_3) \cup (A_3 \cap A_1)$.

A. 1.5π B. 2π C. 3π D. 4π E. NOTA

9. Determine the volume of an octahedron with each face having area $9\sqrt{3}/2$.

A. $18\sqrt{6}$ B. $36\sqrt{6}$ C. 18 D. 36 E. NOTA

10. An acute triangle has two sides of length 7 and 10 and an area of 28. Find the length of its third side.

A. $\sqrt{93}$ B. $\sqrt{37}$ C. $\sqrt{65}$ D. $\sqrt{107}$ E. NOTA

11. Equilateral triangle ABC has points D, E, and F on $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ respectively such that D is $1/3$ of the way from A to B, E is $1/3$ of the way from B to C, and F is $1/3$ of the way from C to A. What is the ratio of the area of ΔDEF to the area of ΔABC?

A. $\frac{1}{9}$ B. $\frac{2}{5}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. NOTA

12. Kev the baker makes some oddly shaped cookies, in the shape of a frustum of a cone. However, he is consistent, so they all have the exact same size and shape. If each cookie has a height of 4 and its bottom and top bases have circumferences of 18π and 12π respectively, what is the surface area of one of Kev's cookies?

A. 165π B. 167π C. 192π D. 194π E. NOTA

13. A quadrilateral has vertices located at $(2,3), (7,3), (0,0), (7,-5)$. Find its area.

A. 8 B. 18 C. 36 D. 35.5 E. NOTA

14. Doug the miner has struck gold but only has 30 feet of fencing to enclose a site. Help him by providing the maximal area he can enclose with his fencing.

A. $\frac{225}{\pi^2}$ B. $\frac{225}{4}$ C. $\frac{225}{9}$ D. $\frac{225}{\pi}$ E. NOTA

15. Flat Stanley's house is made out of a square and has a right isosceles triangle as a roof placed hypotenuse-down on top of the square. Given that his roof is just as wide as rest of his house, and his house has a total height $2+\sqrt{2}$, compute the outer perimeter of Stanley's house.

A. 10 B. 12 C. 14 D. 16 E. NOTA

16. Give the perimeter of a regular pentagon inscribed in a circle of radius 1.

A. $5\sin 36°$ B. $10\sin 36°$ C. $5\sin 72°$ D. $10\sin 18°$ E. NOTA

17. Compute the ratio of the area of a regular hexagon inscribed in a unit circle to the area of a regular hexagon circumscribed about the same circle.

A. 2/3 B. 3/4 C. 3/2 D. 4/3 E. NOTA

18. If the volume of a cone with equal height and radius and the area of a circle with the same radius are equal, what is the circle's radius? (Ignore units)

A. 1 B. 4 C. 3 D. 9 E. NOTA

19. Determine the circumference of the circumcircle of a triangle with side lengths 5, 6, 7.

A. $\frac{35\pi\sqrt{6}}{12}$ B. $\frac{35\sqrt{6}}{24}$ C. 35π D. $\frac{35\pi\sqrt{3}}{12}$ E. NOTA

20. Andrew the ant has to get from one corner of a rectangular prism to the corner that is farthest from him, but he can only crawl along the surface of the prism. If the prism has dimensions 3 by 4 by 5, what is the shortest distance Andrew can traverse to get to the corner?

A. $4\sqrt{5}$ B. $\sqrt{74}$ C. 9 D. 12 E. NOTA

21. Given ΔABC, there is a circle with center O that is tangent at three points: C' on the extension of AC, B' on the extension of AB and a point X on BC. If $CC' = 3, BB' = 5, AB = 6$, and $m\angle XOB = 30°$. Find the area of ΔABC.

A. $6\sqrt{3}$ B. 12 C. $12\sqrt{3}$ D. 18 E. NOTA

22. A white inverted cone cup is filled to half of its height with some white milk. If the volume of the milk is 100π and the cone cup has height 24, then what is the visible surface area of this milk-and-cup drink that is white?

A. 260π B. 285π C. 480π D. 505π E. NOTA

23. ΔTUV has two cevians that emanate from T and U and meet at point Q. The cevian emanating from T meets UV at T' and the cevian from U meets TV at U'. The area of ΔTUV is 24 and $TQ = 10, QT' = 5, UU' = 9, QU' = 3$. Find the area of quadrilateral $QU'VT'$.

A. 8 B. 10 C. 12 D. 16 E. NOTA

24. Determine the area of the annulus bounded by two concentric circles of circumferences 10π and 30π.

A. 50π B. 100π C. 150π D. 200π E. NOTA

25. Bob the builder is building a tower with a unique design: it is formed by stacking cubical boxes on top of each other. The bottom box has a side length of 1, and each box has one-half the side length of the one below it. This pattern continues infinitely. Despite the fact that Bob will never be able to complete the tower, he thinks he can still determine how much volume it would have so he can bill his customer. Help Bob by giving the volume of the completed tower.

A. 3/2 B. 2 C. 7/8 D. ∞ E. NOTA

26. Refer to the previous question, Pam the painter thinks she can paint the exterior of the tower Bob is building. If the tower were to be completed, what is the area that requires painting?

A. 11/2 B. 16/3 C. 19/3 D. ∞ E. NOTA

27. Find the area of ΔDAN if the three sides d, a, and n satisfy the following 3 equations:

$$a + n + d = 16$$
$$ad + nd + an = 82$$
$$dna = 135$$

A. $6\sqrt{2}$ B. $6\sqrt{3}$ C. $6\sqrt{5}$ D. $6\sqrt{7}$ E. NOTA

28. Carol and Connor are running around the 370π-meter long perimeter of a circular track. They start at the same point but run in opposite directions. If Connor runs at a rate of 4 m/s and Carol runs at a rate of 2 m/s, then how far away from their starting point will they be when they first meet again? (Note: straight-line distance, not along the track)

A. $\frac{185\pi}{3}$ B. $185\sqrt{3}$ C. $\frac{185\pi\sqrt{3}}{3}$ D. $\frac{370\pi\sqrt{3}}{3}$ E. NOTA

29. Find the volume of a regular hexagonal pyramid with base side length 6 and height 5.

A. $36\sqrt{3}$ B. $45\sqrt{3}$ C. $90\sqrt{3}$ D. $105\sqrt{3}$ E. NOTA

30. Find the volume of a sphere with radius 3.

A. 36 B. 27 C. 18 D. 72 E. NOTA