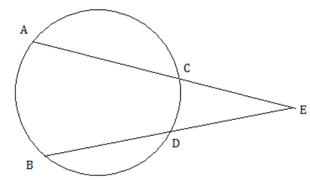
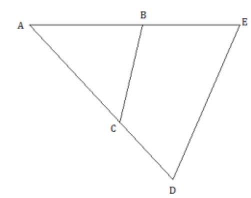
2012 – 2013 Log1 Contest Round 2
Theta Geometry

Name: ________________

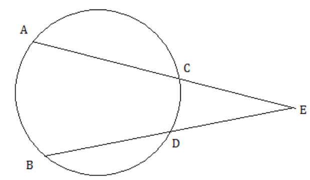
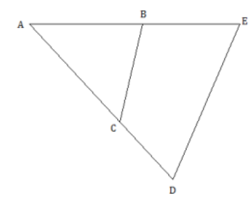
4 points each		
1	Find the area enclosed by a triangle that has one side of length 4 and an altitude to that side that also has length 4.	
2	Refer to the picture to the right. If $mAB = 111^\circ$ and arcs CD and $\angle CED$ have equivalent degree measure, find the measure of $\angle CED$ in degrees. 	
3	Refer to the picture to the right. In $\triangle ADE$, B and C are points on sides $\overline{AE}$ and $\overline{AD}$, respectively, such that $\angle ABC \cong \angle ADE$. If $\overline{AC}$ and $\overline{DE}$ each have length x, $\overline{BC}$ has length 12, $\overline{AB}$ has length 15, and $\overline{CD}$ has length 6, find the value of x. 	
4	A square is inscribed in a circle whose radius has length 16. Find the area enclosed by the circle that is outside the square.	
5	An isosceles trapezoid has bases of lengths 20 and 30 and legs of length 13. Find the area enclosed by the trapezoid.	

5 points each		
6	A triangle has sides with lengths 5, 7, and 8. Find the minimum distance from the center of the triangle's inscribed circle to the side with length 8.	
7	Find the number of regular n-gons whose total number of diagonals is divisible by 3 but not by 9.	
8	In an icosahedron, which has 20 faces, the sum of the number of vertices and edges equals 42. Find the number of edges of an icosahedron.	
9	Two circles that are externally tangent have radii of lengths 5 and 12. A line is drawn, externally tangent to both circles. Consider the points of tangency, one on each circle, of the circles with the line. Find the distance between these two points.	
10	Find the least integer greater than 100 that can be the total number of diagonals in a regular n-gon.	

6 points each		
11	A rhombus has an enclosed area of 442 and diagonals with integral lengths. Find the minimum value of the sum of the lengths of the diagonals of the rhombus.	
12	In a certain ellipse, the distance between the foci is 18, and the major and minor axes have integral lengths. If the two axes' lengths differ by more than 5, find the area enclosed by the ellipse.	
13	A 9 inch by 12 inch rectangular sheet of paper is folded so that one pair of opposite corners coincide. Find the length, in inches, of the crease made in the sheet of paper that results from folding the sheet of paper in this way.	
14	Three distinct larger circles with equal radius length are all externally tangent to each other. A smaller circle is externally tangent to each of the three larger circles. If the smaller circle has radius of length 1, find the length of the radius of one of the larger circles.	
15	Find the area enclosed by a quadrilateral whose vertices are $(-1,-2)$, $(3,3)$, $(2,5)$, and $(6,3)$, connected in that order.	

2012 – 2013 Log1 Contest Round 2
Alpha Geometry

Name: ________________

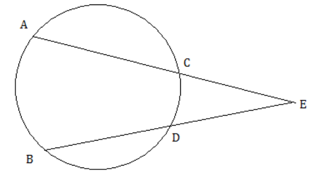
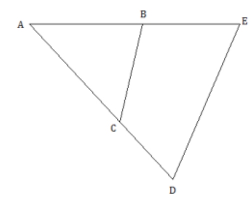
	4 points each	
1	Find the area enclosed by a triangle that has one side of length 4 and an altitude to that side that also has length 4.	
2	Refer to the picture to the right. If $mAB = 111^\circ$ and arcs CD and $\angle CED$ have equivalent degree measure, find the measure of $\angle CED$ in degrees. 	
3	Refer to the picture to the right. In $\triangle ADE$, B and C are points on sides $\overline{AE}$ and $\overline{AD}$, respectively, such that $\angle ABC \cong \angle ADE$. If $\overline{AC}$ and $\overline{DE}$ each have length x, $\overline{BC}$ has length 12, $\overline{AB}$ has length 15, and $\overline{CD}$ has length 6, find the value of x. 	
4	A square is inscribed in a circle whose radius has length 16. Find the area enclosed by the circle that is outside the square.	
5	An isosceles trapezoid has bases of lengths 20 and 30, legs whose lengths are integers, and an enclosed area that is also an integer. Find the enclosed area.	

	5 points each	
6	Find the number of regular n-gons whose total number of diagonals is divisible by 3 but not by 9.	
7	In an icosahedron, which has 20 faces, the sum of the number of vertices and edges equals 42. Find the number of edges of an icosahedron.	
8	Two circles that are externally tangent have radii of lengths 5 and 12. A line is drawn, externally tangent to both circles. Consider the points of tangency, one on each circle, of the circles with the line. Find the distance between these two points.	
9	Find the least integer greater than 100 that can be the total number of diagonals in a regular n-gon.	
10	A rhombus has an enclosed area of 442 and diagonals with integral lengths. Find the minimum value of the sum of the lengths of the diagonals of the rhombus.	

6 points each		
11	A kite is inscribed in a circle. The longer diagonal of the kite is divided by the shorter diagonal into two segments whose lengths are 18 and 32. Find the area enclosed by the circle.	
12	In a certain ellipse, the distance between the foci is 18, and the major and minor axes have integral lengths. If the two axes' lengths differ by more than 5, find the area enclosed by the ellipse.	
13	A 9 inch by 12 inch rectangular sheet of paper is folded so that one pair of opposite corners coincide. Find the length, in inches, of the crease made in the sheet of paper that results from folding the sheet of paper in this way.	
14	Three distinct larger circles with equal radius length are all externally tangent to each other. A smaller circle is externally tangent to each of the three larger circles. If the smaller circle has radius of length 1, find the length of the radius of one of the larger circles.	
15	A regular tetrahedron encloses a volume of 9 cubic units. Find the length of an edge of the tetrahedron.	

2012 – 2013 Log1 Contest Round 2
Mu Geometry

Name: ________________

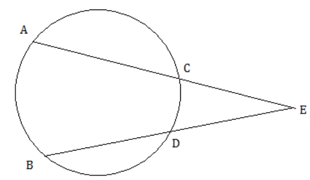
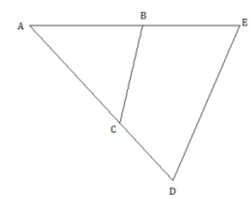
	4 points each	
1	Find the area enclosed by a triangle that has one side of length 4 and an altitude to that side that also has length 4.	
2	Refer to the picture to the right. If $m\overset{\frown}{AB} = 111^\circ$ and arcs CD and $\angle CED$ have equivalent degree measure, find the measure of $\angle CED$ in degrees. 	
3	Refer to the picture to the right. In $\triangle ADE$, B and C are points on sides $\overline{AE}$ and $\overline{AD}$, respectively, such that $\angle ABC \cong \angle ADE$. If $\overline{AC}$ and $\overline{DE}$ each have length x, $\overline{BC}$ has length 12, $\overline{AB}$ has length 15, and $\overline{CD}$ has length 6, find the value of x. 	
4	An isosceles trapezoid has bases of lengths 20 and 30, legs whose lengths are integers, and an enclosed area that is also an integer. Find the enclosed area.	
5	Find the number of regular n-gons whose total number of diagonals is divisible by 3 but not by 9.	

	5 points each	
6	Let $A = \{(x, y) \mid -2 \le x \le 0,\ 0 \le y \le 2\} \cup \{(x, y) \mid 2 \le x \le 4,\ 2 \le y \le 4\}$. Find the volume obtained by rotating A about the y-axis.	
7	In an icosahedron, which has 20 faces, the sum of the number of vertices and edges equals 42. Find the number of edges of an icosahedron.	
8	Two circles that are externally tangent have radii of lengths 5 and 12. A line is drawn, externally tangent to both circles. Consider the points of tangency, one on each circle, of the circles with the line. Find the distance between these two points.	
9	A rhombus has an enclosed area of 442 and diagonals with integral lengths. Find the minimum value of the sum of the lengths of the diagonals of the rhombus.	
10	A kite is inscribed in a circle. The longer diagonal of the kite is divided by the shorter diagonal into two segments whose lengths are 18 and 32. Find the area enclosed by the circle.	

6 points each		
11	A regular n-gon encloses an area of $21/5$. As $n \to \infty$, to what number does the perimeter of the n-gon approach?	
12	In a certain ellipse, the distance between the foci is 18, and the major and minor axes have integral lengths. If the two axes' lengths differ by more than 5, find the area enclosed by the ellipse.	
13	A 9 inch by 12 inch rectangular sheet of paper is folded so that one pair of opposite corners coincide. Find the length, in inches, of the crease made in the sheet of paper that results from folding the sheet of paper in this way.	
14	Three distinct larger circles with equal radius length are all externally tangent to each other. A smaller circle is externally tangent to each of the three larger circles. If the smaller circle has radius of length 1, find the length of the radius of one of the larger circles.	
15	Find the volume when the region $C = \left\{ (x, y) \middle| (x-5)^2 + (y+4)^2 \leq 9 \right\}$ is rotated about the x-axis.	

2012 – 2013 Log1 Contest Round 2
Theta Geometry

Name: ________________

	4 points each	
1	Find the area enclosed by a triangle that has one side of length 4 and an altitude to that side that also has length 4.	8
2	Refer to the picture to the right. If $m\overset{\frown}{AB} = 111^\circ$ and arcs CD and $\angle CED$ have equivalent degree measure, find the measure of $\angle CED$ in degrees. 	37°
3	Refer to the picture to the right. In $\triangle ADE$, B and C are points on sides $\overline{AE}$ and $\overline{AD}$, respectively, such that $\angle ABC \cong \angle ADE$. If $\overline{AC}$ and $\overline{DE}$ each have length x, $\overline{BC}$ has length 12, $\overline{AB}$ has length 15, and $\overline{CD}$ has length 6, find the value of x. 	24
4	A square is inscribed in a circle whose radius has length 16. Find the area enclosed by the circle that is outside the square.	$256\pi - 512$
5	An isosceles trapezoid had bases of lengths 20 and 30 and legs of length 13. Find the area enclosed by the trapezoid.	300

	5 points each	
6	A triangle has sides with lengths 5, 7, and 8. Find the minimum distance from the center of the triangle's inscribed circle to the side with length 8.	$\sqrt{3}$
7	Find the number of regular n-gons whose total number of diagonals is divisible by 3 but not by 9.	0
8	In an icosahedron, which has 20 faces, the sum of the number of vertices and edges equals 42. Find the number of edges of an icosahedron.	30
9	Two circles that are externally tangent have radii of lengths 5 and 12. A line is drawn, externally tangent to both circles. Consider the points of tangency, one on each circle, of the circles with the line. Find the distance between these two points.	$4\sqrt{15}$
10	Find the least integer greater than 100 that can be the total number of diagonals in a regular n-gon.	104

6 points each		
11	A rhombus has an enclosed area of 442 and diagonals with integral lengths. Find the minimum value of the sum of the lengths of the diagonals of the rhombus.	60
12	In a certain ellipse, the distance between the foci is 18, and the major and minor axes have integral lengths. If the two axes' lengths differ by more than 5, find the area enclosed by the ellipse.	180π
13	A 9 inch by 12 inch rectangular sheet of paper is folded so that one pair of opposite corners coincide. Find the length, in inches, of the crease made in the sheet of paper that results from folding the sheet of paper in this way.	11.25 or $\frac{45}{4}$
14	Three distinct larger circles with equal radius length are all externally tangent to each other. A smaller circle is externally tangent to each of the three larger circles. If the smaller circle has radius of length 1, find the length of the radius of one of the larger circles.	$3+2\sqrt{3}$
15	Find the area enclosed by a quadrilateral whose vertices are $(-1,-2)$, $(3,3)$, $(2,5)$, and $(6,3)$, connected in that order.	10.5 or $\frac{21}{2}$

2012 – 2013 Log1 Contest Round 2
Alpha Geometry

Name: ________________

4 points each		
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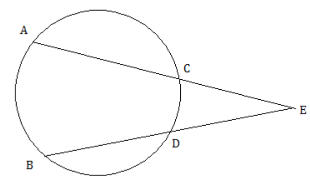
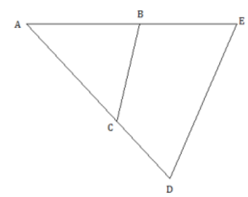
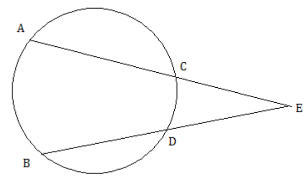
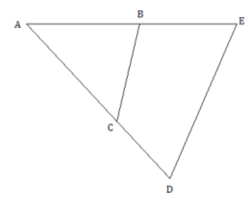
2	Refer to the picture to the right. If $m\overset{\frown}{AB} = 111^\circ$ and arcs CD and $\angle CED$ have equivalent degree measure, find the measure of $\angle CED$ in degrees. 	37°
3	Refer to the picture to the right. In $\triangle ADE$, B and C are points on sides $\overline{AE}$ and $\overline{AD}$, respectively, such that $\angle ABC \cong \angle ADE$. If $\overline{AC}$ and $\overline{DE}$ each have length x, $\overline{BC}$ has length 12, $\overline{AB}$ has length 15, and $\overline{CD}$ has length 6, find the value of x. 	24
4	A square is inscribed in a circle whose radius has length 16. Find the area enclosed by the circle that is outside the square.	$256\pi - 512$
5	An isosceles trapezoid had bases of lengths 20 and 30, legs whose lengths are integers, and an enclosed area that is also an integer. Find the enclosed area.	300

5 points each		
6	Find the number of regular n-gons whose total number of diagonals is divisible by 3 but not by 9.	0
7	In an icosahedron, which has 20 faces, the sum of the number of vertices and edges equals 42. Find the number of edges of an icosahedron.	30
8	Two circles that are externally tangent have radii of lengths 5 and 12. A line is drawn, externally tangent to both circles. Consider the points of tangency, one on each circle, of the circles with the line. Find the distance between these two points.	$4\sqrt{15}$
9	Find the least integer greater than 100 that can be the total number of diagonals in a regular n-gon.	104
10	A rhombus has an enclosed area of 442 and diagonals with integral lengths. Find the minimum value of the sum of the lengths of the diagonals of the rhombus.	60

6 points each		
11	A kite is inscribed in a circle. The longer diagonal of the kite is divided by the shorter diagonal into two segments whose lengths are 18 and 32. Find the area enclosed by the circle.	625π
12	In a certain ellipse, the distance between the foci is 18, and the major and minor axes have integral lengths. If the two axes' lengths differ by more than 5, find the area enclosed by the ellipse.	180π
13	A 9 inch by 12 inch rectangular sheet of paper is folded so that one pair of opposite corners coincide. Find the length, in inches, of the crease made in the sheet of paper that results from folding the sheet of paper in this way.	11.25 or $\frac{45}{4}$
14	Three distinct larger circles with equal radius length are all externally tangent to each other. A smaller circle is externally tangent to each of the three larger circles. If the smaller circle has radius of length 1, find the length of the radius of one of the larger circles.	$3+2\sqrt{3}$
15	A regular tetrahedron encloses a volume of 9 cubic units. Find the length of an edge of the tetrahedron.	$3\sqrt{2}$

2012 – 2013 Log1 Contest Round 2
Mu Geometry

Name: ________________

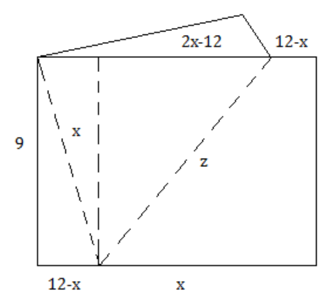
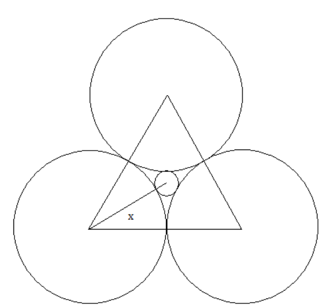
	4 points each	
1	Find the area enclosed by a triangle that has one side of length 4 and an altitude to that side that also has length 4.	8
2	Refer to the picture to the right. If $m\overset{\frown}{AB} = 111^\circ$ and arcs $\overset{\frown}{CD}$ and $\angle CED$ have equivalent degree measure, find the measure of $\angle CED$ in degrees. 	37°
3	Refer to the picture to the right. In $\triangle ADE$, B and C are points on sides $\overline{AE}$ and $\overline{AD}$, respectively, such that $\angle ABC \cong \angle ADE$. If $\overline{AC}$ and $\overline{DE}$ each have length x, $\overline{BC}$ has length 12, $\overline{AB}$ has length 15, and $\overline{CD}$ has length 6, find the value of x. 	24
4	An isosceles trapezoid has bases of lengths 20 and 30, legs whose lengths are integers, and an enclosed area that is also an integer. Find the enclosed area.	300
5	Find the number of regular n-gons whose total number of diagonals is divisible by 3 but not by 9.	0

	5 points each	
6	Let $A = \{(x, y) \mid -2 \le x \le 0,\ 0 \le y \le 2\} \cup \{(x, y) \mid 2 \le x \le 4,\ 2 \le y \le 4\}$. Find the volume obtained by rotating A about the y-axis.	32π
7	In an icosahedron, which has 20 faces, the sum of the number of vertices and edges equals 42. Find the number of edges of an icosahedron.	30
8	Two circles that are externally tangent have radii of lengths 5 and 12. A line is drawn, externally tangent to both circles. Consider the points of tangency, one on each circle, of the circles with the line. Find the distance between these two points.	$4\sqrt{15}$
9	A rhombus has an enclosed area of 442 and diagonals with integral lengths. Find the minimum value of the sum of the lengths of the diagonals of the rhombus.	60
10	A kite is inscribed in a circle. The longer diagonal of the kite is divided by the shorter diagonal into two segments whose lengths are 18 and 32. Find the area enclosed by the circle.	625π

6 points each		
11	A regular n-gon encloses an area of $21/5$. As $n \to \infty$, to what number does the perimeter of the n-gon approach?	$\frac{2\sqrt{105\pi}}{5}$
12	In a certain ellipse, the distance between the foci is 18, and the major and minor axes have integral lengths. If the two axes' lengths differ by more than 5, find the area enclosed by the ellipse.	180π
13	A 9 inch by 12 inch rectangular sheet of paper is folded so that one pair of opposite corners coincide. Find the length, in inches, of the crease made in the sheet of paper that results from folding the sheet of paper in this way.	11.25 or $\frac{45}{4}$
14	Three distinct larger circles with equal radius length are all externally tangent to each other. A smaller circle is externally tangent to each of the three larger circles. If the smaller circle has radius of length 1, find the length of the radius of one of the larger circles.	$3+2\sqrt{3}$
15	Find the volume when the region $C=\left\{(x,y)\middle|(x-5)^2+(y+4)^2\leq 9\right\}$ is rotated about the x-axis.	$72\pi^2$

2012 – 2013 Log1 Contest Round 2
Geometry Solutions

Mu	Al	Th	Solution
1	1	1	The area is $A=\frac{1}{2}(4)(4)=8$.
2	2	2	$\frac{111^\circ - x}{2}=x \Rightarrow 111^\circ - x = 2x \Rightarrow 3x = 111^\circ \Rightarrow x = 37^\circ$
3	3	3	By similar triangles, $\frac{12}{x}=\frac{15}{6+x} \Rightarrow 72+12x=15x \Rightarrow 3x=72 \Rightarrow x=24$.
	4	4	The square has side length $\frac{2(16)}{\sqrt{2}}=16\sqrt{2}$, so the area of the square is $\left(16\sqrt{2}\right)^2=512$. Therefore, the area inside the circle but outside the square is $\pi(16)^2-512$ $=256\pi-512$.
		5	The longer base overlaps the shorter base by 5 on each side, so because the legs have length 13, the altitude must be 12, and the area is $\frac{1}{2}(20+30)(12)=300$.
		6	The shortest distance is the radius of the inscribed circle, which is the area of the triangle divided by its semiperimeter. Using Heron's formula, $r=\frac{\sqrt{10(5)(3)(2)}}{10}=\sqrt{3}$.
4	5		The longer base overlaps the shorter base by 5 on each side, so if L is the leg length and H is the altitude, then $25=L^2-H^2=(L-H)(L+H)$. Since the area is an integer and is equal to $\frac{1}{2}(20+30)H=25H$, H must be an integer. Therefore, since L is an integer, $L-H$ and $L+H$ are both integers. The only possibility is $L-H=1$ and $L+H=25$, making $H=12$ and $L=13$. Therefore, the area is $25(12)=300$.
5	6	7	The total number of diagonals is given by $\frac{n(n-3)}{2}$. Since this number is divisible by 3 and 2 is not divisible by 3, the product $n(n-3)$ is divisible by 3, and since 3 is prime, either n or $n-3$ must be divisible by 3. However, having a difference of 3, both numbers are divisible by 3, meaning that the product is always divisible by 9 if it is divisible by 3. Therefore, there are 0 such polygons.
6			The region is two pieces: a cylinder and a cylindrical shell that the cylinder fits inside, so essential the region's volume is that of a cylinder with base radius 4 and height 2, so the volume is $\pi(4)^2(2)=32\pi$.
7	7	8	$v-e+20=2 \Rightarrow v-e=-18$, and $v+e=42$ was given, so subtracting the first equation from the second yields $2e=60 \Rightarrow e=30$.
8	8	9	The external tangent's length is $\sqrt{(12+5)^2-(12-5)^2}=\sqrt{289-49}=\sqrt{240}=4\sqrt{15}$.

	9	10	Using the same formula as earlier, by inspection, $\frac{(15)(15-3)}{2}=90$ and $\frac{(16)(16-3)}{2}=104$, so the smallest such number is 104.
9	10	11	The product of the diagonals is $2(442)=884=17\cdot 13\cdot 2^2$. To factor this into two integers, the 17 and 13 must be in different factors, so the possibilities occur with placing both 2s with the 17, both with the 13, or one with each. Those factorings are $68\cdot 13,\ 52\cdot 17,$ and $34\cdot 26$, respectively. The sums of the corresponding pairs of factors are 81, 69, and 60, so the minimum value is 60.
10	11		The kite is symmetric about its longer diagonal, which has a total length of 50. Also, by symmetry, this must be the diameter of the circle, so the enclosed area is $\pi(25)^2$ $=625\pi$.
11			As $n\to\infty$, the polygon approaches a circle, so the radius of the circle would be $r=\sqrt{\frac{21/5}{\pi}}=\frac{\sqrt{105\pi}}{5\pi}$, making the circumference $2\pi\frac{\sqrt{105\pi}}{5\pi}=\frac{2\sqrt{105\pi}}{5}$.
12	12	12	Let m and n be the major and minor axes' lengths, respectively. Thus, $81=\left(\frac{m}{2}\right)^2-\left(\frac{n}{2}\right)^2\Rightarrow 324=m^2-n^2=(m-n)(m+n)$, and the only possibilities of factorings where m and n are integers are if $m-n$ and $m+n$ are 2 and 162 or 6 and 54, making m and n 82 and 80 or 30 and 24, respectively. Since the axes' lengths had to differ by more than 5, the second case is the correct one, and the ellipse's enclosed area is $\pi(15)(12)=180\pi$.
13	13	13	Based on the diagram, the right triangle farthest to the right yields $81+(12-x)^2=x^2\Rightarrow 225-24x+x^2=x^2$ $\Rightarrow 24x=225\Rightarrow x=75/8$. Therefore, the right triangle with the crease as its hypotenuse, when solved for z, yields $z=\sqrt{9^2+\left(27/4\right)^2}=\sqrt{\frac{2025}{16}}=\frac{45}{4}$ or 11.25. 
14	14	14	The distance from the center of the smallest circle to the center of a larger circle is $2/3$ the length of the altitude of the triangle, which is $\frac{2}{3}\left(x\sqrt{3}\right)=\frac{2\sqrt{3}}{3}x$. Therefore, $\frac{2\sqrt{3}}{3}x=x+1\Rightarrow x\left(\frac{2\sqrt{3}-3}{3}\right)=1\Rightarrow x=\frac{3}{2\sqrt{3}-3}$ $=3+2\sqrt{3}$. 

		15	Using the shoelace method, $\begin{array}{r\|rr\|r} & -1 & -2 & \\ -6 & 3 & 3 & -3 \\ 6 & 2 & 5 & 15 \\ 30 & 6 & 3 & 6 \\ -3 & -1 & -2 & -12 \\ 27 & & & 6 \end{array} = \frac{1}{2}(27-6) = \frac{21}{2}$ or 10.5.
	15		$9 = \frac{a^3\sqrt{2}}{12} \Rightarrow a^3 = 54\sqrt{2} \Rightarrow a = 3\sqrt{2}$
15			This will be a torus with an "outer" radius of 4 (since the rotation is about the x-axis) and a cross-sectional radius of 3 (since that is the radius of the circular region), so the volume is $2\pi^2(4)(3)^2 = 72\pi^2$.