2012 – 2013 Log1 Contest Round 3
Theta Individual

Name: ________________

	4 points each	
1	A regular hexagon has how many diagonals?	
2	Consider a room containing 1000 people. There is at least one month in which the birthdays of at least n people from this room occur during this month. Find the largest possible value of n that makes this statement true without knowing the people's actual birthdays.	
3	Find the equation of the oblique asymptote of the function $y=\dfrac{30x^3-28x^2+13x+3}{6x^2-2x-1}$.	
4	Find the value of $f(-3)$, given that $f(x)=2x^5-5x^4+17x^2-13x-1$.	
5	My watch loses eight minutes every hour. At 9 am this morning, I set my watch accurately. Now, the time on my watch is 2:25 pm, but what is the actual time? Make sure to include am or pm in your answer.	

	5 points each	
6	Find the number of positive integers less than 50 that are relatively prime with 50.	
7	Two circles with radii of lengths 5 cm and 15 cm are externally tangent. A common external tangent is drawn to both circles. Find the area, in cm^2, of the region bounded by the circles and the common external tangent that is outside both circles.	
8	Solve for x: $3\log_6(x-15)-2\log_6(x+15)=-1$	
9	A rectangle measures 15 inches by 20 inches. What is the distance, in inches, from one corner of the rectangle to the diagonal of the rectangle not containing that corner?	
10	Solve for x: $\left(\dfrac{2}{3}\right)^{2x-3}=\left(\dfrac{9}{4}\right)^{9x-4}$	

6 points each		
11	Write in $a+bi$ form: $\left(-\sqrt{3}+3i\right)^5$	
12	Fred and Wilma alternately draw one dinosaur egg at random from a giant rock bowl that contains seven dinosaur eggs—4 red and 3 green. Drawn dinosaur eggs are not placed back into the bowl. If Fred draws first, what is the probability that Fred draws a red dinosaur egg before Wilma does?	
13	Find the sum of the series: $\sum_{n=1}^{2013}\left(\frac{2016}{n^2+5n+6}\right)$	
14	The graph of $10x^2-13xy-3y^2+18x-10y+8=0$ consists of two lines. Find the coordinates of the point at the intersection of the two lines.	
15	Two of a triangle's interior angles measure 45° and 60°. The side opposite the 45° angle has length 20 cm. Find the area, in cm^2, enclosed by the triangle.	

2012 – 2013 Log1 Contest Round 3
Alpha Individual

Name: ________________

	4 points each	
1	Consider a room containing 1000 people. There is at least one month in which the birthdays of at least n people from this room occur during this month. Find the largest possible value of n that makes this statement true without knowing the people's actual birthdays.	
2	Find the equation of the oblique asymptote of the function $y=\dfrac{30x^3-28x^2+13x+3}{6x^2-2x-1}$.	
3	My watch loses eight minutes every hour. At 9 am this morning, I set my watch accurately. Now, the time on my watch is 2:25 pm, but what is the actual time? Make sure to include am or pm in your answer.	
4	Find the solution, written in interval notation, of the inequality $\dfrac{\lvert x-4\rvert\cdot\sqrt{x+3}}{(x-2)}\geq 0$.	
5	Find the number of positive integers less than 50 that are relatively prime with 50.	

	5 points each	
6	A hyperbola has a transverse axis that is horizontal, and an ellipse has equation $3x^2+2y^2+12x+4y-22=0$. Additionally, the hyperbola and ellipse share a center. If the two conic sections intersect in three or more points, then the transverse axis of the hyperbola cannot equal or exceed a length of n units. Find the least possible value of n.	
7	Solve for x: $3\log_6(x-15)-2\log_6(x+15)=-1$	
8	Solve the system for ordered quintuple (a,b,c,d,e): $\begin{cases} a+b=21 \\ b+c=38 \\ c+d=14 \\ d+e=23 \\ e+a=18 \end{cases}$	
9	Two positive numbers a and b have a harmonic mean of 20 and an arithmetic mean of 45. Find the numerical values of a and b.	
10	Write in $a+bi$ form: $\left(-\sqrt{3}+3i\right)^5$	

6 points each		
11	Find the number of intersections in the xy-plane of the polar graphs with equations $r=1+2\cos\theta$ and $r=3\sin\theta$.	
12	Fred and Wilma alternately draw one dinosaur egg at random from a giant rock bowl that contains five dinosaur eggs—3 red and 2 green. Dinosaur eggs that Wilma draws are not placed back into the bowl, but dinosaur eggs that Fred draws are placed back into the bowl. If Fred draws first, what is the probability that Fred draws a red dinosaur egg before Wilma does?	
13	Find the sum of the series: $\sum_{n=1}^{2013}\left(\frac{2016}{n^2+5n+6}\right)$	
14	The graph of $10x^2-13xy-3y^2+18x-10y+8=0$ consists of two lines. Find the coordinates of the point at the intersection of the two lines.	
15	Find the solutions to the equation $2\sin(2\theta)+2\sqrt{5}\sin\left(\theta-\arccos\left(\frac{\sqrt{10}}{5}\right)\right)-\sqrt{6}=0$ in the interval $[0,2\pi)$.	

2012 – 2013 Log1 Contest Round 3
Mu Individual

Name: ________________

	4 points each	
1	Consider a room containing 1000 people. There is at least one month in which the birthdays of at least n people from this room occur during this month. Find the largest possible value of n that makes this statement true without knowing the people's actual birthdays.	
2	My watch loses eight minutes every hour. At 9 am this morning, I set my watch accurately. Now, the time on my watch is 2:25 pm, but what is the actual time? Make sure to include am or pm in your answer.	
3	Find the solution, written in interval notation, of the inequality $\frac{\lvert x-4\rvert\cdot\sqrt{x+3}}{(x-2)}\geq 0$.	
4	Find the value of $\left.\frac{dy}{dx}\right\rvert_{x=8}$ if $y^3=x^4$.	
5	Find the maximum value of the function $f(x)=6x^5-15x^4-10x^3+30x^2-12$ on the interval $[-2,2]$.	

	5 points each	
6	A hyperbola has a transverse axis that is horizontal, and an ellipse has equation $3x^2+2y^2+12x+4y-22=0$. Additionally, the hyperbola and ellipse share a center. If the two conic sections intersect in three or more points, then the transverse axis of the hyperbola cannot equal or exceed a length of n units. Find the least possible value of n.	
7	Solve for x: $3\log_6(x-15)-2\log_6(x+15)=-1$	
8	I have 20 grape Jolly Ranchers that I want to give to my comprehensive math team class. There are 9 students in the class, and I want to ensure that every student except Madhukar gets at least one grape Jolly Rancher (whether Madhukar gets some Jolly Ranchers or not does not matter to me; I just want to ensure the other eight students get at least one Jolly Rancher each). In how many distinct ways can I distribute the 20 Jolly Ranchers to my students? Assume the Jolly Ranchers are indistinguishable from each other.	
9	Two positive numbers a and b have a harmonic mean of 20 and an arithmetic mean of 45. Find the numerical values of a and b.	
10	Find the interval(s) on which the graph of $y=\int_2^{4x^2-2x+1}\left(t^3e^t\ln t\right)dt$ is increasing.	

6 points each		
11	Find the number of intersections in the xy-plane of the polar graphs with equations $r=1+2\cos\theta$ and $r=3\sin\theta$.	
12	Fred and Wilma alternately draw one dinosaur egg at random from a giant rock bowl that contains five dinosaur eggs—3 red and 2 green. Dinosaur eggs that Wilma draws are not placed back into the bowl, but dinosaur eggs that Fred draws are placed back into the bowl. If Fred draws first, what is the probability that Fred draws a red dinosaur egg before Wilma does?	
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14	Find the solutions to the equation $2\sin(2\theta)+2\sqrt{5}\sin\left(\theta-\arccos\left(\frac{\sqrt{10}}{5}\right)\right)-\sqrt{6}=0$ in the interval $[0,2\pi)$.	
15	Find the area of the region bounded by the graphs of $y=\frac{1}{\sqrt{5x-x^2}}$, the x-axis, $x=\frac{5}{2}$, and $x=\frac{15}{4}$.	

2012 – 2013 Log1 Contest Round 3
Theta Individual

Name: ________________

4 points each		
1	A regular hexagon has how many diagonals?	9
2	Consider a room containing 1000 people. There is at least one month in which the birthdays of at least n people from this room occur during this month. Find the largest possible value of n that makes this statement true without knowing the people's actual birthdays.	84
3	Find the equation of the oblique asymptote of the function $y=\dfrac{30x^3-28x^2+13x+3}{6x^2-2x-1}$.	$y=5x-3$
4	Find the value of $f(-3)$, given that $f(x)=2x^5-5x^4+17x^2-13x-1$.	-700
5	My watch loses eight minutes every hour. At 9 am this morning, I set my watch accurately. Now, the time on my watch is 2:25 pm, but what is the actual time? Make sure to include am or pm in your answer.	3:15 pm

5 points each		
6	Find the number of positive integers less than 50 that are relatively prime with 50.	20
7	Two circles with radii of lengths 5 cm and 15 cm are externally tangent. A common external tangent is drawn to both circles. Find the area, in cm^2, of the region bounded by the circles and the common external tangent that is outside both circles.	$\dfrac{600\sqrt{3}-275\pi}{6}$ or equiv.
8	Solve for x: $3\log_6(x-15)-2\log_6(x+15)=-1$	21
9	A rectangle measures 15 inches by 20 inches. What is the distance, in inches, from one corner of the rectangle to the diagonal of the rectangle not containing that corner?	12
10	Solve for x: $\left(\dfrac{2}{3}\right)^{2x-3}=\left(\dfrac{9}{4}\right)^{9x-4}$	0.55 or $\dfrac{11}{20}$

6 points each		
11	Write in $a+bi$ form: $\left(-\sqrt{3}+3i\right)^5$	$-144\sqrt{3}-432i$
12	Fred and Wilma alternately draw one dinosaur egg at random from a giant rock bowl that contains seven dinosaur eggs—4 red and 3 green. Drawn dinosaur eggs are not placed back into the bowl. If Fred draws first, what is the probability that Fred draws a red dinosaur egg before Wilma does?	$\frac{24}{35}$
13	Find the sum of the series: $\sum_{n=1}^{2013}\left(\frac{2016}{n^2+5n+6}\right)$	671
14	The graph of $10x^2-13xy-3y^2+18x-10y+8=0$ consists of two lines. Find the coordinates of the point at the intersection of the two lines.	$\left(-\frac{14}{17},\frac{2}{17}\right)$
15	Two of a triangle's interior angles measure 45° and 60°. The side opposite the 45° angle has length 20 cm. Find the area, in cm^2, enclosed by the triangle.	$150+50\sqrt{3}$

2012 – 2013 Log1 Contest Round 3
Alpha Individual

Name: ________________

	4 points each	
1	Consider a room containing 1000 people. There is at least one month in which the birthdays of at least n people from this room occur during this month. Find the largest possible value of n that makes this statement true without knowing the people's actual birthdays.	84
2	Find the equation of the oblique asymptote of the function $y=\dfrac{30x^3-28x^2+13x+3}{6x^2-2x-1}$.	$y=5x-3$
3	My watch loses eight minutes every hour. At 9 am this morning, I set my watch accurately. Now, the time on my watch is 2:25 pm, but what is the actual time? Make sure to include am or pm in your answer.	3:15 pm
4	Find the solution, written in interval notation, of the inequality $\dfrac{\lvert x-4\rvert\cdot\sqrt{x+3}}{(x-2)}\geq 0$.	$[-3,-3]\cup(2,\infty)$
5	Find the number of positive integers less than 50 that are relatively prime with 50.	20

	5 points each	
6	A hyperbola has a transverse axis that is horizontal, and an ellipse has equation $3x^2+2y^2+12x+4y-22=0$. Additionally, the hyperbola and ellipse share a center. If the two conic sections intersect in three or more points, then the transverse axis of the hyperbola cannot equal or exceed a length of n units. Find the least possible value of n.	$4\sqrt{3}$
7	Solve for x: $3\log_6(x-15)-2\log_6(x+15)=-1$	21
8	Solve the system for ordered quintuple (a,b,c,d,e): $\begin{cases} a+b=21 \\ b+c=38 \\ c+d=14 \\ d+e=23 \\ e+a=18 \end{cases}$	$(-4,25,13,1,22)$
9	Two positive numbers a and b have a harmonic mean of 20 and an arithmetic mean of 45. Find the numerical values of a and b.	$45\pm15\sqrt{5}$
10	Write in $a+bi$ form: $\left(-\sqrt{3}+3i\right)^5$	$-144\sqrt{3}-432i$

6 points each		
11	Find the number of intersections in the xy-plane of the polar graphs with equations $r=1+2\cos\theta$ and $r=3\sin\theta$.	3
12	Fred and Wilma alternately draw one dinosaur egg at random from a giant rock bowl that contains five dinosaur eggs—3 red and 2 green. Dinosaur eggs that Wilma draws are not placed back into the bowl, but dinosaur eggs that Fred draws are placed back into the bowl. If Fred draws first, what is the probability that Fred draws a red dinosaur egg before Wilma does?	0.73 or $\frac{73}{100}$
13	Find the sum of the series: $\sum_{n=1}^{2013}\left(\frac{2016}{n^2+5n+6}\right)$	671
14	The graph of $10x^2-13xy-3y^2+18x-10y+8=0$ consists of two lines. Find the coordinates of the point at the intersection of the two lines.	$\left(-\frac{14}{17},\frac{2}{17}\right)$
15	Find the solutions to the equation $2\sin(2\theta)+2\sqrt{5}\sin\left(\theta-\arccos\left(\frac{\sqrt{10}}{5}\right)\right)-\sqrt{6}=0$ in the interval $[0,2\pi)$.	$\frac{\pi}{3},\frac{2\pi}{3},\frac{3\pi}{4},\frac{5\pi}{4}$ (in any order)

2012 – 2013 Log1 Contest Round 3
Mu Individual

Name: ________________

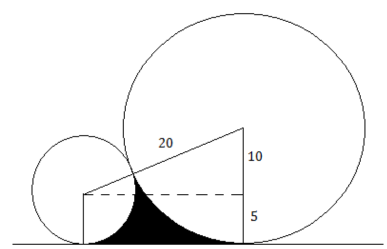
	4 points each	
1	Consider a room containing 1000 people. There is at least one month in which the birthdays of at least n people from this room occur during this month. Find the largest possible value of n that makes this statement true without knowing the people's actual birthdays.	84
2	My watch loses eight minutes every hour. At 9 am this morning, I set my watch accurately. Now, the time on my watch is 2:25 pm, but what is the actual time? Make sure to include am or pm in your answer.	3:15 pm
3	Find the solution, written in interval notation, of the inequality $\frac{\lvert x-4\rvert \cdot \sqrt{x+3}}{(x-2)} \geq 0$.	$[-3,-3]\cup(2,\infty)$
4	Find the value of $\left.\frac{dy}{dx}\right\rvert_{x=8}$ if $y^3 = x^4$.	$\frac{8}{3}$
5	Find the maximum value of the function $f(x) = 6x^5 - 15x^4 - 10x^3 + 30x^2 - 12$ on the interval $[-2,2]$.	7

	5 points each	
6	A hyperbola has a transverse axis that is horizontal, and an ellipse has equation $3x^2 + 2y^2 + 12x + 4y - 22 = 0$. Additionally, the hyperbola and ellipse share a center. If the two conic sections intersect in three or more points, then the transverse axis of the hyperbola cannot equal or exceed a length of n units. Find the least possible value of n.	$4\sqrt{3}$
7	Solve for x : $3\log_6(x-15) - 2\log_6(x+15) = -1$	21
8	I have 20 grape Jolly Ranchers that I want to give to my comprehensive math team class. There are 9 students in the class, and I want to ensure that every student except Madhukar gets at least one grape Jolly Rancher (whether Madhukar gets some Jolly Ranchers or not does not matter to me; I just want to ensure the other eight students get at least one Jolly Rancher each). In how many distinct ways can I distribute the 20 Jolly Ranchers to my students? Assume the Jolly Ranchers are indistinguishable from each other.	125,970
9	Two positive numbers a and b have a harmonic mean of 20 and an arithmetic mean of 45. Find the numerical values of a and b.	$45 \pm 15\sqrt{5}$
10	Find the interval(s) on which the graph of $y = \int_2^{4x^2-2x+1} \left(t^3 e^t \ln t\right) dt$ is increasing.	$\left(0, \tfrac{1}{4}\right)\cup\left(\tfrac{1}{2}, \infty\right)$

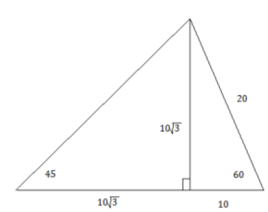
6 points each		
11	Find the number of intersections in the xy-plane of the polar graphs with equations $r=1+2\cos\theta$ and $r=3\sin\theta$.	3
12	Fred and Wilma alternately draw one dinosaur egg at random from a giant rock bowl that contains five dinosaur eggs—3 red and 2 green. Dinosaur eggs that Wilma draws are not placed back into the bowl, but dinosaur eggs that Fred draws are placed back into the bowl. If Fred draws first, what is the probability that Fred draws a red dinosaur egg before Wilma does?	0.73 or $\frac{73}{100}$
13	The graph of $10x^2-13xy-3y^2+18x-10y+8=0$ consists of two lines. Find the coordinates of the point at the intersection of the two lines.	$\left(-\frac{14}{17},\frac{2}{17}\right)$
14	Find the solutions to the equation $2\sin(2\theta)+2\sqrt{5}\sin\left(\theta-\arccos\left(\frac{\sqrt{10}}{5}\right)\right)-\sqrt{6}=0$ in the interval $[0,2\pi)$.	$\frac{\pi}{3},\frac{2\pi}{3},\frac{3\pi}{4},\frac{5\pi}{4}$ (in any order)
15	Find the area of the region bounded by the graphs of $y=\frac{1}{\sqrt{5x-x^2}}$, the x-axis, $x=\frac{5}{2}$, and $x=\frac{15}{4}$.	$\frac{\pi}{6}$

2012 – 2013 Log1 Contest Round 3
Individual Solutions

Mu	Al	Th	Solution
		1	$\frac{(6)(6-3)}{2}=9$ total diagonals
1	1	2	Using the Pigeonhole Principle, there is at least one month with $\left\lceil \frac{1000}{12} \right\rceil = \lceil 83.333... \rceil$ $=84$ people's birthdays. 85 or any other higher number will not work because there is a possible spreading of the birthdays with 8 months with 83 birthdays and 4 months with 84 birthdays (basically spreading the birthdays out as evenly as possible).
	2	3	Performing long division between polynomials, $\frac{30x^3-28x^2+13x+3}{6x^2-2x-1}=5x-3+\frac{12x}{6x^2-2x-1}$, so the oblique asymptote is $y=5x-3$.
		4	$f(-3)=2(-3)^5-5(-3)^4+17(-3)^2-13(-3)-1=-486-405+153+39-1=-700$
2	3	5	My watch progresses 52 minutes when 60 minutes have actually passed, so the watch is perpetually at $13/15$ of the actual time. Since from 9 am on the watch to 2:25 pm on the watch is a total of 325 minutes on the watch, we have $\frac{13}{15}=\frac{325}{x}\Rightarrow x=375$, so 375 actual minutes have passed since 9 am, which would make the actual time 3:15 pm.
3	4		Since both factors in the numerator are non-negative, we only consider where the denominator is positive (since the denominator can't equal 0), which is $(2,\infty)$. Additionally, we must include any number that makes the numerator 0 that we haven't already included—the only such number is -3. Therefore, the solution is $[-3,-3]\cup(2,\infty)$ (you could also accept $\{-3\}\cup(2,\infty)$).
4			If $x=8$, $y=16$. Therefore, $3y^2\frac{dy}{dx}=4x^3\Rightarrow\frac{dy}{dx}=\frac{4x^3}{3y^2}\Rightarrow\left.\frac{dy}{dx}\right\|_{x=8}=\frac{4(8)^3}{3(16)^2}=\frac{8}{3}$.
	5	6	We should exclude all multiples of 2 (24 of them) and multiples of 5 (9 of them), but we have double counted the multiples of 10 (4 of them), so we need to exclude $24+9-4=29$ of those integers. Since there were 49 total, there will be 20 such integers. OR Using number theory, since $50=5^2\cdot 2$ is the prime factorization of 50, $\Phi(50)=50\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)=20$.

5			$f'(x)=30x^4-60x^3-30x^2+60x=30x(x-2)(x-1)(x+1)$, so using the closed interval method, the maximum will be greatest of $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, and $f(2)$. $f(-2)=6(-2)^5-15(-2)^4-10(-2)^3+30(-2)^2-12=-244$ $f(-1)=6(-1)^5-15(-1)^4-10(-1)^3+30(-1)^2-12=7$ $f(0)=6(0)^5-15(0)^4-10(0)^3+30(0)^2-12=-12$ $f(1)=6(1)^5-15(1)^4-10(1)^3+30(1)^2-12=-1$ $f(2)=6(2)^5-15(2)^4-10(2)^3+30(2)^2-12=-20$ So the maximum value on that interval is 7.
6	6		The ellipse's equation in standard form is $\frac{(x+2)^2}{12}+\frac{(y+1)^2}{18}=1$, so the center of the hyperbola is also at $(-2,-1)$. Since the ellipse and hyperbola share a center, and since the ellipse has vertical major axis and the hyperbola has horizontal transverse axis, the hyperbola may not have a tranverse axis as long as or longer than the minor axis of the ellipse (since the two conics had to intersect in three or more places, making the transverse axis longer would yield no intersections). Therefore, n cannot equal or exceed $4\sqrt{3}$, which is the length of the minor axis of the ellipse. n cannot equal or exceed any higher number also, but the minimum value was asked for, and any number less than $4\sqrt{3}$ would not work (if $M<4\sqrt{3}$ was a possible value of n, then the length of the transverse axis of the hyperbola could be T, where $M<T<4\sqrt{3}$, and still intersect in four places, contradicting the given information).
		7	To find the sought region (black in the picture), find the total region enclosed by the trapezoid, then subtract the two sectors of the circles. The area enclosed by the trapezoid is $\frac{(5+15)(10\sqrt{3})}{2}=100\sqrt{3}$, and the smaller circle's sector, having a central angle of 120°, has area $\frac{1}{3}\pi(5)^2=\frac{25\pi}{3}$ while the larger circle's sector, having a central angle of 60°, has area $\frac{1}{6}\pi(15)^2=\frac{75\pi}{2}$. Therefore, the sought region's area is $100\sqrt{3}-\frac{25\pi}{3}-\frac{75\pi}{2}=\frac{600\sqrt{3}-275\pi}{6}$. 
7	7	8	$-1=3\log_6(x-15)-2\log_6(x+15)=\log_6\frac{(x-15)^3}{(x+15)^2}\Rightarrow\frac{1}{6}=\frac{(x-15)^3}{(x+15)^2}$ $=\frac{x^3-45x^2+675x-3375}{x^2+30x+225}\Rightarrow 6x^3-270x^2+4050x-20250=x^2+30x+225$ $\Rightarrow 0=6x^3-271x^2+4020x-20475=(x-21)(6x^2-145x+975)$, and since the discriminant of the quadratic is negative, the only real solution is $x=21$.
	8		Adding all five equations together yields $2(a+b+c+d+e)=114$, or $a+b+c+d+e=57$. Considering the second and fourth equation, $a=-4$, and then plugging into the first equation down, you get $b=25$, $c=13$, $d=1$, and $e=22$, so the ordered quintuple is $(-4,25,13,1,22)$.

8			If you give one Jolly Rancher to each of the other eight students, you then must find the number of ways to distribute 12 Jolly Ranchers among the 9 students, which is $\binom{9+12-1}{12}=\binom{20}{12}=\frac{20!}{12!8!}=125{,}970$.
		9	Drawing the diagonal creates two identical right triangles with legs of length 15 and 20 inches and hypotenuse of length 25 inches. Since the distance to the hypotenuse would be the altitude of the triangle to that side, we must have $15\cdot 20=25\cdot x\Rightarrow x=12$ inches.
9	9		$20=\frac{2ab}{a+b}=\frac{ab}{\left(\frac{a+b}{2}\right)}=\frac{ab}{45}\Rightarrow ab=900$, so a and b are the solutions to $x^2-90x+900=0$. $x=\frac{90\pm\sqrt{8100-4\cdot 900}}{2}=\frac{90\pm\sqrt{4500}}{2}=\frac{90\pm 30\sqrt{5}}{2}=45\pm 15\sqrt{5}$, so these are the values of a and b.
		10	$\left(\frac{3}{2}\right)^{3-2x}=\left(\frac{2}{3}\right)^{2x-3}=\left(\frac{9}{4}\right)^{9x-4}=\left(\frac{3}{2}\right)^{18x-8}\Rightarrow 3-2x=18x-8\Rightarrow 11=20x\Rightarrow x=\frac{11}{20}$
	10	11	$\left(-\sqrt{3}+3i\right)^5=\left(2\sqrt{3}\right)^5\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^5=288\sqrt{3}\left(cis120^\circ\right)^5=288\sqrt{3}\,cis600^\circ$ $=288\sqrt{3}\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=-144\sqrt{3}-432i$
10			According to the Fundamental Theorem of Calculus, $y'=(8x-2)\left(4x^2-2x+1\right)^3e^{4x^2-2x+1}\ln\left(4x^2-2x+1\right)$. The second and third factors are always positive, and the fourth factor is negative only if $4x^2-2x+1<1\Rightarrow 0<x<\frac{1}{2}$. The first factor is negative only if $x<\frac{1}{4}$. Therefore, $y'>0$ if the first and fourth factors are both negative, which is $0<x<\frac{1}{4}$, or if the first and fourth factors are both positive, which is $x>\frac{1}{2}$. Therefore, the intervals where y is increasing are $\left(0,\frac{1}{4}\right)\cup\left(\frac{1}{2},\infty\right)$.
11	11		The first equation is a limaçon with a loop pointed in the positive x-direction, and the second graph is a circle centered on the positive y-axis. Therefore, the two graphs intersect at the origin, once on the inner loop of the limaçon, and once on the outer part of the limaçon for a total of three intersection points in the xy-plane.
		12	Breaking this down into cases, the possibilities are that Fred draws a red egg first (R), which has probability $\frac{4}{7}$; or that Fred and Wilma each draw green, then Fred draws red (GGR), which has a probability of $\frac{3}{7}\cdot\frac{2}{6}\cdot\frac{4}{5}=\frac{4}{35}$. Fred can't be the first to draw a red egg on his third draw because there are only 3 green eggs, so if Fred draws two green eggs, Wilma must draw a red egg on her second draw. Therefore, the probability is $\frac{4}{7}+\frac{4}{35}=\frac{24}{35}$.

12	12		Breaking this down into cases, the possibilities are that Fred draws a red egg first (R), which has probability $\frac{3}{5}$; or that Fred and Wilma each draw green, then Fred draws red (GGR), which has a probability of $\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{3}{4}=\frac{3}{25}$; or that Fred and Wilma each drawn green on their first two turns, then Fred draws red (GGGGR), which has a probability of $\frac{2}{5}\cdot\frac{2}{5}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{3}=\frac{1}{100}$. If Wilma draws two green eggs (a sequence beginning with GGGG), then only red eggs remain, so there are no other possible cases. Therefore, the probability is $\frac{3}{5}+\frac{3}{25}+\frac{1}{100}=\frac{73}{100}$.
	13	13	This is a telescoping sum, so $\sum_{n=1}^{2013}\left(\frac{2016}{n^2+5n+6}\right)=\sum_{n=1}^{2013}\left(\frac{2016}{n+2}-\frac{2016}{n+3}\right)=\frac{2016}{3}-\frac{2016}{4}$ $+\frac{2016}{4}-\frac{2016}{5}+\frac{2016}{5}-\frac{2016}{6}+...+\frac{2016}{2015}-\frac{2016}{2016}=\frac{2016}{3}-\frac{2016}{2016}=672-1=671$.
13	14	14	Since you are told that the equation forms two lines, factor the equation in the following way: $0=10x^2-13xy-3y^2+18x-10y+8=(5x+y+A)(2x-3y+B)$, where A and B are real numbers (you can do this by just factoring the expression formed by the first three terms). Then multiplying out this expression, you get that $-3A+B$ is the y-coefficient (which is -10) and $2A+5B$ is the x-coefficient (which is 18); solving this system yields $A=4$ and $B=2$. Also notice that the constant in the expansion is AB (which is 8), so this is a consistent system, and the factorization is $0=(5x+y+4)(2x-3y+2)$. Therefore, the two lines have equations $5x+y+4=0$ and $2x-3y+2=0$. Solving the first equation for y and plugging into the second equation yields $0=2x-3(-5x-4)+2=17x+14\Rightarrow x=-\frac{14}{17}$, which in turn yields $y=\frac{2}{17}$, so the point is $\left(-\frac{14}{17},\frac{2}{17}\right)$.
		15	Based on the picture, completing both parts of the larger triangle makes the base of the larger triangle $10+10\sqrt{3}$ and the altitude to that side $10\sqrt{3}$, so the area enclosed by this triangle is $\frac{10\sqrt{3}\left(10+10\sqrt{3}\right)}{2}=150+50\sqrt{3}$. 
14	15		$0=2\sin(2\theta)+2\sqrt{5}\sin\left(\theta-\arccos\left(\frac{\sqrt{10}}{5}\right)\right)-\sqrt{6}=4\sin\theta\cos\theta$ $+2\sqrt{5}\left(\sin\theta\cdot\frac{\sqrt{10}}{5}-\cos\theta\cdot\frac{\sqrt{15}}{5}\right)-\sqrt{6}=4\sin\theta\cos\theta+2\sqrt{2}\sin\theta-2\sqrt{3}\cos\theta-\sqrt{6}$ $=\left(2\sin\theta-\sqrt{3}\right)\left(2\cos\theta+\sqrt{2}\right)\Rightarrow\sin\theta=\frac{\sqrt{3}}{2}$ or $\cos\theta=-\frac{\sqrt{2}}{2}\Rightarrow\theta=\frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{3\pi}{4}$, or $\frac{5\pi}{4}$.

15			The area is $\int_{5/2}^{15/4} \frac{1}{\sqrt{5x-x^2}}\,dx = \int_{5/2}^{15/4} \frac{1}{\sqrt{\frac{25}{4}-\left(x-\frac{5}{2}\right)^2}}\,dx$, and using the substitution $u = x - \frac{5}{2}$, we get $\int_0^{5/4} \frac{1}{\sqrt{\frac{25}{4}-u^2}}\,du = \arcsin \frac{u}{5/2}\Big\vert_0^{5/4} = \arcsin \frac{2u}{5}\Big\vert_0^{5/4} = \arcsin \frac{1}{2} - \arcsin 0$ $= \frac{\pi}{6} - 0 = \frac{\pi}{6}$.