2013 – 2014 Log1 Contest Round 1
Theta Functions

Name: ________________

	4 points each	
1	If $g(x) = -7x + 13 + \dfrac{15}{x}$, find the value of $g(-3)$.	
2	What is the range of the real-valued function $f(x) = \sqrt{16 - 4x^2}$?	
3	Find a simplified fractional expression for the reciprocal of $h(x) = \dfrac{2}{2x+3} + \dfrac{3}{x}$.	
4	If $g(x) = 52x - 17$, find the value of $g^{-1}(19)$.	
5	Define a function D, whose domain is all 2×2 matrices with real entries, by $D(A) = \lvert A \rvert$, the determinant of A. Find the value of $D\left(\begin{bmatrix} 5 & 5 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 12 & -1 \end{bmatrix}\right)$.	

	5 points each	
6	If $f(x) = \dfrac{x-3}{x+1}$, find an expression for $f(f(f(x)))$.	
7	$f(x)$ is a function. $g(x)$ is found by translating $f(x)$ down 6 units, then left 4 units, then reflecting the resulting graph over the *y*-axis. Write an expression for $g(x)$ in terms of $f(x)$.	
8	What is the greatest root of the function $h(x) = x^3 + 4x^2 - 3$?	
9	If $f(x) = \dfrac{x^2 - 2}{x^2 - 4}$ with domain $-2 < x < 2$, find the maximum value of f.	
10	Find the greatest integer *n* such that $f(n)$ is an integer for $f(x) = \dfrac{(x+1)(x+4)}{(x+2)(x+3)}$.	

6 points each		
11	What is the inverse of the function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$?	
12	What is the range of function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$, in interval notation?	
13	A rocket is shot upward with an initial velocity of 96 feet/second from a height of 32 feet. The height in feet of the rocket t seconds after it is shot upward is given by the function $g(t)=-16t^2+96t+32$. What is the rocket's maximum height in feet?	
14	f is a real-valued function such that $f\left(f(x+1)\right)=\lvert f(x+3)\rvert+2$. If $f(2)=5$ and $f(5)=4$, find the product of all possible values of $f(4)$.	
15	Define a function h with domain positive integers such that $h(x)=$ the sum of the positive integral divisors of x. Find the value of $h(2013)$.	

2013 – 2014 Log1 Contest Round 1
Alpha Functions

Name: ________________

	4 points each	
1	If $g(x)=-7x+13+\dfrac{15}{x}$, find the value of $g(-3)$.	
2	What is the range of the real-valued function $f(x)=\sqrt{16-4x^2}$?	
3	Find a simplified fractional expression for the reciprocal of $h(x)=\dfrac{2}{2x+3}+\dfrac{3}{x}$.	
4	Define a function D, whose domain is all 2×2 matrices with real entries, by $D(A)=\lvert A\rvert$, the determinant of A. Find the value of $D\left(\begin{bmatrix}5 & 5\\ -3 & 7\end{bmatrix}\begin{bmatrix}0 & -2\\ 12 & -1\end{bmatrix}\right)$.	
5	If $f(x)=\dfrac{x-3}{x+1}$, find an expression for $f\left(f\left(f(x)\right)\right)$.	

	5 points each	
6	A bug starts at the point $(-5,-4)$ and travels at constant velocity in a straight line, reaching the point $(7,5)$ in 10 seconds. At the same time the bug starts traveling, a spider starts at the point $(-4,5)$ and travels at constant velocity in a straight line, reaching the point $(3,-6)$ in 10 seconds. How many times do the bug and spider meet?	
7	$f(x)$ is a function. $g(x)$ is found by translating $f(x)$ down 6 units, then left 4 units, then reflecting the resulting graph over the y-axis. Write an expression for $g(x)$ in terms of $f(x)$.	
8	What is the greatest root of the function $h(x)=x^3+4x^2-3$?	
9	If $f(x)=\dfrac{x^2-2}{x^2-4}$ with domain $-2<x<2$, find the maximum value of f.	
10	What is the inverse of the function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$?	

6 points each		
11	What is the range of function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$, in interval notation?	
12	Find the points where the functions $g(x)=x^2+3x-2$ and $h(x)=-x^2-5x-8$ intersect.	
13	A rocket is shot upward with an initial velocity of 96 feet/second from a height of 32 feet. The height in feet of the rocket t seconds after it is shot upward is given by the function $g(t)=-16t^2+96t+32$. What is the rocket's maximum height in feet?	
14	f is a real-valued function such that $f(f(x+1))=\lvert f(x+3)\rvert+2$. If $f(2)=5$ and $f(5)=4$, find the product of all possible values of $f(4)$.	
15	Define a function h with domain positive integers such that $h(x)=$ the sum of the positive integral divisors of x. Find the value of $h(2013)$.	

2013 – 2014 Log1 Contest Round 1
Mu Functions

Name: ________________

	4 points each	
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2	What is the range of the real-valued function $f(x)=\sqrt{16-4x^2}$?	
3	Find a simplified fractional expression for the reciprocal of $h(x)=\frac{2}{2x+3}+\frac{3}{x}$.	
4	If $f(x)=\frac{x-3}{x+1}$, find an expression for $f(f(f(x)))$.	
5	Let $f(x)=\ln\left(x+\sqrt{x^2+5}\right)$. Find the value of $f'(2)$.	

	5 points each	
6	A bug starts at the point $(-5,-4)$ and travels at constant velocity in a straight line, reaching the point $(7,5)$ in 10 seconds. At the same time the bug starts traveling, a spider starts at the point $(-4,5)$ and travels at constant velocity in a straight line, reaching the point $(3,-6)$ in 10 seconds. How many times do the bug and spider meet?	
7	$f(x)$ is a function. $g(x)$ is found by translating $f(x)$ down 6 units, then left 4 units, then reflecting the resulting graph over the y-axis. Write an expression for $g(x)$ in terms of $f(x)$.	
8	What is the greatest root of the function $h(x)=x^3+4x^2-3$?	
9	Find the maximum value of the function $f(x)=-3x^3-18x^2+45x+12$ on the interval $[-3,2]$.	
10	What is the inverse of the function $f(x)=\frac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$?	

6 points each		
11	What is the range of function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$, in interval notation?	
12	Find the points where the functions $g(x)=x^2+3x-2$ and $h(x)=-x^2-5x-8$ intersect.	
13	If $g(x)=2x^3-3x^2+6x-1-\dfrac{1}{x}$, then $\int_2^4 g(x)dx=a-\ln b$, where a and b are integers. Find the value of $\sqrt{a+b}$.	
14	f is a real-valued function such that $f(f(x+1))=\lvert f(x+3)\rvert+2$. If $f(2)=5$ and $f(5)=4$, find the product of all possible values of $f(4)$.	
15	If $F(x)=\int\tan(4x)dx$ such that $F(0)=0$, then $F\left(\dfrac{\pi}{6}\right)=\ln A$, where $A>0$ is real. Find the value of $\sum_{i=2}^{\infty}\left(\dfrac{1}{A^{4i}}\right)$.	

2013 – 2014 Log1 Contest Round 1
Theta Functions

Name: ________________

	4 points each	
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2	What is the range of the real-valued function $f(x) = \sqrt{16 - 4x^2}$?	$0 \le y \le 4$ or $[0,4]$
3	Find a simplified fractional expression for the reciprocal of $h(x) = \frac{2}{2x+3} + \frac{3}{x}$.	$\frac{2x^2+3x}{8x+9}$
4	If $g(x) = 52x - 17$, find the value of $g^{-1}(19)$.	$\frac{9}{13}$
5	Define a function D, whose domain is all 2×2 matrices with real entries, by $D(A) = \lvert A \rvert$, the determinant of A. Find the value of $D\left(\begin{bmatrix} 5 & 5 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 12 & -1 \end{bmatrix}\right)$.	1200

	5 points each	
6	If $f(x) = \frac{x-3}{x+1}$, find an expression for $f(f(f(x)))$.	x
7	$f(x)$ is a function. $g(x)$ is found by translating $f(x)$ down 6 units, then left 4 units, then reflecting the resulting graph over the *y*-axis. Write an expression for $g(x)$ in terms of $f(x)$.	$g(x) = f(-x+4) - 6$ or equiv.
8	What is the greatest root of the function $h(x) = x^3 + 4x^2 - 3$?	$\frac{-3+\sqrt{21}}{2}$
9	If $f(x) = \frac{x^2-2}{x^2-4}$ with domain $-2 < x < 2$, find the maximum value of f.	$\frac{1}{2}$
10	Find the greatest integer *n* such that $f(n)$ is an integer for $f(x) = \frac{(x+1)(x+4)}{(x+2)(x+3)}$.	-1

6 points each		
11	What is the inverse of the function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$?	$f^{-1}(x)=x+4$
12	What is the range of function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$, in interval notation?	$(-\infty,-10)\cup(-10,-7)\cup(-7,3)\cup(3,\infty)$
13	A rocket is shot upward with an initial velocity of 96 feet/second from a height of 32 feet. The height in feet of the rocket t seconds after it is shot upward is given by the function $g(t)=-16t^2+96t+32$. What is the rocket's maximum height in feet?	176
14	f is a real-valued function such that $f(f(x+1))=\lvert f(x+3)\rvert+2$. If $f(2)=5$ and $f(5)=4$, find the product of all possible values of $f(4)$.	2
15	Define a function h with domain positive integers such that $h(x)=$ the sum of the positive integral divisors of x. Find the value of $h(2013)$.	2976

2013 – 2014 Log1 Contest Round 1
Alpha Functions

Name: ________________

	4 points each	
1	If $g(x)=-7x+13+\frac{15}{x}$, find the value of $g(-3)$.	29
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3	Find a simplified fractional expression for the reciprocal of $h(x)=\frac{2}{2x+3}+\frac{3}{x}$.	$\frac{2x^2+3x}{8x+9}$
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5	If $f(x)=\frac{x-3}{x+1}$, find an expression for $f(f(f(x)))$.	x

	5 points each	
6	A bug starts at the point $(-5,-4)$ and travels at constant velocity in a straight line, reaching the point $(7,5)$ in 10 seconds. At the same time the bug starts traveling, a spider starts at the point $(-4,5)$ and travels at constant velocity in a straight line, reaching the point $(3,-6)$ in 10 seconds. How many times do the bug and spider meet?	0
7	$f(x)$ is a function. $g(x)$ is found by translating $f(x)$ down 6 units, then left 4 units, then reflecting the resulting graph over the y-axis. Write an expression for $g(x)$ in terms of $f(x)$.	$g(x)=$ $f(-x+4)-6$ or equiv.
8	What is the greatest root of the function $h(x)=x^3+4x^2-3$?	$\frac{-3+\sqrt{21}}{2}$
9	If $f(x)=\frac{x^2-2}{x^2-4}$ with domain $-2<x<2$, find the maximum value of f.	$\frac{1}{2}$
10	What is the inverse of the function $f(x)=\frac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$?	$f^{-1}(x)=x+4$

6 points each		
11	What is the range of function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$, in interval notation?	$(-\infty,-10)\cup(-10,-7)\cup(-7,3)\cup(3,\infty)$
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14	f is a real-valued function such that $f(f(x+1))=\lvert f(x+3)\rvert+2$. If $f(2)=5$ and $f(5)=4$, find the product of all possible values of $f(4)$.	2
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2013 – 2014 Log1 Contest Round 1
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Name: ________________

	4 points each	
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2	What is the range of the real-valued function $f(x)=\sqrt{16-4x^2}$?	$0\le y\le 4$ or $[0,4]$
3	Find a simplified fractional expression for the reciprocal of $h(x)=\frac{2}{2x+3}+\frac{3}{x}$.	$\frac{2x^2+3x}{8x+9}$
4	If $f(x)=\frac{x-3}{x+1}$, find an expression for $f(f(f(x)))$.	x
5	Let $f(x)=\ln\left(x+\sqrt{x^2+5}\right)$. Find the value of $f'(2)$.	$\frac{1}{3}$

	5 points each	
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8	What is the greatest root of the function $h(x)=x^3+4x^2-3$?	$\frac{-3+\sqrt{21}}{2}$
9	Find the maximum value of the function $f(x)=-3x^3-18x^2+45x+12$ on the interval $[-3,2]$.	36
10	What is the inverse of the function $f(x)=\frac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$?	$f^{-1}(x)=x+4$

6 points each		
11	What is the range of function $f(x)=\dfrac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}$, in interval notation?	$(-\infty,-10)\cup(-10,-7)\cup(-7,3)\cup(3,\infty)$
12	Find the points where the functions $g(x)=x^2+3x-2$ and $h(x)=-x^2-5x-8$ intersect.	$(-3,-2)$ and $(-1,-4)$
13	If $g(x)=2x^3-3x^2+6x-1-\dfrac{1}{x}$, then $\int_2^4 g(x)dx=a-\ln b$, where a and b are integers. Find the value of $\sqrt{a+b}$.	10
14	f is a real-valued function such that $f(f(x+1))=\lvert f(x+3)\rvert+2$. If $f(2)=5$ and $f(5)=4$, find the product of all possible values of $f(4)$.	2
15	If $F(x)=\int\tan(4x)dx$ such that $F(0)=0$, then $F\left(\dfrac{\pi}{6}\right)=\ln A$, where $A>0$ is real. Find the value of $\sum_{i=2}^{\infty}\left(\dfrac{1}{A^{4i}}\right)$.	$\dfrac{1}{2}$

2013 – 2014 Log1 Contest Round 1
Functions Solutions

Mu	Al	Th	Solution
1	1	1	$g(-3)=-7(-3)+13+\frac{15}{-3}=21+13-5=29$
2	2	2	Since f is a root function, the minimum value of the function is 0. Also, $4x^2$ has minimum value 0, so the maximum value of f is $\sqrt{16}=4$. Since all values in between those two are also achievable, the range is $0\le y\le 4$ or $[0,4]$.
3	3	3	$h(x)=\frac{2}{2x+3}+\frac{3}{x}=\frac{2x+3(2x+3)}{x(2x+3)}=\frac{8x+9}{2x^2+3x}$, so the reciprocal expression is $\frac{2x^2+3x}{8x+9}$.
		4	$52g^{-1}(19)-17=19\Rightarrow 52g^{-1}(19)=36\Rightarrow g^{-1}(19)=\frac{36}{52}=\frac{9}{13}$
	4	5	$D\left(\begin{bmatrix}5 & 5\\-3 & 7\end{bmatrix}\begin{bmatrix}0 & -2\\12 & -1\end{bmatrix}\right)=D\left(\begin{bmatrix}5 & 5\\-3 & 7\end{bmatrix}\right)D\left(\begin{bmatrix}0 & -2\\12 & -1\end{bmatrix}\right)=50\cdot 24=1200$
4	5	6	$f(f(x))=\frac{\frac{x-3}{x+1}-3}{\frac{x-3}{x+1}+1}=\frac{x-3-3(x+1)}{x-3+(x+1)}=\frac{-2x-6}{2x-2}=\frac{-x-3}{x+1}$, so $f(f(f(x)))=f\left(\frac{-x-3}{x-1}\right)$ $=\frac{\frac{-x-3}{x-1}-3}{\frac{-x-3}{x-1}+1}=\frac{-x-3-3(x-1)}{-x-3+(x-1)}=\frac{-4x}{-4}=x$
5			$f'(x)=\frac{1+\frac{2x}{2\sqrt{x^2+5}}}{x+\sqrt{x^2+5}}=\frac{1}{\sqrt{x^2+5}}\Rightarrow f'(2)=\frac{1}{3}$
6	6		The bug's parametric equations of motion are $x=-5+\frac{6}{5}t$ and $y=-4+\frac{9}{10}t$. The spider's parametric equations of motion are $x=-4+\frac{7}{10}t$ and $y=5-\frac{11}{10}t$. Setting the two *x* equations equal yields $t=2$ while setting the two *y* equations equal yields $t=\frac{9}{2}$. Therefore, the bug and spider never occupy the same space at the same time.
7	7	7	The resulting functions, in order, are $f(x)-6$, $f(x+4)-6$, and $f(-x+4)-6$, so $g(x)=f(-x+4)-6$.
8	8	8	$h(x)=x^3+4x^2-3=(x+1)(x^2+3x-3)$, so the roots of *h* are -1 and $\frac{-3\pm\sqrt{21}}{2}$. The only positive root of the three is $\frac{-3+\sqrt{21}}{2}$, so it is the greatest.

	9	9	For $f(x)=\frac{x^2-2}{x^2-4}$, vertical asymptotes are at $x=2$ and $x=-2$. Further, the graph of f opens downward and is symmetric with respect to the y-axis, so the maximum value is at $x=0$, making the maximum value $\frac{1}{2}$.
9			$f'(x)=-9x^2-36x+45=-9(x+5)(x-1)$, so the only critical number for f in the interval is 1. $f(-3)=-204$, $f(2)=6$, and $f(1)=36$, so the maximum value is 36.
		10	$f(x)=\frac{(x+1)(x+4)}{(x+2)(x+3)}=\frac{x^2+5x+4}{x^2+5x+6}=1-\frac{2}{(x+2)(x+3)}$, and the only consecutive integers whose product divides 2 are -2 and -1 or 1 and 2, so setting the denominator of the expression equal to those two cases yields $x=-4$ and $x=-1$, respectively. Therefore, the greatest value of n would be -1.
10	10	11	$f(x)=\frac{x^4-2x^3-53x^2+54x+504}{x^3+2x^2-45x-126}=\frac{(x-7)(x+6)(x+3)(x-4)}{(x-7)(x+6)(x+3)}=x-4$, so the inverse is $f^{-1}(x)=x+4$.
11	11	12	From the previous problem, $f(x)=x-4$ as long as $x\neq-6,-3,7$. Without the restriction, the range would be all reals, so the range is all reals except the y-values achieved by plugging in those x-values. Therefore, the range of the function is $(-\infty,-10)\cup(-10,-7)\cup(-7,3)\cup(3,\infty)$.
12	12		$-x^2-5x-8=x^2+3x-2\Rightarrow 0=2x^2+8x+6=2(x+3)(x+1)\Rightarrow x=-3$ or $x=-1$. $g(-3)=h(-3)=-2$ and $g(-1)=h(-1)=-4$, so the points are $(-3,-2)$ and $(-1,-4)$.
	13	13	Since the graph of g is a parabola opening downward, the maximum height is at the vertex. The t-value at the vertex is $-\frac{96}{2(-16)}=3$, so the maximum height is $g(3)=-16(3)^2+96(3)+32=176$ feet.
13			$\int_2^4 g(x)dx=\left(\frac{1}{2}x^4-x^3+3x^2-x-\ln\lvert x\rvert\right)\Big\rvert_2^4=(128-64+48-4-\ln 4)-(8-8+12-2-\ln 2)=98-\ln 2$, so $a=98$ and $b=2$. Therefore, $\sqrt{a+b}=\sqrt{98+2}=\sqrt{100}=10$.
14	14	14	$4=f(5)=f(f(2))=f(f(1+1))=\lvert f(1+3)\rvert+2\Rightarrow\lvert f(4)\rvert=2\Rightarrow f(4)=\pm2$. However, $f(4)=f(f(5))=f(f(4+1))=\lvert f(4+3)\rvert+2>0$, implying that $f(4)=2$ only.
	15	15	Since $2013=3\cdot11\cdot61$ is the prime factorization of 2013, $h(2013)=(1+3)(1+11)(1+61)=2976$.
15			$F(x)=\int\tan(4x)dx=\frac{1}{4}\ln\lvert\sec 4x\rvert+C\Rightarrow 0=F(0)=\frac{1}{4}\ln\lvert\sec 0\rvert+C=C$. Therefore, $F(x)=\frac{1}{4}\ln\lvert\sec 4x\rvert$. $F\left(\frac{\pi}{6}\right)=\frac{1}{4}\ln\left\lvert\sec\frac{2\pi}{3}\right\rvert=\frac{1}{4}\ln 2=\ln\sqrt[4]{2}\Rightarrow A=\sqrt[4]{2}$. Therefore, $\sum_{i=2}^{\infty}\left(\frac{1}{A^{4i}}\right)=\sum_{i=2}^{\infty}\left(\frac{1}{2^i}\right)=\frac{1/4}{1-1/2}=\frac{1}{2}$