Mu

BC Calculus

Test #132

Directions:

1. Fill out the top left section of the scantron. Do not abbreviate your school name.

2. In the Student ID Number grid, write your 9-digit ID# and bubble.

3. In the Test Code grid, write the 3-digit test# on this test cover and bubble.

4. Scoring for this test is 5 times the number correct plus the number omitted.

5. TURN OFF ALL CELL PHONES.

6. No calculators may be used on this test.

7. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future National Conventions, disqualification of the student and/or school from this Convention, at the discretion of the Mu Alpha Theta Governing Council.

8. If a student believes a test item is defective, select "E) NOTA" and file a dispute explaining why.

9. If an answer choice is incomplete, it is considered incorrect. For example, if an equation has three solutions, an answer choice containing only two of those solutions is incorrect.

10. If a problem has wording like "which of the following could be" or "what is one solution of", an answer choice providing one of the possibilities is considered to be correct. Do not select "E) NOTA" in that instance.

11. If a problem has multiple equivalent answers, any of those answers will be counted as correct, even if one answer choice is in a simpler format than another. Do not select "E) NOTA" in that instance.

12. Unless a question asks for an approximation or a rounded answer, give the exact answer.

If the correct answer is not one of the answers presented, select E "None of the Above". All the best!

1. Compute the number of inflection points to the function:

$$f(t) = \frac{t^4}{12} - t^3 + \frac{9t^2}{2} + 15t + 8$$

A. 0 B. 1 C. 2 D. 3 E. NOTA

2. Evaluate:

$$\lim_{x \to \sqrt{3}} \sqrt{4 - x^2} + \lim_{x \to 2} \sqrt{4 - x^2} + \lim_{x \to -\sqrt{3}} \sqrt{4 - x^2} - \lim_{x \to -2} \sqrt{4 - x^2}$$

A. -1 B. 0 C. 1 D. 2 E. NOTA

3. Let c_0 the particular value that satisfies the MVT for derivatives for the function:

$$f(x) = 231x^2 + 299x + 202202056$$

on the interval $(1331, 2022)$.

Given that $c_0 = \frac{m}{n}$, where m and n are relatively prime positive integers, then compute the remainder when $m + n$ is divided by 1000.

A. 153 B. 355 C. 537 D. 791 E. NOTA

4. Consider the differential equation:

$$y' = \sqrt{1 + y^2}.$$

Given that $y(0) = 0$, then $y(\ln(2)) = \frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

A. 7 B. 9 C. 11 D. 13 E. NOTA

5. Consider the nonnegative sequence $\{a_n\}$ such that for $n \geq 0$,:

$$2a_{n+1}^2 = a_n + 1.$$

Given that $a_0 = \frac{1}{2}$, compute $\lim_{n\to\infty} a_n$

A. 0 B. ½ C. 1 D. DNE E. NOTA

For Questions 6 - 10 :

Denote $g(x) = -x^2 + 8x - 15$. Denote R to be the region of the area bound above by $g(x)$ and below by the x axis.

6. Given that the area of R can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers, then compute $m + n$.

A. 7 B. 13 C. 29 D. 57 E. NOTA

7. Denote S to be the set of all rectangles that can be inscribed in R. Given that the maximum area of an element of S can be written in the form $\frac{a\sqrt{b}}{c}$, where a, b, c are positive integers, b is not divisible by the square of any prime and a, c are relatively prime. Compute $a + b + c$.

A. 14 B. 16 C. 18 D. 20 E. NOTA

8. Given that the volume of the solid formed when R is revolved about the line $y = 4$ can be written in the form $\frac{m\pi}{n}$, where m and n are relatively prime positive integers, then compute $m + n$.

A. 21 B. 53 C. 125 D. 189 E. NOTA

9. Given that the volume of the solid formed when R is revolved about the line $y = -\frac{4x}{3} + 12$ can be written in the form $\frac{m\pi}{n}$, where m and n are relatively prime positive integers, then compute the remainder when $m + n$ is divided by 1000.

A. 767 B. 787 C. 807 D. 827 E. NOTA

10. Denote L to be the set of lines with positive slope that pass through the origin and divide R into two pieces of equal area. Given that the maximum value of the slope of an element in L can be written in the form:

$$A + B\sqrt[C]{D\sqrt[E]{F} + G},$$

where all variables are integers, C and E are minimized, the greatest common factor of G and D is not divisible by the square of any prime, the absolute value of D is maximized, and A is positive.

Compute $A + B + C + D + E + F + G$.

A. 60 B. 62 C. 76 D. 78 E. NOTA

11. Evaluate:

$$\lim_{x \to -\infty} \left(\sqrt{x^2 + 8x} - \sqrt{x^2 + 4x}\right)$$

A. -2 B. 0 C. 2 D. 4 E. NOTA

12. Evaluate:

$$\lim_{x \to \infty} x\left(\sqrt{x^2 + 4x + 5} - \sqrt{x^2 + 4x + 3}\right).$$

A. -1 B. 0 C. 1 D. 2 E. NOTA

13. Denote:

$$S = \sum_{k=0}^{\infty} \frac{(k^2 + k)}{k!}.$$

Given that $S = a + be$, where a, b are integers, then compute $a + b$.

A. 1 B. 2 C. 3 D. 4 E. NOTA

14. Denote:

$$g(x) = \int_0^x (1 - t^2 + t^4 - t^6 + \cdots)dt$$

Compute $g(\sqrt{3})$.

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$ C. $\frac{\pi}{3}$ D. $\frac{\pi}{2}$ E. NOTA

15. A certain bacteria's population P follows the differential equation:

$$\frac{dP}{dt} = P(50 - P).$$

If the initial population of the bacteria is 48, then the time it takes for the bacteria's population to be cut in half can be written in the form $\frac{A\ln(B)}{C}$, where A, C are relatively prime integers, and B, C are positive integers. Compute $A + B + C$.

A. 75 B. 77 C. 97 D. 99 E. NOTA

16. Denote R to be the region in the first quadrant such that each point (x_r, y_r) in R satisfies the relation:

$$\left(\sum_{j=0}^{5} x^{2j}y^{10-2j}\binom{5}{j}\right)^{1/2} = 2(x+y)\left(\sum_{j=0}^{2} x^j y^{2-j}(-1)^j\right).$$

Given that the area of R can be written in the form $\frac{p}{q}$ where p, q are relatively prime positive integers, then compute $p + q$.

A. 5 B. 7 C. 11 D. 19 E. NOTA

17. Consider the system of differential equations:

$$8f'(x) + 3g'(x) + g(x) = 0$$
$$9g'(x) + f(x) + 4f'(x) = 0$$

Given that $f(0) = 5 - g(0)$, then $f\left(6\ln\left(\frac{16}{25}\right)\right) + g\left(6\ln\left(\frac{16}{25}\right)\right)$ can be written in the form $\frac{m}{n}$, where m, n are relatively prime positive integers. Compute $m + n$.

A. 5 B. 13 C. 21 D. 29 E. NOTA

For Questions 18-20 :
ABC is a triangle that is changing with respect to time. The length of side AC is increasing at 2 units per second, the length of side AB is decreasing at 3 units per second, and the length of side BC is decreasing at 1 unit per second. At time $t = t_1$, sides AC, AB, BC have side lengths 13,14, and 15 respectively.

18. Calculate the rate of change of the perimeter of triangle ABC in units per second.

A. -2 B. 0 C. 2 D. 4 E. NOTA

19. Denote R to be the rate of change of $\angle A$ in radians per second at time t_1. Given that R can be written in the form $\frac{p}{q}$ where $p < q$ and p, q are relatively prime integers, compute $p + q$.

A. 38 B. 40 C. 51 D. 53 E. NOTA

20. Denote M to be the midpoint of BC. Denote S to be the rate of change of the length of AM in units per second at time t_1. Given that $S^2 = \frac{m}{n}$ where m, n are relatively prime positive integers, then compute the remainder when $m + n$ is divied by 1000.

A. 299 B. 309 C. 319 D. 329 E. NOTA

21. Define $\{F_n\}, n \geq 0$ to be a sequence such that $F_{n+2} = F_{n+1} + F_n$. Compute:

$$\lim_{n\to\infty} \frac{F_n}{F_{n+1}}.$$

A. 0 B. $\frac{1+\sqrt{5}}{2}$ C. 1 D. DNE E. NOTA

22. Denote r_1, r_2, r_3 to be the complex roots of:

$$h(x) = 5x^3 + 12x^2 + 14x + 2.$$

Denote $H = h'(r_1) + h'(r_2) + h'(r_3)$.

Given that $|H|$ can be written in the form $\frac{m}{n}$ where m, n are relatively prime positive integers, then compute $m + n$.

A. 9 B. 71 C. 141 D. 211 E. NOTA

23. Let

$$h(x) = \int_0^x 3x^3 dt.$$

Compute $h'(1)$.

A. 0 B. 3 C. 6 D. 9 E. NOTA

For Questions 24 –25 denote ellipse A to be the ellipse with major axis of length 6 and minor axis of length 4.

24. Compute the volume of the solid whose base is ellipse A and has semi circular cross sections perpendicular to the minor axis.

A. 12π B. 18π C. 24π D. 36π E. NOTA

25. Compute the volume of the solid formed when the ellipse is revolved about its minor axis.

A. 24π B. 36π C. 48π D. 72π E. NOTA

26. Let:

$$f(x) = \frac{\left(e^{x^2} + 5x + 6\right)\sqrt[4]{(2x^2 - 1)(\sin(x) - 1)}}{\left((1 + x) \cdot \sqrt{1 - x}\right)^5}$$

Given that $|f'(0)| = \frac{p}{q}$ where p, q are relatively prime positive integers then compute $p + q$.

A. 40 B. 61 C. 82 D. 103 E. NOTA

27. Denote

$$I = \int_0^{\frac{\pi}{2}} \frac{26dx}{(5\cos(x) + \sin(x))^2}$$

Given that I can be written in the form $\frac{m}{n}$ where m, n are relatively prime positive integers, then compute $m + n$.

A. 30 B. 31 C. 32 D. 33 E. NOTA

28. Let:

$$S = \sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{6^n}.$$

Given that $S^2 = \frac{m}{n}$ where m, n are relatively prime positive integers, then compute $m + n$.

A. 4 B. 6 C. 8 D. 10 E. NOTA

29. Let

$$S = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{n^4 m^3 + m^4 n^3}.$$

Given that $S = \frac{A\pi^B}{C}$, where A, B, C are positive integers and A, C are relatively prime. Compute $A + B + C$.

A. 38 B. 39 C. 74 D. 75 E. NOTA

30. Let:

$$S = \sum_{n=1}^{\infty} \frac{3^n + 1}{n^2 \cdot 4^n}.$$

Given that S can be written in the form:

$$\frac{A\pi^B + \mathrm{C} \cdot \ln(D) \cdot \ln(E) + F \cdot \ln^{\mathrm{H}}(G)}{I},$$

where $A - I$ are integers, A, C, F, I are relatively prime, $A > 0$.
Compute $A + B + C + D + E + F + G + H + I$.

A. 20 B. 21 C. 22 D. 23 E. NOTA