

2018 – 2019 Log1 Contest Round 2
Theta Equations and Inequalities

Name: _____

Units do not have to be included.

4 points each		
1	Solve for x. $-3(x + 2) < 15$	
2	What is the sum of the roots of the equation $x^2 + 5x = -6$	
3	What is the coefficient of the x^5 term of the expansion $\left(\frac{1}{2}x + 4\right)^7$?	
4	Danielle and Kristal go to Mal-Mart to buy school supplies. Kristal buys five notebooks and ten pencils for \$3.95 and Danielle buys two notebooks and twelve pencils for \$2.78. How much does a notebook cost?	
5	20% of \$1.50 is what percent of a quarter?	

5 points each		
6	Find the quotient $\frac{y}{x}$ of the coordinates of the point at which the lines $4x - y = 2$ and $3x + 24 = 5y$ intersect.	
7	June is 14 years less than three times Rose's age. In four years, Rose will be half June's age. What age, in years, will Rose be when June is 69 years old?	
8	A metal rod with an initial temperature of 50 degrees Fahrenheit ($^{\circ}\text{F}$) is heated so that its temperature increases at a constant rate of 4.5°F per minute. At the same time, another rod with an initial temperature of 70°F is being heated so that its temperature increases at a constant rate of 0.75°F per minute. After how many seconds will the two metal rods be at the same temperature?	
9	If $x^2 = 17$, then what is the value of $2x^4$?	
10	Given the system of equations $2x = 5y - 4$ and $5x = 3y + 9$ find the value of $\frac{y^x}{5y}$	

6 points each

11	A rectangular hedge surrounds a garden. The hedge has a uniform width of 0.5 foot and the length of the garden is 1.5 times longer than its width. Find the perimeter of the garden if the area of the hedge (including the garden) is 651 square feet and the width of the garden is an integer.	
12	Determine the values of x for which the following inequality is valid. $\frac{6x^2 - x - 35}{24x^2 + 14x - 3} > 0$	
13	Let W represent the amount of 80% copper alloy, in pounds, that should be mixed with a fixed amount of 40% copper alloy to end up with fifty pounds of an alloy containing an allowable percentage of copper. If the new copper alloy mixture must contain between 50% and 70% copper, by weight, for what values of W , in pounds, will this be achieved?	
14	The mean of three positive integers, a , b , and c , is 40. If the median is $a + 15$ and $a < b < c$, what is the lowest possible value of c ?	
15	B is the coefficient of the x^2 term in the expansion $(4x - 3)^5$. Given the equation $y = 3x^2 + \frac{B}{10}x + 12$, let m = the sum of the roots of y and n = the product of the roots of y . Calculate $\frac{m}{n}$	

2018 - 2019 Log1 Contest Round 2
Alpha Equations and Inequalities

Name: _____

Units do not have to be included.

4 points each	
1	Solve for x. $-3(x + 2) < 15$
2	What is the sum of the roots of the equation $x^2 + 5x = -6$
3	What is the coefficient of the x^5 term of the expansion $\left(\frac{1}{2}x + 4\right)^7$?
4	Find the positive difference of roots of the equation $2x^2 = 3 - 5x$
5	20% of \$1.50 is what percent of a quarter?

5 points each	
6	Find the quotient $\frac{y}{x}$ of the coordinates of the point at which the lines $4x - y = 2$ and $3x + 24 = 5y$ intersect.
7	June is 14 years less than three times Rose's age. In four years, Rose will be half June's age. What age, in years, will Rose be when June is 69 years old?
8	A metal rod with an initial temperature of 50 degrees Fahrenheit ($^{\circ}\text{F}$) is heated so that its temperature increases at a constant rate of 4.5°F per minute. At the same time, another rod with an initial temperature of 70°F is being heated so that its temperature increases at a constant rate of 0.75°F per minute. After how many seconds will the two metal rods be at the same temperature?
9	Solve for x. $4^{2x} = 9^{(x-5)}$
10	Given the system of equations $2x = 5y - 4$ and $5x = 3y + 9$ find the value of $\frac{y^x}{5y}$

6 points each

11	A rectangular hedge surrounds a garden. The hedge has a uniform width of 0.5 foot and the length of the garden is 1.5 times longer than its width. Find the perimeter of the garden if the area of the hedge (including the garden) is 651 square feet and the width of the garden is an integer.	
12	Determine the values of x for which the following inequality is valid. $\frac{6x^2 - x - 35}{24x^2 + 14x - 3} > 0$	
13	Let W represent the amount of 80% copper alloy, in pounds, that should be mixed with a fixed amount of 40% copper alloy to end up with fifty pounds of an alloy containing an allowable percentage of copper. If the new copper alloy mixture must contain between 50% and 70% copper, by weight, for what values of W , in pounds, will this be achieved?	
14	Yadira's math tests are graded out of 15 points. Her scores on her math tests are shown with the following data set: 12, 13, 8, 10, 14, 7, a , and b . The mean is 10 and the variance is 8.5. Calculate $\frac{ab}{a+b}$.	
15	B is the coefficient of the x^2 term in the expansion $(4x - 3)^5$. Given the equation $y = 3x^2 + \frac{B}{10}x + 12$, let $m =$ the sum of the roots of y and $n =$ the product of the roots of y Calculate $\frac{m}{n}$	

2018 – 2019 Log1 Contest Round 2
Mu Equations and Inequalities

Name: _____

Units do not have to be included.

4 points each		
1	Solve for x. $-3(x + 2) < 15$	
2	What is the sum of the roots of the equation $x^2 + 5x = -6$	
3	What is the coefficient of the x^5 term of the expansion $\left(\frac{1}{2}x + 4\right)^7$?	
4	Find the positive difference of roots of the equation $2x^2 = 3 - 5x$	
5	Evaluate $f'\left(\frac{\pi}{6}\right)$ given the equation $f(x) = \sqrt{7 \sin x}$	

5 points each		
6	Find the quotient $\frac{y}{x}$ of the coordinates of the point at which the lines $4x - y = 2$ and $3x + 24 = 5y$ intersect.	
7	June is 14 years less than three times Rose's age. In four years, Rose will be half June's age. What age, in years, will Rose be when June is 69 years old?	
8	A metal rod with an initial temperature of 50 degrees Fahrenheit ($^{\circ}\text{F}$) is heated so that its temperature increases at a constant rate of 4.5°F per minute. At the same time, another rod with an initial temperature of 70°F is being heated so that its temperature increases at a constant rate of 0.75°F per minute. After how many seconds will the two metal rods be at the same temperature?	
9	Solve for x. $4^{2x} = 9^{(x-5)}$	
10	Suppose that the minute hand of a clock is 15 mm long and the hour hand is 12 mm. How fast is the distance between the tip of the hour hand and the tip of the minute hand changing at 2 pm, in $\frac{\text{mm}}{\text{s}}$?	

6 points each

11	A rectangular hedge surrounds a garden. The hedge has a uniform width of 0.5 foot and the length of the garden is 1.5 times longer than its width. Find the perimeter of the garden if the area of the hedge (including the garden) is 651 square feet and the width of the garden is an integer.	
12	Determine the values of x for which the following inequality is valid. $\frac{6x^2 - x - 35}{24x^2 + 14x - 3} > 0$	
13	Let W represent the amount of 80% copper alloy, in pounds, that should be mixed with a fixed amount of 40% copper alloy to end up with fifty pounds of an alloy containing an allowable percentage of copper. If the new copper alloy mixture must contain between 50% and 70% copper, by weight, for what values of W , in pounds, will this be achieved?	
14	Yadira's math tests are graded out of 15 points. Her scores on her math tests are shown with the following data set: 12, 13, 8, 10, 14, 7, a , and b . The mean is 10 and the variance is 8.5. Calculate $\frac{ab}{a+b}$.	
15	C is the sum of the roots of the cubic $y = x^3 - 5x^2 - 4x + 20$. Using the expansion of $(Cx + 2)^C$, divide the coefficient of the x^2 term by the coefficient of the x^4 term then multiply by the coefficient of the x term.	

2018 – 2019 Log1 Contest Round 2
Theta Equations and Inequalities – Answer Key

Name: _____

Units do not have to be included.

4 points each		
1	Solve for x. $-3(x + 2) < 15$	$x > -7$
2	What is the sum of the roots of the equation $x^2 + 5x = -6$	-5
3	What is the coefficient of the x^5 term of the expansion $\left(\frac{1}{2}x + 4\right)^7$?	$\frac{21}{2}$
4	Danielle and Kristal go to Mal-Mart to buy school supplies. Kristal buys five notebooks and ten pencils for \$3.95 and Danielle buys two notebooks and twelve pencils for \$2.78. How much does a notebook cost?	\$0.49
5	20% of \$1.50 is what percent of a quarter?	120

5 points each		
6	Find the quotient $\frac{y}{x}$ of the coordinates of the point at which the lines $4x - y = 2$ and $3x + 24 = 5y$ intersect.	3
7	June is 14 years less than three times Rose's age. In four years, Rose will be half June's age. What age, in years, will Rose be when June is 69 years old?	47
8	A metal rod with an initial temperature of 50 degrees Fahrenheit ($^{\circ}\text{F}$) is heated so that its temperature increases at a constant rate of 4.5°F per minute. At the same time, another rod with an initial temperature of 70°F is being heated so that its temperature increases at a constant rate of 0.75°F per minute. After how many seconds will the two metal rods be at the same temperature?	320
9	If $x^2 = 17$, then what is the value of $2x^4$?	578
10	Given the system of equations $2x = 5y - 4$ and $5x = 3y + 9$ find the value of $\frac{y^x}{5y}$	$\frac{4}{5}$

6 points each		
11	A rectangular hedge surrounds a garden. The hedge has a uniform width of 0.5 foot and the length of the garden is 1.5 times longer than its width. Find the perimeter of the garden if the area of the hedge (including the garden) is 651 square feet and the width of the garden is an integer.	100
12	Determine the values of x for which the following inequality is valid. $\frac{6x^2 - x - 35}{24x^2 + 14x - 3} > 0$	$x < -\frac{7}{3}$ OR $-\frac{3}{4} < x < \frac{1}{6}$ OR $x > \frac{5}{2}$
13	Let W represent the amount of 80% copper alloy, in pounds, that should be mixed with a fixed amount of 40% copper alloy to end up with fifty pounds of an alloy containing an allowable percentage of copper. If the new copper alloy mixture must contain between 50% and 70% copper, by weight, for what values of W, in pounds, will this be achieved?	$12.5 \leq W \leq 37.5$
14	The mean of three positive integers, a, b, and c, is 40. If the median is a + 15 and a < b < c, what is the lowest possible value of c?	47
15	B is the coefficient of the x^2 term in the expansion $(4x - 3)^5$. Given the equation $y = 3x^2 + \frac{B}{10}x + 12$, let m = the sum of the roots of y and n = the product of the roots of y. Calculate $\frac{m}{n}$	36

2018 - 2019 Log1 Contest Round 2
Alpha Equations and Inequalities - Answer Key

Name: _____

Units do not have to be included.

4 points each		
1	Solve for x. $-3(x + 2) < 15$	$x > -7$
2	What is the sum of the roots of the equation $x^2 + 5x = -6$	-5
3	What is the coefficient of the x^5 term of the expansion $\left(\frac{1}{2}x + 4\right)^7$?	$\frac{21}{2}$
4	Find the positive difference of roots of the equation $2x^2 = 3 - 5x$	$\frac{7}{2}$
5	20% of \$1.50 is what percent of a quarter?	120

5 points each		
6	Find the quotient $\frac{y}{x}$ of the coordinates of the point at which the lines $4x - y = 2$ and $3x + 24 = 5y$ intersect.	3
7	June is 14 years less than three times Rose's age. In four years, Rose will be half June's age. What age, in years, will Rose be when June is 69 years old?	47
8	A metal rod with an initial temperature of 50 degrees Fahrenheit ($^{\circ}\text{F}$) is heated so that its temperature increases at a constant rate of 4.5°F per minute. At the same time, another rod with an initial temperature of 70°F is being heated so that its temperature increases at a constant rate of 0.75°F per minute. After how many seconds will the two metal rods be at the same temperature?	320
9	Solve for x: $4^{2x} = 9^{(x-5)}$	$\frac{5 \log 9}{\log 9 - \log 16}$
10	Given the system of equations $2x = 5y - 4$ and $5x = 3y + 9$ find the value of $\frac{y^x}{5y}$	$\frac{4}{5}$

6 points each		
11	A rectangular hedge surrounds a garden. The hedge has a uniform width of 0.5 foot and the length of the garden is 1.5 times longer than its width. Find the perimeter of the garden if the area of the hedge (including the garden) is 651 square feet and the width of the garden is an integer.	100
12	Determine the values of x for which the following inequality is valid. $\frac{6x^2 - x - 35}{24x^2 + 14x - 3} > 0$	$x < -\frac{7}{3}$ OR $-\frac{3}{4} < x < \frac{1}{6}$ OR $x > \frac{5}{2}$
13	Let W represent the amount of 80% copper alloy, in pounds, that should be mixed with a fixed amount of 40% copper alloy to end up with fifty pounds of an alloy containing an allowable percentage of copper. If the new copper alloy mixture must contain between 50% and 70% copper, by weight, for what values of W, in pounds, will this be achieved?	$12.5 \leq W \leq 37.5$
14	Yadira's math tests are graded out of 15 points. Her scores on her math tests are shown with the following data set: 12, 13, 8, 10, 14, 7, a, and b. The mean is 10 and the variance is 8.5. Calculate $\frac{ab}{a+b}$.	$\frac{55}{16}$
15	B is the coefficient of the x^2 term in the expansion $(4x - 3)^5$. Given the equation $y = 3x^2 + \frac{B}{10}x + 12$, let m = the sum of the roots of y and n = the product of the roots of y Calculate $\frac{m}{n}$	36

2018 – 2019 Log1 Contest Round 2
Mu Equations and Inequalities – Answer Key

Name: _____

Units do not have to be included.

4 points each		
1	Solve for x. $-3(x + 2) < 15$	$x > -7$
2	What is the sum of the roots of the equation $x^2 + 5x = -6$	-5
3	What is the coefficient of the x^5 term of the expansion $\left(\frac{1}{2}x + 4\right)^7$?	$\frac{21}{2}$
4	Find the positive difference of roots of the equation $2x^2 = 3 - 5x$	$\frac{7}{2}$
5	Evaluate $f' \left(\frac{\pi}{6}\right)$ given the equation $f(x) = \sqrt{7 \sin x}$	$\frac{\sqrt{42}}{4}$

5 points each		
6	Find the quotient $\frac{y}{x}$ of the coordinates of the point at which the lines $4x - y = 2$ and $3x + 24 = 5y$ intersect.	3
7	June is 14 years less than three times Rose's age. In four years, Rose will be half June's age. What age, in years, will Rose be when June is 69 years old?	47
8	A metal rod with an initial temperature of 50 degrees Fahrenheit ($^{\circ}\text{F}$) is heated so that its temperature increases at a constant rate of 4.5°F per minute. At the same time, another rod with an initial temperature of 70°F is being heated so that its temperature increases at a constant rate of 0.75°F per minute. After how many seconds will the two metal rods be at the same temperature?	320
9	Solve for x: $4^{2x} = 9^{(x-5)}$	$\frac{5 \log 9}{\log 9 - \log 16}$
10	Suppose that the minute hand of a clock is 15 mm long and the hour hand is 12 mm. How fast is the distance between the tip of the hour hand and the tip of the minute hand changing at 2 pm, in $\frac{\text{mm}}{\text{s}}$?	$-\frac{11\pi}{720\sqrt{7}}$ Or $-\frac{11\pi}{5040}\sqrt{7}$

6 points each		
11	A rectangular hedge surrounds a garden. The hedge has a uniform width of 0.5 foot and the length of the garden is 1.5 times longer than its width. Find the perimeter of the garden if the area of the hedge (including the garden) is 651 square feet and the width of the garden is an integer.	100
12	Determine the values of x for which the following inequality is valid. $\frac{6x^2 - x - 35}{24x^2 + 14x - 3} > 0$	$x < -\frac{7}{3}$ OR $-\frac{3}{4} < x < \frac{1}{6}$ OR $x > \frac{5}{2}$
13	Let W represent the amount of 80% copper alloy, in pounds, that should be mixed with a fixed amount of 40% copper alloy to end up with fifty pounds of an alloy containing an allowable percentage of copper. If the new copper alloy mixture must contain between 50% and 70% copper, by weight, for what values of W, in pounds, will this be achieved?	$12.5 \leq W \leq 37.5$
14	Yadira's math tests are graded out of 15 points. Her scores on her math tests are shown with the following data set: 12, 13, 8, 10, 14, 7, a, and b. The mean is 10 and the variance is 8.5. Calculate $\frac{ab}{a+b}$.	$\frac{55}{16}$
15	C is the sum of the roots of the cubic $y = x^3 - 5x^2 - 4x + 20$. Using the expansion of $(Cx + 2)^C$, divide the coefficient of the x^2 term by the coefficient of the x^4 term then multiply by the coefficient of the x term.	128

2018 – 2019 Log1 Contest Round 2
Equations and Inequalities Solutions

Mu	Al	Th	Solution
1	1	1	$-3(x + 2) < 15$ $-3x - 6 < 15$ $-3x < 21$ $x > -7$
2	2	2	$x^2 + 5x + 6 = 0$ <p>For a quadratic in standard form, $ax^2 + bx + c = 0$ the sum of the roots is</p> $-\frac{b}{a} = -\frac{5}{1} = -5$ <p>Alternatively, you could factor the equation.</p> $x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3, -2$ <p>The sum of the roots is $-3 + -2 = -5$</p>
3	3	3	<p>Use the Binomial Theorem for the nth term of the binomial expansion. Let m be the coefficient of the x^5 term of the expansion.</p> $m = \binom{7}{2} \left(\frac{1}{2}\right)^5 (4)^2 = \left(\frac{7!}{2!5!}\right) \left(\frac{1}{32}\right) (16) = \frac{21}{2}$
4	4		<p>Move all terms to one side of the equation then factor.</p> $2x^2 + 5x - 3 = 0$ $(2x - 1)(x + 3) = 0$ <p>The roots are $\frac{1}{2}$ and -3. The positive difference is $\frac{7}{2}$</p>
		4	<p>Setup a system of linear equations.</p> $5N + 10P = 3.95 \quad \text{and} \quad 2N + 12P = 2.78$ <p>Re-write the equations.</p> $30N + 60P = 23.70 \quad \text{and} \quad 10N + 60P = 13.90$ <p>Subtract the equations and solve for N.</p> $20N = 9.80$ $N = 0.49$

5			$f'(x) = \frac{1}{2} (7 \sin x)^{-\frac{1}{2}} (7 \cos x)$ $f'(x) = \frac{1}{2} (7 \cos x) (7 \sin x)^{-\frac{1}{2}}$ $f' \left(\frac{\pi}{6} \right) = \frac{1}{2} \left(7 \cos \frac{\pi}{6} \right) \left(7 \sin \frac{\pi}{6} \right)^{-\frac{1}{2}}$ $f' \left(\frac{\pi}{6} \right) = \frac{7\sqrt{3}}{2} \left(\frac{7}{2} \right)^{-\frac{1}{2}}$ $f' \left(\frac{\pi}{6} \right) = \frac{\sqrt{7 * 3 * 2}}{4} = \frac{\sqrt{42}}{4}$
5	5	5	<p>Calculate the percentages.</p> $X = 0.20 * 1.50 = \frac{3}{2} * \frac{1}{5} = \frac{3}{10}$ $\frac{3}{10} = Y * \frac{1}{4} \rightarrow y = \frac{3/10}{1/4} = \frac{3}{10} * \frac{4}{1} = \frac{12}{10} = 1.2$ $Y = 120\%$
6	6	6	<p>Solve the systems of equations. $4x - y = 2$ and $3x - 5y = -24$</p> <p>Re-write the equations $20x - 5y = 10$ and $3x - 5y = -24$</p> <p>Subtract the equations</p> $17x = 34$ $x = 3$ <p>Solving for y: $y = 6$</p> <p>To get the quotient, you must divide y by x.</p> $\frac{y}{x} = 3$
7	7	7	<p>$J = \text{June's age}$ and $R = \text{Rose's age}$ Setup the equations for Rose's and June's ages. $J = 3R - 14$ and $R + 4 = \frac{1}{2}(J + 4) \rightarrow R = \frac{1}{2}J - 2$</p> <p>Use substitution to find June's age.</p> $J = 3 \left(\frac{1}{2}J - 2 \right) - 14 \rightarrow \frac{3}{2}J - 6 - 14 = \frac{3}{2}J - 20$ $J = 40$ <p>Solve for Rose's age. $R = 18$ Since June is 22 years older than Rose, Rose must be 47</p>

8	8	8	<p>The temperature of each rod, after a time t, may be expressed by the following equations.</p> $T_1 = 50 + 4.5t \quad \text{and} \quad T_2 = 70 + 0.75t$ <p>Setting these equations equal yields</p> $50 + 4.5t = 70 + 0.75t \rightarrow 3.75t = 20$ $t = \frac{20}{3.75} = \frac{20}{3\frac{3}{4}} = \frac{20}{\frac{15}{4}} = 20 \left(\frac{4}{15} \right) = \frac{16}{3} \text{ min} = 320 \text{ s}$
9	9		<p>Take the log of both sides of the equation.</p> $2x \log(4) = (x - 5) \log(9)$ $x(2 \log(4) - \log(9)) = -5 \log(9)$ $x(\log(4)^2 - \log(9)) = -5 \log(9)$ $x = \frac{5 \log 9}{\log 9 - \log 16}$ <p>Alternative forms of this answer are acceptable if they are equivalent. The use of logs in other bases is also acceptable.</p>
		9	<p>Substitute. $2(x^2)^2 = 2(17)^2 = 2(289) = 578$</p>

Define the clock hand angular separation and rates.

$$\frac{d\theta_m}{dt} = \frac{2\pi}{3600} = \frac{\pi}{1800} \text{ s}^{-1} \quad \text{and} \quad \frac{d\theta_h}{dt} = \frac{2\pi}{12 \cdot 3600} = \frac{\pi}{21600} \text{ s}^{-1}$$

At 2 p.m., $\theta_h > \theta_m$. Therefore, let $\theta = \theta_h - \theta_m = \frac{\pi}{3}$

$$\frac{d\theta}{dt} = \frac{d\theta_h}{dt} - \frac{d\theta_m}{dt} = -\frac{11\pi}{21600} \text{ s}^{-1}$$

From the Law of Cosines, the distance between the tips is

$$s^2 = 15^2 + 12^2 - 2(15)(12) \cos \theta$$

At 2 p.m., $s^2 = 225 + 144 - 360 \cos \frac{\pi}{3} = 189$

$$s = \sqrt{189} = 3\sqrt{21} \text{ mm}$$

Method 1: Standard Differentiation

$$\begin{aligned} s &= \sqrt{369 - 360 \cos \theta} \\ \frac{ds}{d\theta} &= \frac{180 \sin \theta}{\sqrt{369 - 360 \cos \theta}} \\ \frac{ds}{d\theta} \cdot \frac{d\theta}{dt} &= \frac{180 \sin \theta}{\sqrt{369 - 360 \cos \theta}} \cdot \frac{d\theta}{dt} \\ \frac{ds}{dt} &= \frac{180 \sin \frac{\pi}{3}}{\sqrt{369 - 360 \cos \frac{\pi}{3}}} \left(\frac{-11\pi}{21600} \right) \\ \frac{ds}{dt} &= \frac{90\sqrt{3}}{\sqrt{189}} \left(\frac{-11\pi}{21600} \right) = \frac{-11\pi}{720\sqrt{7}} \\ \frac{ds}{dt} &= -\frac{11\pi}{720\sqrt{7}} = -\frac{11\pi}{5040} \sqrt{7} \text{ mm s}^{-1} \end{aligned}$$

Method 2: Implicit Differentiation

$$\begin{aligned} 2s ds &= 360 \sin \theta d\theta \\ \frac{ds}{dt} &= \left(\frac{180 \sin \theta}{s} \right) \frac{d\theta}{dt} \\ \frac{ds}{dt} &= \left(\frac{180 \sin \frac{\pi}{3}}{3\sqrt{21}} \right) \left(\frac{-11\pi}{21600} \right) = \left(\frac{90\sqrt{3}}{3\sqrt{21}} \right) \left(-\frac{11\pi}{21600} \right) \\ \frac{ds}{dt} &= \left(\frac{30\sqrt{3}}{\sqrt{21}} \right) \left(-\frac{11\pi}{21600} \right) = -\frac{11\pi}{720\sqrt{7}} \end{aligned}$$

This is the same answer obtained in Method 1.

Accept answers even if they have not rationalized the denominator

	10	10	<p>Through any method for solving a system of two equations, the solutions are $x = 3$ and $y = 2$</p> $\frac{2^3}{5 * 2} = \frac{8}{10} = \frac{4}{5}$
11	11	11	<p>Area = $\left(\frac{3}{2}x + 1\right)(x + 1) = 651$, where x is the width of the garden. This equation, with some effort, can be factored.</p> $\frac{3}{2}x^2 + \frac{5}{2}x + 1 = 651$ $3x^2 + 5x + 2 = 1302$ $3x^2 + 5x - 1300 = 0$ $(3x + 65)(x - 20) = 0$ <p>Alternatively, use the quadratic formula to solve for x. The solutions are $x = \left(20, -\frac{65}{3}\right)$. The only sensible answer is 20. Since the length is 1.5 times more than 20, the perimeter will be $2(20) + 2(30) = 100$</p>
12	12	12	<p>Factor the quadratics and check for solutions.</p> $\frac{6x^2 - x - 35}{24x^2 + 14x - 3} > 0 \rightarrow \frac{(2x - 5)(3x + 7)}{(6x - 1)(4x + 3)} > 0$ <p>The following solutions are disallowed.</p> $x = \frac{1}{6} \text{ and } x = -\frac{3}{4}$ <p>Checking the possible combinations of factors to produce positive results would be extremely exhaustive. However, a graphical approach reveals the answer more intuitively.</p> <p>The numerator may be represented by an equation that is concave upward, when graphed on an x-y coordinate system, with x-intercepts at $\left(-\frac{7}{3}, \frac{5}{2}\right)$. The equation is everywhere positive on the open intervals $x < -\frac{7}{3}$ and $x > \frac{5}{2}$. Likewise, the denominator is everywhere positive on the open interval $x < -\frac{3}{4}$ and $x > \frac{1}{6}$. The interval for which the inequality is everywhere positive must allow for numerator and denominator to be simultaneously positive or negative. This can only happen on three open intervals.</p> $x < -\frac{7}{3} \text{ OR } -\frac{3}{4} < x < \frac{1}{6} \text{ OR } x > \frac{5}{2}$

13	13	13	<p>There are 40 pounds of copper in 50 pounds of 80% copper alloy. There are 20 pounds of copper in 50 pounds of 40% copper alloy.</p> <p>To achieve a copper alloy that is at least 50% but no more than 70% copper, the mixing percentages for 80% and 40% alloys is $25 \leq 40x + 20(1 - x) \leq 35$</p> $25 \leq 20x + 20 \leq 35$ $5 \leq 20x \leq 15$ $\frac{1}{4} \leq x \leq \frac{3}{4}$ <p>Thus, you need to have 25% (or 12.5 lbs.) and 75% (or 37.5 lbs.) of the 80% mixture to achieve 50% and 70% copper alloys, respectively.</p>
14	14		<p>Let \bar{x} = the mean of the data set.</p> $\bar{x} = \frac{12 + 13 + 8 + 10 + 14 + 7 + a + b}{8} = 10$ <p>Simplifying:</p> $a + b = 16$ <p>Also,</p> $(a + b)^2 = 256$ $a^2 + b^2 + 2ab = 256$ $a^2 + b^2 = 256 - 2ab$ <p>Let σ^2 = the variance</p> $\sigma^2 = \frac{2^2 + 3^2 + 2^2 + 0^2 + 4^2 + 3^2 + (a-10)^2 + (b-10)^2}{8} = \frac{17}{2}$ <p>Simplifying:</p> $a^2 + b^2 - 20(a + b) = -174$ <p>Substituting:</p> $256 - 2ab - 20(16) = -174$ $2ab = 110$ $ab = 55$ <p>Therefore,</p> $\frac{ab}{a + b} = \frac{55}{16}$

	14	<p>Let $a + b + c = 120$ since their average equals 40. If the median is equal to $a + 15$, then $b = a + 15$ since $a < b < c$.</p> <p>Substituting,</p> $a + (a + 15) + c = 120$ $c = 105 - 2a$ <p>If $b < c$, then $a + 15 < c$.</p> <p>Substituting,</p> $a + 15 < 105 - 2a$ $a < 30$ <p>Thus, $a = 29$</p> <p>Substituting,</p> $c = 105 - 2(29) = 47$
15		<p>One may try to factor the cubic polynomial after setting it equal to zero. However, the general formula for the sum of the roots of a cubic polynomial is the same as it is for a quadratic. Therefore,</p> $C = -\frac{b}{a} = 5$ <p>The binomial expansion is then expressed as</p> $z = (5x + 2)^5$ <p>Let $a_1 =$ coefficient of the x^1 term $a_2 =$ coefficient of the x^2 term $a_4 =$ coefficient of the x^4 term</p> $a_1 = \binom{5}{4} 5^1 2^4 = \left(\frac{5!}{1! 4!}\right) 5 * 16 = 5 * 80 = 400$ $a_2 = \binom{5}{3} 5^2 2^3 = \left(\frac{5!}{2! 3!}\right) 25 * 8 = 10 * 200 = 2000$ $a_4 = \binom{5}{1} 5^4 2^1 = \left(\frac{5!}{4! 1!}\right) 625 * 2 = 5 * 1250 = 6250$ $\frac{a_1 a_2}{a_4} = \frac{400 * 2000}{6250} = \frac{80000}{625} = 128$

	15	15	<p>The formula for the sum and product of the roots of the equation $y = ax^2 + bx + c$ are $m = -\frac{b}{a}$ and $n = \frac{c}{a}$</p> <p>Let $B = a_4 = \binom{5}{2}4^2(-3)^3 = \left(\frac{5!}{2!3!}\right)(16)(-27) = -4320$</p> <p>Thus, $b = \frac{4320}{10} = -432$</p> $m = -\frac{b}{a} = \frac{432}{3} = 144$ $n = \frac{c}{a} = \frac{12}{3} = 4$ $\frac{m}{n} = \frac{144}{4} = 36$
--	----	----	---