1. B
2. C
3. A
4. C
5. D
6. A
7. A
8. C
9. E
10. B
11. A
12. D
13. A
14. D
15. E
16. B
17. D
18. E
19. A
20. B
21. C
22. C
23. D
24. B
25. B
26. B
27. C
28. B
29. C
30. C
1. **B.** MDCCLXXVI, so L.

2. **C.** \( A = \sqrt{s(s-s_1)(s-s_2)(s-s_3)} \), where \( s \) is the semiperimeter and \( s_k \) are the lengths of the three sides. Plugging in what we know, the perimeter is \( 3a \), so we get
   \[
   18a = \sqrt{\frac{3a}{2}(3a/2-a+3)(a/2)(3a/2-a-3)} \Rightarrow 36 = \sqrt{3\left(\frac{a^2}{4}-9\right)} \Rightarrow a = \pm 42, \text{ but only the plus root works. So the perimeter is } 3a = 126.
   \]

3. **A.** \( 4*C(13,5)/C(52,5) \) There are \( C(13,5) \) ways to get a flush of one suit (and 4 suits) and \( C(52,5) \) ways to get 5 cards from a deck. The multiplication comes out to
   \[
   4*13*12*11*10*9/52*51*50*49*48 \text{ or } 33/16660
   \]

4. **C.** The possible domain of \( \ln(x) \) is \( x > 0 \). For \( \ln(\ln(x)) \) the only values of \( \ln(x) \) that would work must be for \( x > 1 \). Then for \( \ln(\ln(\ln(x))) \), the only values of \( \ln(\ln(x)) \) that would work would be for \( x > e \).

5. **D.** A quick solution to this problem would be to let \( a = 1 \) and \( x \) and \( y \) both equal 0, then you get \( 1/2 + 1/2 = 1 \). For a more thorough solution,
   \[
   \frac{1}{1+a^{x-y}} + \frac{1}{1+a^{y-x}} = \frac{1+a^{y-x}+1+a^{x-y}}{(1+a^{x-y})(1+a^{y-x})} = \frac{2+a^{y-x}+a^{x-y}}{2+a^{y-x}+a^{x-y}} = 1
   \]

6. **A.** The formula for a lemniscate is \( r^2 = a^2 \cos 2\theta \), thus A is correct.

7. **A.** \( \tan(a-b) = \left[\tan(a) - \tan(b)\right]/\left[1 + \tan(a)\tan(b)\right] \). \( \tan\left(\cos^{-1}(3/5)\right) = 4/3 \) and \( \tan\left(\sin^{-1}(5/13)\right) = 5/12 \). Plugging into the formula and simplifying gives \( 33/56 \).

8. **C.** \( f = k \frac{m_1 m_2}{d^2} \). For this question we have
   \[
   k \frac{3m_1 (4m_2)}{(d/2)^2} = 48 \left( k \frac{m_1 m_2}{d^2} \right) = 48f.
   \]

9. **E.** The matrix is singular, and thus it has no inverse.

10. **B.** If you draw the points out on a graph, you’ll see that \( 5\sqrt{2} \) is the hypotenuse and 5 is one of the legs of a 45-45-90 triangle. Thus the distance between the two points is the other leg, 5.
11. A. \( \sum_{n=1}^{\infty} (2n-1)^{-1}(2n+1)^{-1} \), or with partial sums \( \sum_{n=1}^{\infty} \frac{1}{2(n-1)}-\frac{1}{2(n+1)} \). If you expand out the first several terms of this series, you see that all terms cancel except the first 1/2, which is the answer.

12. D. The two lines are 160 miles apart. They are moving toward each other at 20 + 40 = 60 mph. Thus in 160/60 hours, or 8/3 hour at 5:40 PM, they will collide. In 8/3 hr, the line north of his location will move 20*8/3=160/3 miles in his direction. This is less than 60 miles, the distance between him and the north cloud line at the beginning...by a difference of 20/3 mile. So the collision will occur 20/3 miles north of his current location.

13. A. See solution for question number 12.

14. D. Writing out the formula from the determinant gets \((a-x)(b-x)-ab=0 \). Simplifying gives \(ab-(a+b)x+x^2-ab=0 \), or \(x^2-(a+b)x=0 \), or \(x(x-(a+b))=0 \). So the solutions are \(x=\{0, a+b\} \).

15. E. The simplest way to answer this question is to convert each number to base 10, do the multiplication, then convert back to base 9. \(33_9=30 \) and \(44_9=40 \). 30 times 40 is 1200. This converts back to \(1*9^3+5*9^2+7*9+3 \), or \(1573_9 \).

16. B. If \(b^2-4ac>0 \), where \(b \) is the coefficient on the \(xy \) term and \(a \) and \(c \) are the coefficients on the \(x^2 \) and \(y^2 \) terms, respectively, then the conic section is a hyperbola...if less than 0 an ellipse, and if equal to 0 a parabola. Here we have \(3^2-4*(-4)=65>0 \). Thus it is a hyperbola.

17. D. If you have 1 choice only, then you could only have 1 addition. If you have 2 choices, then you can have 1 of each, or a combination...3 additions. For 3 additions, you have the 3 choices, then 3 combinations of 2, then 1 combination of all 3...7 additions. Note the pattern is \(2^n-1 \). Thus for 10 addition choices, the answer is 1023.

18. E. A quarter of a mile is 440 yards. Thus the two semicircle ends have to be a total of 440 minus 2*150 = 140 yards. So the radius of each semicircle has length 140/\(\pi \). If you have 6 lanes, then you must add 5 yards to the radius for Lane 6. For one lap, he would then walk \(2\left(\frac{70}{\pi}\right)+5\pi +2(150) = 440 +10\pi \). For 4 laps, this would be 1760+40\(\pi \). Someone walking 4 laps around Lane 1 would go 1760 yards, thus the extra amount walked is 40\(\pi \).

19. A. The full expansion of the dot product gives \((x^2 +3x)(x-3)+4x(x+3) = 0 \), or \(x^3 + 4x^2 + 3x = 0 \Rightarrow x(x+1)(x+3) = 0 \), so the solutions are \(x = \{0,-1,-3\} \).

20. B. All the snail has to do is reach the top. After its climb on day 1, it has made it to 5 feet. On day 2, it has made it to 6 feet. Following the progression, on day 96 it makes it to 100 feet.
21. C. Let \( y = \sin(2x) \), then the problem comes down to solving a quadratic equation. You get 
\[ 2y^2 + y - 1 = 0 \Rightarrow (2y-1)(y+1) = 0 \] . Then replace \( y \) back in and solve for \( x \). \( \sin(2x) = 1/2 \) and \( \sin(2x) = -1 \). The solutions to the first are \( x = \{\pi/12, 5\pi/12, 13\pi/12, 17\pi/12\} \) in the given domain and to the second are \( x = \{3\pi/4, 7\pi/4\} \). So there are 6 solutions.

22. C. The resultant of the two vectors is the sum \( \langle -7, 6, -6 \rangle \), which has a magnitude of 
\[ \sqrt{(-7)^2 + 6^2 + (-6)^2} = \sqrt{121} = 11 \] . To make this vector a unit vector the magnitude must be 1. If you want it parallel to the resultant, then divide \( \langle -7, 6, -6 \rangle \) by 11, to get \( \langle -7/11, 6/11, -6/11 \rangle \).

23. D. Represent the base \( i \) as \( e^{(\pi/2+2\pi k)i} \), where \( |k| \) is a whole number. Then you have 
\[ (e^{(\pi/2+2\pi k)i})^i = e^{(\pi/2+2\pi k)i^2} = e^{-(\pi/2+2\pi k)}. \] For \( k = 0 \), the only correct answer choice is \( e^{-\pi/2} \).

24. B. For the 2 2-digit squares, you have 6 ways to choose the first and then 5 to choose the second (because they have to be different square numbers), so there are 30 combinations here. Finally there are 68 numbers whose squares come out to 4 digits (32^2 through 99^2). This makes for a total of 98. Next you have to look for repeats! The easiest way to check for this is to write out the 30 combinations of 2 2-digit squares. The number 1681 is both 41^2 and 43^2. This turns out to be the only repeat, as you’ll note that 50*50=2500 and then 51*51=2601, and the same is true for all of the higher 4-digit numbers. Thus the answer is 97.

25. B. If the roots are going to be rational, then the only possible roots are \( \pm 1 \), \( \pm p \), \( \pm q \), and \( \pm pq \). Now the sum of the roots must be -1, so the absolute difference between two of the roots must be 1. Only the prime numbers 2 and 3 fit this profile, as 1 is not a prime number. Now you must find out which roots work in the equation (go through all of the plus/minus combinations). You find that only the choice with \( x = \{2 \text{ or } -3\} \) works, so there is only 1 set of such roots.

26. B. Average speed is harmonic mean of 50 & 70, or \( 2/[1/(50)+(1/70)] = 175/3 \).

27. C. The equation breaks down to an ellipse with form: 
\[ \frac{(x-2)^2}{1^2} + \frac{(y-1)^2}{3^2} = 1 \] . The area of an ellipse is given by \( \pi ab \), where \( a \) is 3 in this case and \( b \) is 1. Thus the area is \( 3\pi \).

28. B. For \( y = \tan(Ax-B) \), the periodicity is \( \pi/A \), so the answer is \( \pi/8 \).

29. C. \( f[g(x)] = 6x^2 + 15x - 14 = 0 \). Using quadratic equation gives \( x = -15 \pm \sqrt{561}/12 \).
30. C. The volume is $4x^2$, where $x$ is the length of a side of the base. Thus $x = 2\sqrt{3}$. The surface area will then be the four sides plus the base times 2 (both the inside and the outside of the box has exposed surface area… $2 \left( 4 \times 8\sqrt{3} + \left(2\sqrt{3}\right)^2 \right) = 64\sqrt{3} + 24.$