Answer E. NOTA means "none of the above answers is correct".

1. What is the greatest common factor of 72 and 210?

    (A) 4   (B) 6   (C) 7   (D) 9   (E) NOTA

2. Convert the base 7 number 56 to base 8.

    (A) 46   (B) 49   (C) 48   (D) 51   (E) NOTA

3. What is the remainder when 5^5 is divided by 7?

    (A) 1   (B) 2   (C) 3   (D) 4   (E) NOTA

4. How many positive integral factors does 1555200 have? (Note: $1555200 = 2^8\, 3^5\, 5^2$)

    (A) 80   (B) 124   (C) 162   (D) 196   (E) NOTA

5. Which of the following is divisible by 11?

    (A) 138157795   (B) 474438620   (C) 740276835   (D) 941025426   (E) NOTA

6. The four-digit number 6A6B is divisible by 72. Find the product of all possible values of A.

    (A) 10   (B) 16   (C) 12   (D) 14   (E) NOTA

7. Which of the following numbers is congruent to 3 mod 8?

    (A) 61944012   (B) 43292611   (C) 6170465   (D) 18001874   (E) NOTA

8. Let x equal the smallest positive integer that leaves a remainder of 2, 3, and 2 upon division by 3, 5, and 7, respectively. Find the product of the digits of x.

    (A) 16   (B) 32   (C) 6   (D) 24   (E) NOTA

9. Find the sum of the positive integral factors of 196.

    (A) 397   (B) 399   (C) 401   (D) 403   (E) NOTA

10. What is the product of the positive integral factors of 196?

    (A) 14^{16} (B) 14^{18} (C) 14^8 (D) 14^9 (E) NOTA

Answer E. NOTA means "none of the above answers is correct".

11. Let x be the greatest common factor of 1001 and 2007. Let y be the least common multiple of 1001 and 2007. Find the value of xy.

    (A) 2009007   (B) 2009006   (C) 2009005   (D) 2009004   (E) NOTA

12. How many integers strictly between 100 and 10,000 have an odd number of positive integral factors?

    (A) 91   (B) 90   (C) 89   (D) 88   (E) NOTA

13. The Euler Phi Function, denoted by $\phi(n)$, is defined as the number of positive integers less than or equal to n that are relatively prime to n. Find the value of $\phi(196)$.

    (A) 84   (B) 82   (C) 80   (D) 86   (E) NOTA

14. Using each of the digits 2, 3, 4, and 9 exactly once, make two positive integers such that one is 19 times the other. Subtract the smaller number from the larger and call this result x. Find the sum of the digits of x.

    (A) 11   (B) 12   (C) 13   (D) 14   (E) NOTA

15. Let x and y be nonnegative integers. Find the largest integer that *cannot* be expressed in the form 7x + 13y.

    (A) 40   (B) 71   (C) 111   (D) 182   (E) NOTA

16. The Fibonacci Numbers F(n), where n is a natural number, are defined as F(1) = 1, F(2) = 1, and for n > 2, defined recursively by F(n) = F(n – 1) + F(n – 2). Let x be the sum of the ten smallest Fibonacci numbers. What is the remainder when x is divided by 3?

    (A) 3   (B) 2   (C) 1   (D) 0   (E) NOTA

17. Find the remainder when 1! + 2! + 3! + ... + 2006! + 2007! is divided by 5.

    (A) 1   (B) 0   (C) 4   (D) 3   (E) NOTA

18. Let P(x) be a quadratic polynomial with integer coefficients. Which of the following is *not* a possible value for the discriminant of P?

    (A) 2007   (B) 5   (C) 2008   (D) 16   (E) NOTA

Answer E. NOTA means "none of the above answers is correct".

19. Find the largest prime factor of x if $\log_2(\log_3(\log_5 x)) = 2007$.

    (A) 2   (B) 3   (C) 5   (D) 7   (E) NOTA

20. For integers x, let F(x) be a polynomial with integer coefficients. Given that F(2) is divisible by 5 and F(5) is divisible by 2, it follows that F(7) is divisible by which of the following?

    (A) 5   (B) 10   (C) 7   (D) 2   (E) NOTA

21. Find the number of *ordered pairs* of positive integers (x, y) whose least common multiple is equal to 36.

    (A) 23   (B) 25   (C) 27   (D) 29   (E) NOTA

22. Let σ(n) denote the sum of the positive integral factors of n. Let y be the largest integer such that σ(y) = 38. Find the sum of the digits of y.

    (A) 9   (B) 10   (C) 11   (D) 12   (E) NOTA

23. Which of the following base-7 numbers is divisible by 6?

    (A) 33634_7 (B) 5553555_7 (C) 1111111_7 (D) 2534352_7 (E) NOTA

24. A random number x is randomly chosen from the set {5, 1, 16, 1000, 23}, where each element is equally likely to be chosen. What is the probability, as a percent, that x is prime?

    (A) 40%   (B) 60%   (C) 80%   (D) 20%   (E) NOTA

25. Given that x, y, and z are positive integers, find the number of solutions to the equation $x^4 + y^4 = z^4$.

    (A) Infinite   (B) 3   (C) 0   (D) 1   (E) NOTA

26. Solve for the second smallest positive value of x: $3x \equiv 5 \mod 7$.

    (A) 14   (B) 13   (C) 12   (D) 11   (E) NOTA

Answer E. NOTA means "none of the above answers is correct".

27. Anna has a favorite polynomial whose domain is the natural numbers. She tells her friend Catherine that it is of degree 2007, has integer coefficients, has a leading coefficient of 17, and a constant term of 41. Anna claims that her favorite polynomial outputs only primes. Indeed, evaluating the polynomial at small values results in primes. What is the largest value Catherine must evaluate the polynomial in order to refute Anna's claim with certainty?

   (A) 24   (B) 35   (C) 46   (D) 58   (E) NOTA

28. What is the smallest positive prime factor of $3^{2007} + 35^{1000}$?

   (A) 7   (B) 5   (C) 2   (D) 3   (E) NOTA

29. Let n be the smallest three-digit positive integer whose square ends in three 4s when written in base 10. Find the sum of the digits of n. (Hint: $38^2 = 1444$)

   (A) 14   (B) 13   (C) 11   (D) 12   (E) NOTA

30. A *lattice point* is a point with integer coordinates. A straight line through the origin is drawn in the plane and passes through infinitely many lattice points. Which of the following is a possible slope for this line?

   (A) π (B) 2007   (C) e (D) $\sqrt{2}$ (E) NOTA