Alpha Probability
2007 National Mu Alpha Theta Convention

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For each question, the choice E. NOTA stands for "None Of These Answers"

1. If A, B, and C are independent events with probabilities of 1, 0.5, and 0.7 of occurring, respectively, what is the probability that exactly 2 of those events occur?

   A. 0.2 B. 0.3 C. 0.35 D. 0.5 E. NOTA

2. Suppose X and Y are numbers selected at random from the real numbers on the interval [-1,1]. What is the probability that $|X+Y| > 0.5$?

   A. 0.75 B. 0.5 C. 0.5625 D. 0.25 E. NOTA

3. Suppose that in a very highly populated country, all parents who decide to have children will always continue having children until they have their first girl, at which point they will always stop. Suppose also that the probability of having a boy is equal to that of having a girl. If this process has gone on for many years, what should be the current ratio of boys to girls in the population?

   A. 1 : 1 B. 2 : 1 C. 1 : 2 D. 3 : 2 E. NOTA

4. If you draw a card at random from a regular deck of 52 playing cards, what is the probability that it is a jack, queen, or king?

   A. $\frac{3}{13}$ B. $\frac{3}{52}$ C. $\frac{9}{52}$ D. $\frac{3}{4}$ E. NOTA

5. Suppose you are presented with a fair 6-sided die, with the numbers 1 through 6 on the faces. You first roll the die at time = 0. If you do not roll a 6, you must roll again, but you must wait X minutes to roll again, where X is the value of your most recent roll. This process continues until you have rolled a six, at which point you are allowed to stop. How long should you expect to spend rolling this die?

   A. 3 minutes B. 15 minutes C. 21 minutes D. Forever E. NOTA

6. There exist 2 urns, Urn A and Urn B. Urn A contains 5 red balls and 3 green balls. Urn B contains 4 red balls and 5 green balls. Suppose you blindly place one ball from Urn A into Urn B, and then choose a ball randomly from Urn B. If this ball is red, what is the probability that the ball you moved from Urn A to Urn B was also red?

   A. $\frac{5}{11}$ B. $\frac{5}{8}$ C. $\frac{8}{11}$ D. $\frac{25}{37}$ E. NOTA

7. Suppose 3 boys and 3 girls are to be seated at random around a circular table. What is the probability that no boy sits adjacent to any other boy?

A. $\frac{1}{6}$ B. $\frac{1}{10}$ C. $\frac{3}{5}$ D. $\frac{1}{12}$ E. NOTA

8. Suppose A, B, C, and D are each independently chosen at random from the set of integers {1,2}. What is the probability that the matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ has an inverse?

A. $\frac{3}{4}$ B. $\frac{3}{8}$ C. $\frac{1}{2}$ D. $\frac{5}{8}$ E. NOTA

9. Suppose the event V occurs with probability $\frac{3-\sqrt{3}}{3}$. What is the probability that after two independent trials, V has not yet occurred?

A. $\frac{2}{9}$ B. $\frac{4-2\sqrt{3}}{3}$ C. $\frac{1}{3}$ D. $\frac{1}{9}$ E. NOTA

10. Suppose there is a 20% chance of rain tomorrow in North Carolina, and a 40% chance of snow tomorrow in New York. Assuming these events are independent, what is the probability that it rains in North Carolina and snows in New York tomorrow?

A. 0.8 B. 0.6 C. 0.08 D. 0.06 E. NOTA

11. Suppose a subset T is chosen randomly from the set of ALL subsets of the set S = {1,2,3, … , 19, 20}. What is the probability that the largest element of T is 18?

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{1}{10}$ D. $\frac{9}{10}$ E. NOTA

12. What is the probability that the coefficient of a term selected at random from the terms of the complete binomial expansion of $(2x+3y)^{100}$ is divisible by 12?

A. $\frac{49}{50}$ B. $\frac{98}{101}$ C. $\frac{99}{100}$ D. $\frac{99}{101}$ E. NOTA

13. Suppose a spinner of length 1 unit has one endpoint pinned to the origin, and it will be spun around this fixed point. What is the probability that when it stops, the free end of this spinner is greater than ½ units from the x-axis?

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{2}{\pi}\arcsin\left(\frac{1}{4}\right)$ D. $\frac{2}{\pi}\arctan\left(\frac{1}{4}\right)$ E. NOTA

14. There is a dartboard which consists of two concentric circles. A dart that hits within the inner circle is worth 10 points, and a dart that hits within the outer circle but not within the inner circle is worth 4 points. If the radius of the inner circle is 6 inches, and the radius of the entire dartboard is 12 inches, what is the expected amount of points garnered by a dart that hits the dartboard, if the dart is equally likely to hit anywhere on the dartboard?

A . 5.5 B. 6 C. 7 D. 9 E. NOTA

15. JJ decides to go on a strange roller coaster. It consists of one car on a straight 1-mile track. At time 0, the car shoots off at 70 miles per hour (mph). At some randomly determined point along the 1-mile track, the car immediately reduces its speed to 50 mph, and it remains at this speed until the end of the ride (at the end of the track). What is the probability that this roller coaster takes longer than 1 minute to ride?

A. $\frac{1}{2}$ B. $\frac{5}{12}$ C. $\frac{7}{12}$ D. $\frac{25}{48}$ E. NOTA

16. A and B are events. If $P(A \cup B)^c = .1$ and $P(A) = .55$, what is the smallest possible value of P(B)? (Note: the notation X^c indicates the complement of the event X)

A. 0.45 B. 0.35 C. 0.15 D. 0.25 E. NOTA

17. Suppose Sean repeatedly rolls a fair 6-sided die, on which 3 of the sides are labeled "1" and 3 of the sides are labeled "2". Sean begins rolling, and keeps track of the sum of the results of his rolls. What is the probability that at some point, this sum is equal to 6?

A. $\frac{13}{64}$ B. $\frac{1}{2}$ C. $\frac{11}{16}$ D. $\frac{43}{64}$ E. NOTA

18. Suppose 4 ants are sitting on the 4 corners of a square (one per corner), and each is about to travel along one of the 2 sides that meet at its corner, to another corner. If the ants decide which side to travel down at random, what is the probability that no ant crosses paths with another ant by the time the ants reach their new corners?

A. $\frac{1}{6}$ B. $\frac{1}{12}$ C. $\frac{1}{3}$ D. $\frac{1}{16}$ E. NOTA

19. If Marty picks 2 distinct numbers in {1,2,3 … ,9,10}, what is the probability that their sum is divisible by 5?

A. $\frac{6}{45}$ B. $\frac{13}{90}$ C. $\frac{7}{45}$ D. $\frac{1}{6}$ E. NOTA

20. Let S = {1,2,3, … , 9,10}. Suppose one subset T is randomly chosen from the set of ALL subsets of S that contain exactly 3 elements. What is the probability that the largest element of T is less than or equal to 8?

A. $\frac{2}{5}$ B. $\frac{7}{15}$ C. $\frac{8}{15}$ D. $\frac{3}{5}$ E. NOTA

21. Josh and Jamal play a game wherein they take turns shooting free throws (which are independent), and the first one to miss loses the game. Josh always goes first, and the probability that Jamal makes a free throw is 0.75. If there is an equal chance that Josh and Jamal will win the game, what must be the probability that Josh makes a free throw?

A. 0.7 B. 0.75 C. 0.8 D. 0.825 E. NOTA

22. Ten students, including Ross and Jordan, are to be seated at random in a row of ten chairs. What is the probability that Ross and Jordan are seated next to one another?

A. $\frac{2}{9}$ B. $\frac{1}{5}$ C. $\frac{1}{10}$ D. $\frac{1}{9}$ E. NOTA

23. Suppose there exists an infinitely long and infinitely wide sheet of graph paper, whose vertical lines and horizontal lines are both evenly spread in intervals of 6 cm. If Patrick is standing on this sheet, and he tosses a disc of radius 2 cm in the air, what is the probability that it does not intersect any line on the sheet when it lands?

A. $\frac{1}{3}$ B. $\frac{4}{9}$ C. $\frac{2}{3}$ D. $\frac{1}{9}$ E. NOTA

24. Suppose the Florida Marlins play the New York Mets in a best-of-3 series. (That is, the first team to win 2 games wins the series). There are no ties. If each game is independent, and the chance that the Marlins win any given game is 2/3, what is the probability that the Marlins win the series?

A. $\frac{2}{3}$ B. $\frac{26}{27}$ C. $\frac{20}{27}$ D. $\frac{8}{9}$ E. NOTA

25. Consider a bag containing one white marble and seven black marbles. If you draw two marbles from this bag without replacement, what is the probability that one of the two marbles you have drawn is the white marble?

A. $\frac{15}{56}$ B. $\frac{41}{56}$ C. $\frac{1}{56}$ D. $\frac{1}{7}$ E. NOTA

26. Every day, a candy store decides to either increase or decrease the previous day's price of chocolate bars by 10%. Each day, there is an equal chance of an increase or a decrease, and this is independent from the decisions on all other days. If on day 0, a chocolate bar costs $1.00, what is the probability that on day 4 a chocolate bar costs less than $1.00?

A. $\frac{11}{16}$ B. $\frac{1}{2}$ C. $\frac{5}{16}$ D. $\frac{7}{8}$ E. NOTA

27. An ant starts at the origin, and every minute it moves one unit either north, south, east, or west. What is the probability that, after 4 minutes (4 moves), the ant is back at the origin?

A. $\frac{3}{32}$ B. $\frac{9}{64}$ C. $\frac{3}{16}$ D. $\frac{1}{4}$ E. NOTA

28. If four pennies and a quarter are flipped, what is the probability that at least three of the pennies land on the same side (head or tails) as the quarter?

A. $\frac{1}{2}$ B. $\frac{1}{4}$ C. $\frac{5}{16}$ D. $\frac{11}{16}$ E. NOTA

29. Suppose that the probability that lightning hits a very tall building on any given day is 2/5. On any day, it can either rain or not rain. If the probability that lightning hits the building goes up to 3/5 given that it rains, but goes down to 3/10 given that it does not rain, what must be the probability of rain?

A. $\frac{1}{2}$ B. $\frac{1}{5}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$ E. NOTA

30. If X is randomly chosen from the integers {1,2,3,…,100}, what is the probability that X! (X factorial) ends with at least 7 zeroes?

A. 0.71 B. 0.7 C. 0.65 D. 0.66 E. NOTA