Question #1 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Using $9x^2 + 16y^2 + 18x = 64y + 71$:

Let A = the area of the ellipse.

Let B = the sum of the x-coordinates of the foci of the ellipse.

Let C = the product of the abscissa and the ordinate of the center of the ellipse.

Let D = the eccentricity of the ellipse.

Find the exact value of $A \bullet B \bullet C \bullet D$.

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Question #2 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let $R = \begin{vmatrix} 10 & 3 & 2 \\ -1 & 4 & 0 \\ 7 & -1 & 5 \end{vmatrix}$.

Let S = The factor by which the determinant of the matrix shown above will be multiplied if the elements in the first column are multiplied by 2.

Let T = the greatest integer such that 8^T is an integral factor of 44^{44}.

In triangle ABC, a median divides the triangle into two triangles of equal perimeter. The length of the median is 21 and the length of the side to which the median is drawn is 56. Let U be the perimeter of triangle ABC.

Find $R + S + T + U$.

Question #2 Alpha School Bowl
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Find $R + S + T + U$.

Question #3 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let (A, B) be the solution to $\begin{cases} \dfrac{A+3}{4}+\dfrac{B-1}{3}=1 \\ 2A-B=12 \end{cases}$.

Let (C, D) be the solution to $\begin{cases} \dfrac{12}{C}-\dfrac{12}{D}=7 \\ \dfrac{3}{C}+\dfrac{4}{D}=0 \end{cases}$.

Let E be the degree measure of the smallest possible positive angle for which

$$\sin x\cos E+\cos x\sin E=\frac{\sqrt{3}}{2}\text{, if } x=22^\circ.$$

Find $A \bullet B \bullet C \bullet D \bullet E$.

Question #3 Alpha School Bowl
2007 Mu Alpha Theta National Convention

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Find $A \bullet B \bullet C \bullet D \bullet E$.

Question #4 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let A = the degree of the polynomial $\left(x^3+1\right)^4\left(x^4+1\right)^5$.

Let B = the base 7 representation of 298.

Let C = the twelfth term of an arithmetic sequence whose sixth term is 20 and whose tenth term is 320.

Let $D = \begin{vmatrix} {}_7P_3 & 0 & 0 & 0 \\ 0 & \frac{1}{6!} & 0 & 0 \\ 0 & 0 & {}_4C_2 & 0 \\ 0 & 0 & 0 & {}_4P_2 \end{vmatrix}$.

Find $B-A-C-D$.

Question #4 Alpha School Bowl
2007 Mu Alpha Theta National Convention

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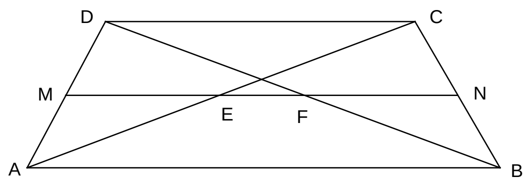
Find $B-A-C-D$.

Question #5 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let A = the product of m and n, if $\left(3x^3 + mx^2 - 5x + n\right)$ is divisible by both $(x-2)$ and $(x+1)$.

Let B = $(-1+i)^{-4}$.

Let C = the length of $\overline{EF}$ if $\overline{MN}$ is a median in trapezoid $ABCD$ where $DC = 8$ and $AB = 20$. The figure is not drawn to scale.



Let D = $a+b+c$ if the graph of $y = \dfrac{ax+b}{x-c}$ has y-intercept 1, vertical asymptote at 4, and horizontal asymptote at 2.

Find $A \bullet B \bullet C \bullet D$.

Question #5 Alpha School Bowl
2007 Mu Alpha Theta National Convention

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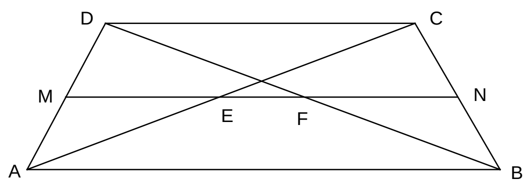

Let D = $a+b+c$ if the graph of $y = \dfrac{ax+b}{x-c}$ has y-intercept 1, vertical asymptote at 4, and horizontal asymptote at 2.

Find $A \bullet B \bullet C \bullet D$.

Question #6 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let A = the value of $\dfrac{\log_3 4 + \log_3 8 + \log_3 16 + \log_3 32}{\log_3 2}$.

Let B = the area of a quadrilateral inscribed in a circle if the sides of the quadrilateral measure 1, 5, 9, and 11. Find B^2.

Find C, if $\sqrt{0.005}$ is C % of $\sqrt{2}$.

Let D = the number of distinct integers that are in the solution set of $\dfrac{1}{3} \le \dfrac{x}{2007} \le \dfrac{4}{9}$.

Find the largest digit from your answers to A, B^2, C, and D.

Question #6 Alpha School Bowl
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Find the largest digit from your answers to A, B^2, C, and D.

Question #7 Alpha School Bowl
2007 Mu Alpha Theta National Convention

In the complete factorization of $x^7 + 125x^4 - x^3 - 125$ using only integral coefficients, the number of factors which are linear polynomials is *A*. Find *A*.

Find *B* if $4^B - 4^{B-1} = 24$.

Let *C* be the least integer such that $\log_5 |3x+7| \le 0$.

Find $A \bullet B \bullet C$.

Question #7 Alpha School Bowl
2007 Mu Alpha Theta National Convention

In the complete factorization of $x^7 + 125x^4 - x^3 - 125$ using only integral coefficients, the number of factors which are linear polynomials is *A*. Find *A*.

Find *B* if $4^B - 4^{B-1} = 24$.

Let *C* be the least integer such that $\log_5 |3x+7| \le 0$.

Find $A \bullet B \bullet C$.

Question #8 Alpha School Bowl
2007 Mu Alpha Theta National Convention

$A =$ the number of positive perfect squares will evenly divide 8400?

In the partial fraction decomposition for $\dfrac{x+7}{x^2-x-6}$, the partial fractions are $\dfrac{B}{x-3}+\dfrac{C}{x+2}$. Find B and C.

Find $A \div B \div C$.

Question #8 Alpha School Bowl
2007 Mu Alpha Theta National Convention

$A =$ the number of positive perfect squares will evenly divide 8400?

In the partial fraction decomposition for $\dfrac{x+7}{x^2-x-6}$, the partial fractions are $\dfrac{B}{x-3}+\dfrac{C}{x+2}$. Find B and C.

Find $A \div B \div C$.

Question #9 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let A = the distance from (3, 17) to the intersection of $y = 3x^2 - 2x + 5$ and $y = 4x + 2$. Find A^2.

Let $B = ab$, if $a^2 + b^2 = 181$ and $a + b = 19$.

Let C = the number of ordered triples of positive integers (x, y, z) that satisfy $\left(x^y\right)^z = 64$.

Find the sum of the units digits of the answers to A^2, B, and C.

Question #9 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let A = the distance from (3, 17) to the intersection of $y = 3x^2 - 2x + 5$ and $y = 4x + 2$. Find A^2.

Let $B = ab$, if $a^2 + b^2 = 181$ and $a + b = 19$.

Let C = the number of ordered triples of positive integers (x, y, z) that satisfy $\left(x^y\right)^z = 64$.

Find the sum of the units digits of the answers to A^2, B, and C.

Question #10 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let $\frac{A}{B} = \lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{x - 1}$, where A and B are relatively prime integers.

The area of triangle PET is 84 square inches and the geometric mean between sides $\overline{PE}$ and $\overline{PT}$ is 16 inches. Let $\frac{C}{D} = \sin P$, where C and D are relatively prime integers.

Find the sum of the digits in the answers to A, B, C, and D.

Question #10 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let $\frac{A}{B} = \lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{x - 1}$, where A and B are relatively prime integers.

The area of triangle PET is 84 square inches and the geometric mean between sides $\overline{PE}$ and $\overline{PT}$ is 16 inches. Let $\frac{C}{D} = \sin P$, where C and D are relatively prime integers.

Find the sum of the digits in the answers to A, B, C, and D.

Question #11 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let $A = \left[\cos x\right]$, where $\log_{\cos x} \sin x = \frac{1}{2}$, and $0° \le x < 90°$ and $[\ ]$ denotes the greatest integer function.

Let B = the numerical value of k, where the roots of $9x^2 - 2x + k = 0$ are $\sin u$ and $\cos u$.

Find $A + B$.

Question #11 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let $A = \left[\cos x\right]$, where $\log_{\cos x} \sin x = \frac{1}{2}$, and $0° \le x < 90°$ and $[\ ]$ denotes the greatest integer function.

Let B = the numerical value of k, where the roots of $9x^2 - 2x + k = 0$ are $\sin u$ and $\cos u$.

Find $A + B$.

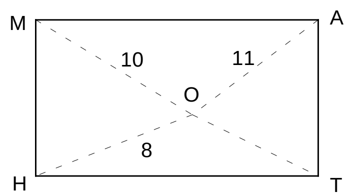
Question #12 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let A= the integer closest to $e+\pi$.

Let B = the number of zeros at the end of 40!.

Let C = twice the number of distinct real roots of $x^3+x-8=\frac{8}{x^2}$.

Let D = the length of $\overline{OT}$, to the nearest whole number, in the rectangle shown:



Find the mode of the set $\{A,\ B,\ C,\ D\}$

Question #12 Alpha School Bowl
2007 Mu Alpha Theta National Convention

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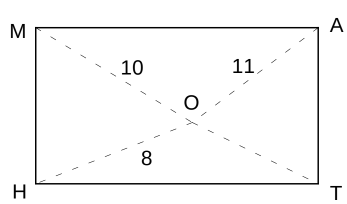

Find the mode of the set $\{A,\ B,\ C,\ D\}$

Question #13 Alpha School Bowl
2007 Mu Alpha Theta National Convention

A disoriented armadillo moves north, east, south, and west and repeats this movement continuously. It travels north 180 meters, and each subsequent move is one-third the distance of the previous move. If the armadillo's starting point is (20, 7), what is its final point of destination, (*A*, *B*)?

Let *C* be the solution to the equation $x^{\log x} = 100x$, where $x > 0$, with the largest absolute value.

Find $\dfrac{B-A}{C}$, written as a percent.

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2007 Mu Alpha Theta National Convention

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Find $\dfrac{B-A}{C}$, written as a percent.

Question #14 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let $A = x^2$ if $x = \sqrt{7+2\sqrt{6}} - \sqrt{7-2\sqrt{6}}$.

Let B = the smallest natural number such that B! is evenly divisible by 990.

Find $A+B$.

Question #14 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let $A = x^2$ if $x = \sqrt{7+2\sqrt{6}} - \sqrt{7-2\sqrt{6}}$.

Let B = the smallest natural number such that B! is evenly divisible by 990.

Find $A+B$.

Question #15 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let A = the smaller of two positive numbers, neither of which has 0 as a digit, that multiply to give 1,000,000,000.

Let B = the square number such that B + 43 is in the set $\{1,\ 4,\ 9,\ 16,\ 25,\ ...\}$.

Let C = the number of distinct arrangements of all the letters in the word *TAMPA*.

Find $A+B+C-1$.

Question #15 Alpha School Bowl
2007 Mu Alpha Theta National Convention

Let A = the smaller of two positive numbers, neither of which has 0 as a digit, that multiply to give 1,000,000,000.

Let B = the square number such that B + 43 is in the set $\{1,\ 4,\ 9,\ 16,\ 25,\ ...\}$.

Let C = the number of distinct arrangements of all the letters in the word *TAMPA*.

Find $A+B+C-1$.