1. Common difference = 36 – 10 = 26  B
2. Common ratio = \( \frac{\pi}{2} \div \frac{1}{2} = \pi \)  A
3. \( a_4 = \frac{2(4)}{4+2} = \frac{4}{3} \)  B
4. \( b_3 = \sin\left(\frac{3 \cdot \pi}{2}\right) = -1 \)  A

5. We can see that this sequence is increasing exponentially, and the ratio of sequential terms is getting closer to 3. \( c_n = 3^n - 2 \) fits this model  C

6. Let \( x = \sqrt{30} + \sqrt{30} + \sqrt{30} + \ldots \), it follows that \( x = \sqrt{30} + x \rightarrow (x-6)(x+5) = 0 \rightarrow x = 6 \) since it must be a positive number  C

\[
\sum_{k=4}^{9} (5k^2 - 2k + 1) = \sum_{k=1}^{9} (5k^2 - 2k + 1) - \sum_{k=1}^{3} (5k^2 - 2k + 1)
\]

7. \[
= \frac{5(9)(9+1)(2(9)+1)}{6} - \frac{2(9)(9+1)}{2} + 9 - \left( \frac{5(3)(3+1)(2(3)+1)}{6} - \frac{2(3)(3+1)}{2} + 3 \right) = 1283
\]

\[
1 > 3 \quad 3 > 2
\]

8. Order of differences \( 6 > 3 > 1 \) shows that \( T(n) \) is a 2\(^{nd}\) degree polynomial

\[
T(n) = an^2 + bn + c\,, \text{ solving simultaneously for } a, b, c:
\]

\[
1 = a + b + c
\]

\[
3 = 4a + 2b + c \rightarrow a = \frac{1}{2}, b = \frac{1}{2}, c = 0 \rightarrow T(n) = \frac{1}{2}n^2 + \frac{1}{2}n \rightarrow \sum_{n=1}^{7} T(n) = 84 \)  D

9. \( 34 – 6 = 28 \)  E

<table>
<thead>
<tr>
<th>n</th>
<th>a(_n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>34</td>
</tr>
</tbody>
</table>

10. Rewrite the equation as \( a_{k+1} - 3a_k = -2 \), giving the characteristic equation \( r - 3 = 0 \)
\( \rightarrow r = 3 \rightarrow a_k = p \cdot 3^k + q \)
\[
3 = 3p + q \quad \Rightarrow \quad p = \frac{2}{3}, \quad q = 1 \quad \Rightarrow \quad a_k = \frac{2}{3}(3^k) + 1 = 2(3^{k-1}) + 1. \quad 2 + 3 - 1 = 4 \quad \text{B}
\]

11. Total distance = \[15 + 10 + \frac{10}{3} + \frac{10}{9} \cdots = 15 + \frac{10}{1 - \frac{1}{3}}\] 12. \[2 \cdot \sum_{k=1}^{15} \sin\left(\frac{\pi \cdot k}{4}\right) \cos\left(\frac{\pi \cdot k}{4}\right) = \sum_{k=1}^{15} \sin\left(\frac{\pi \cdot k}{2}\right) = 0 \quad \text{B}
\]

13. Let \[x = 1 + \frac{1}{3 + \frac{1}{3 + \frac{1}{3 + \cdots}}} \quad \text{then} \quad x = 1 + \frac{1}{3 + \frac{1}{x}} = 1 + \frac{x}{3x + 1} \quad \Rightarrow \quad 3x^2 + x = 4x + 1 \quad \Rightarrow \quad 3x^2 - 3x - 1 = 0 \quad \Rightarrow \quad x = \frac{3 \pm \sqrt{21}}{6}.
\]

\[x \text{ must be positive so } x = \frac{3 + \sqrt{21}}{6} \quad \text{C} \]

14. Solving the equation for \(K\): \[50 = K\left(\frac{1}{2}\right)^{\frac{t}{2}} \quad \text{when} \quad t = 12 \quad \text{gives} \quad K = \frac{50}{\left(\frac{1}{2}\right)^{\frac{12}{2}}} = 100\sqrt{2} \quad \text{E}
\]

15. The last two digits follow a pattern:

<table>
<thead>
<tr>
<th>Number</th>
<th>Last two digits</th>
<th>The last digit is always 1, and the second to last goes through the sequence 2, 4, 6, 8, 0, 2, 4, 6, 8, 0, ...</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>21^2</td>
<td>41</td>
<td>So every 5th power gives the same final two digits. Powers of 5, 10, 15, ... 90, 95, 100 will all have 01 as the final two digits.</td>
</tr>
<tr>
<td>21^3</td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>21^4</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>21^5</td>
<td>01</td>
<td>The last two digits of 21^{100} are 01 C</td>
</tr>
<tr>
<td>21^6</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>21^7</td>
<td>41</td>
<td></td>
</tr>
</tbody>
</table>

16. This is the Fibonacci sequence. The next term is the sum of the previous two.
\[21 + 34 = 55 \quad \text{C} \]

17. \[499,000 = 49,900(1 - 0.55)^{\frac{t}{30}} \quad \Rightarrow \quad t = 30 \cdot \frac{\log(0.1)}{\log(0.45)} \quad \text{B}
\]

18. Using the Pythagorean theorem, \[1 + r^2 = r^4 \quad \Rightarrow \quad \]
\[
r^4 - r^2 - 1 = 0 \rightarrow \left(r^2\right)^2 - r^2 - 1 = 0 \rightarrow r^2 = \frac{1 + \sqrt{5}}{2}
\]

\[
\sin(\theta) = \frac{1}{r^2} = \frac{2}{1 + \sqrt{5}} = \frac{\sqrt{5} - 1}{2} \quad \text{B}
\]

19. We can see that for \( n > 3 \), 
\[
\begin{bmatrix}
0 & a & b \\
0 & 0 & c
\end{bmatrix}^n = 
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\]
So \( \left[X, 3\right] = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} = 0 \quad \text{B}
\]

20. The coefficient of \( x^a y^b \) is \( \frac{(a + b)!}{a!b!} \), so for \( x^{12} y^{13} \) the answer is \( \frac{25!}{13!12!} \quad \text{D} \)

\[
\sum_{n=0}^{k} (n \cdot n!) = (k + 1)! \rightarrow \sum_{n=1}^{k} (n \cdot n!) = (k + 1)! - 1 \rightarrow
\]

21. \sum_{n=2}^{500} (n \cdot n!) = (500 + 1)! - 1 - 1 = 50! - 2 \quad \text{A}

22. Rationalizing the denominators, we have 
\[
\left(\sqrt{2} - \sqrt{1}\right) + \left(\sqrt{3} - \sqrt{2}\right) + \left(\sqrt{4} - \sqrt{3}\right) + ... + \left(\sqrt{25} - \sqrt{24}\right) = 4 \quad \text{D}
\]

23. Only II and III are correct. \text{E}

24. Observe the sum for small values of \( n \), you will see that 
\[
\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}
\]

\[
\sum_{k=1}^{16} \frac{1}{k(k+1)} = \frac{16}{17} \quad \text{B}
\]

25. \( \frac{10^9}{9!} = \frac{10^{10}}{10!} \rightarrow a_0 = a_{10}. \) Only II and III are true. \text{B}

26. Using partial fraction decomposition, 
\[
\frac{2}{k^2 + 2k} = \frac{1}{k} - \frac{1}{k + 2}. \quad \text{For the series, all terms except for 1} + \frac{1}{2} \text{ are subtracted out.} \sum_{i=1}^{\infty} \frac{2}{k^2 + 2k} = \frac{3}{2} \quad \text{C}
\]

27. Order of differences shows that \( S(n) \) is a 3rd degree polynomial.
\[
\begin{align*}
1 > 4 > 6 & \quad 1 = a + b + c + d \\
5 > 10 > 9 > 3 & \quad 5 = 8a + 4b + 2c + d \\
15 > 19 > 12 > 3 & \quad 15 = 27a + 9b + 3c + d \\
34 > 31 > 15 > 3 & \quad 34 = 64a + 16b + 4c + d
\end{align*}
\]

\[
S(n) = \frac{n^3}{2} + \frac{n}{2}. \quad S(9) - S(8) = 109 \quad \text{C}
\]
28. Let \( d_A, d_B \) be the common differences of the two sequences. Then \( a_{11} = a_1 + 10d_A \) and \( b_{11} = b_1 + 10d_B \).

\[
\frac{a_{11}}{b_{11}} = \frac{a_1 + 10d_A}{b_1 + 10d_B}
\]

Using the formula for the sum of an arithmetic series:

\[
(4n + 27) \left( \frac{n}{2} \left( 2a_1 + (n-1)d_A \right) \right) = (7n + 1) \left( \frac{n}{2} \left( 2b_1 + (n-1)d_B \right) \right)
\]

\[
\frac{(2a_1 + (n-1)d_A)}{(2b_1 + (n-1)d_B)} = \frac{(7n + 1)}{(4n + 27)}
\]

Using \( n = 21 \) gives the ratio we are looking for:

\[
\frac{a_1 + 10d_A}{b_1 + 10d_B} = \frac{7(21) + 1}{4(21) + 27} = \frac{4}{3} \quad B
\]

29. Let \( a \) be the first number and \( b \) be the last number. \( a \) is 4005. We can see that \( b + 9 = 5004 \), so it follows that \( n = \frac{5004 - 4005}{9} = 111 \quad B \)

30. \( 2 = x^{3^{\ldots}} \rightarrow 2 = x^2 \rightarrow x = \sqrt{2} \quad A \)