a. Find the largest prime number that is less than the greatest common factor of 252, 308, and 504.

b. If $\left(a+b+c+d+e+f+g+h+j\right)^2$ is expanded and simplified, how many different terms are in the final answer?

c. Find x^2+1 if $x=\log_{\left(\frac{3^2\cdot 22222^2+99999^2}{11111^2\cdot 3^2}\right)}\left(2^2+3^2\right)^5$.

d. 2007 consecutive integers sum to 22,077. Find the 1004th integer in the sequence.

Question #1 Alpha State Bowl 2007 MAΘ National Convention

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d. 2007 consecutive integers sum to 22,077. Find the 1004th integer in the sequence.

a. What is the sum of all integers x for which $\frac{3x+25}{2x-5}$ is an integer?

b. Solve for x if $\begin{vmatrix} 1 & -2 & -3 \\ 7 & 5 & 0 \\ x & 2x & 1 \end{vmatrix} = -35$.

c. Write the equation in slope-intercept form of the line tangent to the circle $x^2 + y^2 + 4x - 46 = 0$ at the point (3, 5).

d. A palindrome number is a number that reads the same backwards as forwards, such as 31713. On a digital watch that does not show a leading 0, how many times during a 12-hour period will the projected time (neglecting the colon) be a palindrome number?

Question #2 Alpha State Bowl 2007 MAΘ National Convention

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a. Find the sum of all x for $\tan 2x + 2\sin x = 0$, $[0, 2\pi)$.

b. The superhero group Super Six recently bought a very small sports car to quickly assist innocent victims. Unfortunately, the car will only hold two people in the front seat and one person in the back seat. Among the Super Six, only two are legally able to drive. In how many ways can the car be legally filled by three of the Super Six, in order to get these three superheroes to their emergencies?

c. Find the positive solution to $3x^2 - 5x + \sqrt{3x^2 - 5x + 4} = 16$.

d. Evaluate $\dfrac{\log_3 5 \bullet \log_7 3 \bullet \log_2 7}{\log_{11} 5 \bullet \log_{17} 11 \bullet \log_8 17}$.

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a. A basketball player throws a basketball that follows the path $y = -(x-6)^2 + 36$, where *y* is the height above the ground and *x* is the distance from the player to the basket. If the ball went in the basket while on its downward arc, how far was the player from the basket if the height of the basket is 10 feet? (Both *x* and *y* are measured in feet.)

b. Simplify $\dfrac{\cos A \cot A - \sin A \tan A}{\csc A - \sec A}$, where $A \neq 45°$, in terms of sine and cosine only.

c. Find the fourth term in the expansion of $\left(\dfrac{\sqrt{x}}{y^2} - \dfrac{y}{\sqrt{x}}\right)^6$.

d. Simplify $(x-a)(x-b)(x-c)...(x-z)$.

Question #4 Alpha State Bowl 2007 MAΘ National Convention

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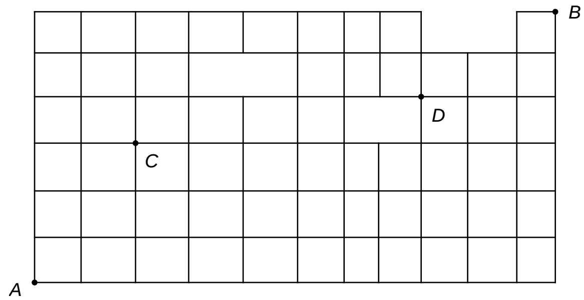
a. How many ways are there to go from point *A* to point *B* through *both* points *C* and *D*, moving only up or to the right? (You may only "turn" at lattice points.)



b. Write the expression $\text{Cot}^{-1}\frac{3}{7}-\text{Tan}^{-1}\frac{1}{4}$ as a single function in terms of Cos^{-1}.

c. Solve for $a+b$ **in terms of** ***x***: $\left(3^{9x+11}\right)\left(7^{5x+9}\right)\left(4^{10x+18}\right)=\left(336^{a}\right)\left(3^{b}\right)$.

d. Euclid High School is building a new wing for its math department. The shape of the building is a regular polygon. A student determined that the new building would have 299 diagonals. How many sides will the building have?

Question #5 Alpha State Bowl 2007 MAΘ National Convention

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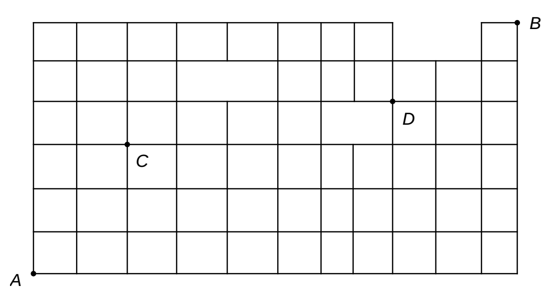

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a. Box *X* contains 5 gold trophies and 4 bronze trophies. Box *Y* contains 8 gold trophies and 11 bronze trophies. If 1 trophy is randomly drawn from box *X* and placed in box *Y*, and then a trophy is randomly drawn from box *Y*, what is the probability that the trophy drawn from box *Y* will be gold?

b. Find the number of even integer factors of 3,450.

c. Find *y* if $y = \sqrt{\sin\frac{\pi}{2} + \sqrt{\sin\frac{\pi}{2} + \sqrt{\sin\frac{\pi}{2} + \ldots}}}$.

d. How many nine-digit numbers in the form __ __ __ __ __ __ 4 __ __ are a multiple of 8 when the blanks are replaced with the eight digits 1, 2, 3, 5, 6, 7, 8, and 9 (in any order, and no number may be used more than once)?

Question #6 Alpha State Bowl 2007 MAΘ National Convention

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a. Two fair, 8-sided dice are rolled. What is the probability that the sum is prime, if the faces of each die are numbered 1 through 8?

b. How many integer pairs (x, y) satisfy the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$?

c. Find the smaller solution to $(x+3)\left(x-\frac{7}{6}\right)+\left(x-\frac{7}{6}\right)\left(x-\frac{14}{11}\right)=0$.

d. Find the product of the solutions to $x^{\log x}=1000x^2$, if $x > 0$.

Question #7 Alpha State Bowl 2007 MAΘ National Convention

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a. What is the largest prime that divides 87! + 88!?

b. If $\log 2 = M$, $\log 5 = N$, and $\log 7 = P$, express log78.4 in terms of *M*, *N*, and *P*.

c. An ellipse whose equation is $\frac{x^2\pi^2}{9} + \frac{y^2}{144} = 1$ is drawn on a football field. The number of people standing on the field is equal to the number of square units included in the area of the ellipse. If each person shakes hands with every other person once, how many handshakes occur?

d. Find the area of a 13-14-15 triangle.

Question #8 Alpha State Bowl 2007 MAΘ National Convention

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a. Find the degree measure of *x* for which $3\tan\frac{x}{2}+3=0$, using the interval $0° < x < 360°$.

b. Find both solutions (*x*, *y*), where *x* and *y* are positive integers, such that *x*! = 20*y*!

c. Find all possible values for $x+y+z$ if $x^2+y^2+z^2=xy+xz+yz=3$.

d. In what base is $44 \times 55 = 3506$?

Question #9 Alpha State Bowl 2007 MAΘ National Convention

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Question #10 Alpha State Bowl 2007 MAΘ National Convention

a. Find the product of the values of *x* for which $5-\frac{1}{x}=\sqrt{5-\frac{1}{x}}$.

b. Determine the maximum value *xy* from the pairs of integers (*x*, *y*) that satisfy the equation $6x^2-3xy-13x+5y=-11$. (There are only two possible pairs.)

c. Find the least positive angle *x*, in radians, for which $3^{\sin^2 x}\bullet 3^{\cos^2 x}\bullet 3^{\tan^2 x}=3^2$.

d. What is the maximum number of regions, not necessarily congruent, into which the interior of a circle can be divided by four chords?

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