

1. **(B)** All choices but (B) are based on elementary or sine or tangent sum identities. Choice (B) should have a minus instead of a plus, based on the identity  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$ .
2. **(A)** Any polar graph with an equation of the form  $r = a \pm b \cos \theta$  with  $|a| < |b|$  is a limaçon with an inner loop. This can easily be confirmed graphically.
3. **(C)** The graph of  $r = a \cos(n\theta)$  has  $n$  petals when  $n$  is odd and  $2n$  petals when  $n$  is even.
4. **(C)** Only (C) is true: the range of  $\text{Arccot } x$  is  $[0, \pi)$ .
5. **(A)**  $\sin \theta = \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{20^2}{29^2}} = \sqrt{1 - \frac{400}{841}} = \sqrt{\frac{441}{841}} = \frac{21}{29}$   
 $\tan \theta + \sec \theta = \frac{\sin \theta + 1}{\cos \theta} = \frac{\frac{21}{29} + 1}{-\frac{20}{29}} = \frac{-21 - 29}{20} = -\frac{5}{2}$   
 $a - b = -5 - 2 = -7$
6. **(B)** Using the half-angle identity  $\cos \theta = \pm \sqrt{\frac{1 + \cos 2\theta}{2}}$ ,  
 $\cos 15^\circ = \sqrt{\frac{1 + \cos 30^\circ}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \frac{1}{2} \sqrt{2 + \sqrt{3}} \Rightarrow \left(\frac{c}{a}\right)^{\frac{b}{a}} = (3 \cdot 2)^{2 \cdot 2} = 1296$   
Sum of digits =  $1 + 2 + 9 + 6 = 18$
7. **(B)** Area =  $\frac{1}{2} ab \sin C = \frac{1}{2}(14)(26) \sin 30^\circ = \frac{1}{2}(14)(26)\left(\frac{1}{2}\right) = 91 = 91\sqrt{1} \Rightarrow ab = (91)(1) = 91 \Rightarrow$   
Sum of digits =  $9 + 1 = 10$
8. **(B)** The period of  $f(x)$  is the smallest  $\lambda$  such that  $f(x) = f(x + \lambda)$ . Since it is known that the smallest such  $\lambda$  for  $g(x) = \cos x$  is  $2\pi$  and  $f(x) = g(2007\pi x)$ , we need  $2007\pi\lambda = 2\pi \Rightarrow \lambda = \frac{2}{2007}$ .
9. **(A)** Since we have the lengths of two adjacent sides,  $x$  and  $y$ , we need only the measure of the included angle  $\angle ABQ$  to compute the area of the parallelogram as  $xy \sin(\angle ABQ)$ . Using the law of sines within triangle  $ABQ$ ,  
 $\frac{\sin(\angle BAQ)}{y} = \frac{\sin(\angle AQB)}{x} \Rightarrow \frac{z}{y} = \frac{\sin(\angle AQB)}{x} \Rightarrow \angle AQB = \text{Arcsin}\left(\frac{xz}{y}\right)$   
 $\angle ABQ + \angle BAQ + \angle AQB = \angle ABQ + z + \text{Arcsin}\left(\frac{xz}{y}\right) = \pi \Rightarrow \angle ABQ = \pi - z - \text{Arcsin}\left(\frac{xz}{y}\right)$   
Area of  $ABQT = xy \sin(\angle ABQ) = xy \sin\left(\pi - z - \text{Arcsin}\left(\frac{xz}{y}\right)\right) = xy \sin\left(z + \text{Arcsin}\left(\frac{xz}{y}\right)\right)$
10. **(E)**  $\cot^2 x + 1 = \csc^2 x \Rightarrow \cot^2 x - 1 = \csc^2 x - 2$
11. **(D)** Four of the functions ( $\sin x$ ,  $\tan x$ ,  $\csc x$ , and  $\cot x$ ) are odd functions, which means they satisfy the condition  $f(-x) = -f(x)$ .  $\frac{4}{6} = \frac{2}{3}$ .
12. **(C)**  $2007 > 2008 - 3 \cot^2 \theta \Rightarrow 3 \cot^2 \theta > 1 \Rightarrow |\cot \theta| > \frac{1}{\sqrt{3}} \Rightarrow |\tan \theta| < \sqrt{3}$ , which is true for  $\theta \in [0, \frac{\pi}{3}) \cup (\frac{2\pi}{3}, \frac{4\pi}{3}) \cup (\frac{5\pi}{3}, 2\pi)$ , a total length of  $\frac{4\pi}{3}$  out of  $2\pi$ .

13. **(D)**  $(-2007, 6267^\circ) = (-2007, 17 \cdot 360^\circ + 147^\circ) = (-2007, 147^\circ) = (2007, 327^\circ)$
14. **(B)** The minimum value of a function of the form  $f(x) = a \sin x + b \cos x$  is  $-\sqrt{a^2 + b^2}$ .
15. **(D)** By the law of cosines,  
 $c^2 = a^2 + b^2 - 2ab \cos C = 15^2 + 16^2 - 2(15)(16)(-\frac{1}{2}) = 225 + 256 + 240 = 721$   
 Sum of digits =  $7 + 2 + 1 = 10$
16. **(A)**  $\cos \theta = -\sqrt{1 - \sin^2 \theta} = -\sqrt{1 - (\frac{3}{5})^2} = -\frac{4}{5} \Rightarrow \cot \theta = \frac{\cos \theta}{\sin \theta} = -\frac{4}{3}$
17. **(D)** (A) and (B) do not guarantee  $\cos \alpha = \cos \beta$ , and (C) does not guarantee that both values be in the interval  $[0, 2\pi)$ . (D) guarantees both.
18. **(D)** The minimum value of the sine function is  $-1$  and its maximum value is  $1$ , so the minimum value of  $f(x)$  is  $-2007(1) + 2007 = 0$  and its maximum value is  $-2007(-1) + 2007 = 4014$ .
19. **(D)** By the cosine sum formula,  $\cos(3\theta) \cos(28\theta) - \sin(3\theta) \sin(28\theta) = \cos(3\theta + 28\theta) = \cos(31\theta)$
20. **(C)**  $f(\frac{3\pi}{4}) - f(\frac{7\pi}{6}) = (\cos \frac{3\pi}{4} - \sin \frac{3\pi}{4}) - (\cos \frac{7\pi}{6} - \sin \frac{7\pi}{6}) = (-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}) - (-\frac{\sqrt{3}}{2} + \frac{1}{2}) = \frac{\sqrt{3}}{2} - \sqrt{2} - \frac{1}{2}$
21. **(B)** The coordinates represent the Cartesian points  $(1, 1)$  and  $(1, -1)$ , respectively, which are separated by a distance of  $2$ .
22. **(A)** Let  $\vec{u} = i + 2j$  and  $\vec{v} = 3i - 5j$ . Then  $\cos^2 \theta = (\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|})^2 = (\frac{1 \cdot 3 + 2 \cdot (-5)}{\sqrt{1^2 + 2^2} \sqrt{3^2 + (-5)^2}})^2 = \frac{(-7)^2}{5 \cdot 34} = \frac{49}{170}$   
 $a + b = 49 + 170 = 219 \Rightarrow$  Sum of digits =  $2 + 1 + 9 = 12$
23. **(C)** The period of  $\sin(\frac{4x}{5})$  is  $2\pi \cdot \frac{5}{4} = \frac{5\pi}{2}$ , and the period of  $\cos(\frac{x}{3})$  is  $2\pi \cdot 3 = 6\pi$ . The least common multiple of these two periods is  $30\pi$ .
24. **(A)** The range of  $\sin x$  is  $[-1, 1]$ ; no value of  $x$  can produce a sine of  $\frac{3}{2}$ .
25. **(A)** Since the range of  $\tan x$  is all real numbers, so is the domain of  $f(x) = \text{Arctan } x$ .
26. **(A)**  $\cot x \cos 2x + \sin 2x = \frac{\cos x}{\sin x}(\cos^2 x - \sin^2 x) + 2 \cos x \sin x = \frac{\cos^3 x}{\sin x} - \cos x \sin x + 2 \cos x \sin x$   
 $= \frac{\cos^3 x}{\sin x} + \cos x \sin x = \frac{\cos^3 x + \cos x \sin^2 x}{\sin x} = \frac{\cos x(\cos^2 x + \sin^2 x)}{\sin x} = \frac{\cos x}{\sin x} = \cot x$
27. **(B)** Since  $\alpha$  and  $\beta$  sum to  $\pi$ ,  $\cos \alpha = \cos(\pi - \beta) = -\cos \beta$  and  $\sin \alpha = \sin(\pi - \beta) = \sin \beta$ .  
 (A), (C), and (D) follow directly from these relations; (B) contradicts  $\sin \alpha = \sin \beta$ .
28. **(D)**  $(\cos \theta + \sin \theta)^3 = (\cos \theta + \sin \theta)(\cos \theta + \sin \theta)^2 = (\cos \theta + \sin \theta)(\cos^2 \theta + 2 \cos \theta \sin \theta + \sin^2 \theta)$   
 $= (\cos \theta + \sin \theta)(1 + \sin 2\theta)$

29. **(B)**  $\cos \alpha = \frac{7}{8}$  and  $0 < \alpha < \frac{\pi}{2}$ , so  $\sin \alpha = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \frac{49}{64}} = \frac{\sqrt{15}}{8}$ .  
 $\sin \beta = \frac{12}{13}$  and  $\frac{\pi}{2} < \beta < \pi$ , so  $\cos \beta = -\sqrt{1 - \sin^2 \beta} = -\sqrt{1 - \frac{144}{169}} = -\frac{5}{13}$ .  
 $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = \left(\frac{7}{8}\right)\left(-\frac{5}{13}\right) + \left(\frac{\sqrt{15}}{8}\right)\left(\frac{12}{13}\right) = \frac{12\sqrt{15}-35}{104}$   
 $a + b + q + t + 90 = 12 + 15 + 35 + 104 + 90 = 256 \Rightarrow$  Sum of digits =  $2 + 5 + 6 = 13$
30. **(B)**  $\sin \pi = 0$