Unless otherwise specified, equations are to be solved over the real numbers.

A capitalized inverse trigonometric function (e.g. Arcsin) denotes that the range of the function is restricted to its traditional subset of $\mathbb{R}$ in order that the function return a unique output for each input.

Answer choice (E) NOTA means “none of these answers.”

No calculators are permitted on this test.

1. Which of the following is not a trigonometric identity (where both sides are defined)?
   (A) $1 - \sin^2 \alpha = \cos^2 \alpha$
   (B) $\cos(\alpha + \beta) - \sin \alpha \sin \beta = \cos \alpha \cos \beta$
   (C) $\sin(\alpha + \beta) - \sin \alpha \cos \beta = \cos \alpha \sin \beta$
   (D) $\tan \alpha = \frac{1}{2}(1 - \tan^2 \alpha) \tan 2\alpha$
   (E) NOTA

2. Which of the following best describes the polar graph of $r = 3 - 28 \cos \theta$?
   (A) Limacon with an inner loop
   (B) Cardioid
   (C) Dimpled limacon
   (D) Lemniscate
   (E) NOTA

3. How many petals does the polar graph of $r = 7 \cos(2007\theta)$ have?
   (A) 7
   (B) 14
   (C) 2007
   (D) 4014
   (E) NOTA

4. Which of the following is true?
   (A) $\arctan(-\sqrt{3}) = \frac{2\pi}{3}$
   (B) $\arctan 0 = \frac{\pi}{2}$
   (C) $\arccot(-\sqrt{3}) = \frac{5\pi}{6}$
   (D) $\arccot(-\sqrt{3}) = -\frac{\pi}{6}$
   (E) NOTA

5. An angle $\theta$ with its terminal side in the second quadrant has a cosine of $-\frac{20}{29}$. If $\tan \theta + \sec \theta = \frac{a}{b}$ with $a$ and $b$ relatively prime integers and $b > 0$, find $a - b$.
   (A) $-7$
   (B) $-3$
   (C) 3
   (D) 7
   (E) NOTA

6. $\cos 15^\circ$ can be written as $a\sqrt{b + \sqrt{c}}$ with $b$ and $c$ integers with no square factors other than 1. Find the sum of the digits of $(\frac{a}{b})^2$.
   (A) 9
   (B) 18
   (C) 19
   (D) 27
   (E) NOTA

7. In triangle $ABC$, side $a$ has length 14, side $b$ has length 26, and angle $C$, opposite side $c$, has measure $30^\circ$. If the area of triangle $ABC$ can be written as $a\sqrt{b}$ with $b$ an integer with no square factors other than 1, find the sum of the digits of the product $ab$.
   (A) 9
   (B) 10
   (C) 11
   (D) 12
   (E) NOTA
8. What is the period of the function $f(x) = \cos(2007\pi x)$?
   (A) $\frac{1}{2007}$  (B) $\frac{2}{2007}$  (C) $\frac{1}{2007\pi}$  (D) $\frac{2}{2007\pi}$  (E) NOTA

9. In parallelogram $ABQT$, the side connecting $A$ to $B$ has length $x$, the side connecting $B$ to $Q$ has length $y$, and the sine of angle $BAQ$ is $z$. Which of the following is equivalent to the area of parallelogram $ABQT$?
   (A) $xy \sin(z + \arcsin(\frac{xz}{y}))$  (B) $xy \sin(\arcsin z + \arcsin(\frac{xz}{y}))$
   (C) $xy \sin(z + \arcsin(\frac{xz}{y}))$  (D) $xy \sin(z + \arcsin(\frac{xz}{y}))$
   (E) NOTA

10. Where defined, $\cot^2 x - 1$ is equivalent to which of the following?
     (A) $\csc x$  (B) $\csc^2 x$  (C) $1 - \csc x$  (D) $2 \cos^2 x$  (E) NOTA

11. If one of the six trigonometric functions $\sin x$, $\cos x$, $\tan x$, $\sec x$, $\csc x$, or $\cot x$ is chosen at random, what is the probability that for the chosen function $f$, $f(x) + f(-x) = 0$ for any $x$ in the domain of $f$?
     (A) $\frac{1}{6}$  (B) $\frac{1}{3}$  (C) $\frac{1}{2}$  (D) $\frac{2}{3}$  (E) NOTA

12. If a value of $\theta$ is chosen at random from $(0, 2\pi)$, what is the probability that $2007 > 2008 - 3 \cot^2 \theta$?
     (A) $\frac{1}{6}$  (B) $\frac{1}{3}$  (C) $\frac{2}{3}$  (D) $\frac{5}{6}$  (E) NOTA

13. In which quadrant does the point with polar coordinates $(-2007, 6267^\circ)$ lie?
     (A) I  (B) II  (C) III  (D) IV  (E) NOTA

14. What is the minimum value of the function $f(x) = 2007 \cos x - 2007 \sin x$?
     (A) $-4014$  (B) $-2007\sqrt{2}$  (C) $-2007$  (D) $0$  (E) NOTA

15. In triangle $ABC$, side $a$ has length 15, side $b$ has length 16, and the measure of angle $C$, opposite side $c$, has measure $120^\circ$. What is the sum of the digits in the square of the length of side $c$?
     (A) 7  (B) 8  (C) 9  (D) 10  (E) NOTA

16. If $\sin \theta = \frac{3}{5}$ and the terminal side of $\theta$ lies in the second quadrant, find $\cot \theta$.
     (A) $-\frac{4}{3}$  (B) $-\frac{2}{5}$  (C) $\frac{3}{4}$  (D) $\frac{4}{3}$  (E) NOTA

17. If $\alpha$ and $\beta$ are distinct values in the interval $[0, 2\pi)$ and $\cos \alpha = \cos \beta$, which of the following must be true?
     (A) $\alpha - \beta = \pi$  (B) $|\alpha - \beta| = \pi$  (C) $\alpha - \beta = 2\pi$  (D) $\alpha + \beta = 2\pi$  (E) NOTA
18. What is the range of the function \( f(x) = -2007 \sin(2007x + 4014) + 2007 \)?
   (A) \([-4014, 4014]\)  (B) \([-2007, 2007]\)  (C) \([0, 2007]\)  (D) \([0, 4014]\)  (E) NOTA

19. Which of the following is equivalent to \( \cos(3\theta) \cos(28\theta) - \sin(3\theta) \sin(28\theta) \)?
   (A) \(\sin(25\theta)\)  (B) \(\cos(25\theta)\)  (C) \(\sin(31\theta)\)  (D) \(\cos(31\theta)\)  (E) NOTA

20. If \( f(x) = \cos x - \sin x \), find \( f\left(\frac{3\pi}{4}\right) - f\left(\frac{7\pi}{6}\right) \).
   (A) \(\sqrt{2} - \frac{1}{2}\)  (B) \(\frac{1}{2} - \sqrt{2}\)  (C) \(\sqrt{2} - \sqrt{2} - \frac{1}{2}\)  (D) \(\frac{1}{2} - \sqrt{2} - \sqrt{2}\)  (E) NOTA

21. What is the distance between the polar coordinates \((\sqrt{2}, 45^\circ)\) and \((\sqrt{2}, 315^\circ)\)?
   (A) \(\sqrt{2}\)  (B) 2  (C) \(2\sqrt{2}\)  (D) 270  (E) NOTA

22. If the square of the cosine of the smaller angle between the vectors \(i + 2j\) and \(3i - 5j\) is \(\frac{a}{b}\) with \(a\) and \(b\) relatively prime natural numbers, find the sum of the digits of \(a + b\).
   (A) 12  (B) 13  (C) 14  (D) 15  (E) NOTA

23. What is the period of the function \( f(x) = \sin\left(\frac{4x}{5}\right) + \cos\left(\frac{x}{3}\right) \)?
   (A) \(5\pi\)  (B) \(15\pi\)  (C) \(30\pi\)  (D) \(60\pi\)  (E) NOTA

24. How many values of \( x \) are there in \([0, 2\pi]\) such that \( \sin x = \frac{3}{2} \)?
   (A) 0  (B) 2  (C) 4  (D) Infinitely many  (E) NOTA

25. What is the domain of \( f(x) = \arctan x \)?
   (A) Real numbers  (B) \(\{x \mid x \neq \frac{(2n+1)\pi}{2}, \text{for all integers } n\}\)
   (C) \(\{x \mid x \neq \frac{n\pi}{2}, \text{for all integers } n\}\)  (D) \(\{x \mid |x| \leq \frac{\pi}{2}\}\)
   (E) NOTA

26. Where defined, \( \cot x \cos 2x + \sin 2x \) is equivalent to which of the following?
   (A) \(\cot x\)  (B) \(\tan x\)  (C) \(\cos x\)  (D) \(\csc x\)  (E) NOTA

27. If \( \alpha \) and \( \beta \) are the measures of two consecutive angles of a parallelogram, which of the following is not true?
   (A) \(\cos \alpha + \cos \beta = 0\)  (B) \(\sin \alpha + \sin \beta = 0\)
   (C) \(\sin^2 \alpha - \sin^2 \beta = \cos^2 \alpha - \cos^2 \beta\)  (D) \(\sin 2\alpha = \sin \beta \cos \alpha - \sin \alpha \cos \beta\)
   (E) NOTA
28. Which of the following is equivalent to \((\cos \theta + \sin \theta)^3\)?
   (A) \(\cos \theta + \sin \theta\) \hspace{1cm} (B) \(\cos^3 \theta + \sin^3 \theta\)
   (C) \((\cos \theta + \sin \theta)(1 - \frac{1}{2}\sin 2\theta)\) \hspace{1cm} (D) \((\cos \theta + \sin \theta)(1 + \sin 2\theta)\)
   (E) NOTA

29. Suppose \(\cos \alpha = \frac{7}{8}\), \(\sin \beta = \frac{12}{13}\), and \(0 < \alpha < \frac{\pi}{2} < \beta < \pi\). If \(\cos(\alpha - \beta)\) is expressed as \(\frac{a\sqrt{b} - q}{t}\),
   with \(a, b, q, t\) natural numbers with no common divisor and \(a\sqrt{b}\) in its most simplified form,
   find the sum of the digits of \(a + b + q + t + 90\).
   (A) 12 \hspace{1cm} (B) 13 \hspace{1cm} (C) 14 \hspace{1cm} (D) 15 \hspace{1cm} (E) NOTA

30. Evaluate: \(\sin \pi\)
   (A) \(-1\) \hspace{1cm} (B) 0 \hspace{1cm} (C) \(\frac{\sqrt{2}}{2}\) \hspace{1cm} (D) 1 \hspace{1cm} (E) NOTA