For all questions, choice E) NOTA denotes **N**one **O**f **T**he **A**bove are correct.
The use of calculators is not permitted.

1) Find the greatest common divisor of 1,596 and 2,907.
A) 1 B) 19 C) 57 D) 153 E) NOTA

2) Given the following multiplication problem of two 2-digit numbers where *A*, *B*, and *C* are digits and *A* and *B* are nonzero, find the sum of all possible values of *C* for which the multiplication holds true.

$$\begin{array}{cccc} & & A & A \\ \times & & B & A \\ \hline A & C & C & A \end{array}$$

A) 5 B) 9 C) 12 D) 15 E) NOTA

3) Determine the value of *a* for any nonnegative integer, *n*, such that $10^n \equiv a \pmod{9}$
A) 0 B) 1 C) 8 D) 9 E) NOTA

4) All primes that can be written in the form $p = 2^{2^n} + 1$ where *n* is a positive integer are called which of the following?
A) Mersenne primes
B) Primorial primes
C) Gaussian primes
D) Fermat primes
E) NOTA

5) Which of the following numbers are NOT prime?
I) 299 II) 399 III) 499 IV) 899
A) II only B) I and II only C) II and III only D) I, II, and IV
E) NOTA

6) Find the sum of all possible positive integers, *n*, such that n^2 is between 10^3 and 11^3.
A) 165 B) 170 C) 175 D) 201 E) NOTA

7) What is the ten's digit of 2007^{2007}?
A) 0 B) 1 C) 4 D) 9 E) NOTA

8) For how many positive integers, *n*, less than or equal to 30 is $(n-1)!$ divisible by *n*?
A) 10 B) 11 C) 19 D) 20 E) NOTA

9) Let the number of positive divisors of a positive integer *n* be denoted as $d(n)$. For example, $d(12)=6$ since the integers 1, 2, 3, 4, 6, and 12 are all positive divisors of 12. For how many integers, n, between 1 and 1000, inclusive, does $d(n)=3$?
A) 11 B) 12 C) 13 D) 14 E) NOTA

10) Find the largest positive integer, n, such that n! is NOT congruent to $0 \pmod{200}$.
A) 10 B) 50 C) 100 D) 200 E) NOTA

11) The n^{th} term of a given sequence is $a_n = n^n$. What is the unit's digit of $\sum_{n=1}^{10} a_n$?
A) 3 B) 4 C) 7 D) 8 E) NOTA

12) Find the number of zeros at the end of $(2007!)^2$.
A) 499 B) 500 C) 999 D) 1,000 E) NOTA

13) Given a finite group, G, the *order* of an element a, denoted $ord(a)$, is defined to be the smallest positive integer, m, such that $a^m = 1$. For example, let G_1 be the set of integers $\{1,2,...,10\}$ under multiplication mod 11. Then $ord(10) = 2$ because $10^2 \equiv 1 \pmod{11}$. How many elements in G_1 have an order of 3?
A) 0 B) 2 C) 4 D) 6 E) NOTA

14) Start with any positive integer (ex. 4,229,301) and count the number of even digits, E, the number of odd digits, O, and their sum, $E + O$. Create a new number by concatenating the three values (in the order E, O, $E+O$) into one number and dropping any leading 0's (for our example it would be 437). For any starting number, what is the most accurate description of the behavior of repeated application of this procedure?
A) Enters a nontrivial cycle for all starting numbers.
B) May enter a nontrivial cycle or converge depending upon the starting number.
C) Converges to a number dependent upon the starting number.
D) Converges to the same number for all starting numbers.
E) NOTA

15) Let F_n denote the sequence of Fibonacci numbers with $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$. For how many Fibonacci numbers less than 2007 is ${F_n}^2 + 1$ divisible by 10?
A) 3 B) 4 C) 5 D) 6 E) NOTA

16) Each positive integer is coded using the following process: encode 1 as *a*, 2 as *b*, 3 as *c*, and continuing so that 26 is coded as *z*. After 26, encode 27 as *aa*, 28 as *ab*, and so forth. What is the code for 2007?
A) *bay* B) *bug* C) *bye* D) *bzi* E) NOTA

17) Find the sum of all odd squares less than 2007.
A) 14,190 B) 15,180 C) 16,114 D) 16,552 E) NOTA

18) Find the sum of $723_8 + 124_5$ in base 9.
A) 506_9 B) 622_9 C) 812_9 D) 1141_9 E) NOTA

19) Find the greatest common divisor of 100! and 102!.
A) 2 B) 100 C) 10200 D) 102! E) NOTA

20) How many pairs of positive integers (m, n) satisfy the equation
$$\frac{1}{m!} + \frac{1}{n!} = \frac{1}{20} \text{ ?}$$
A) 0 B) 1 C) 2 D) 3 E) NOTA

21) An item in a store has 4% sales tax, which turns out to be an exact number of cents (no rounding necessary). The total price including sales tax happens to be an exact number of dollars. The smallest possible pre-tax cost can be written as \$*AB*.*CD* where *A*, *B*, *C*, and *D* digits. Find $A + B + C + D$.
A) 8 B) 10 C) 15 D) 18 E) NOTA

22) For the given problem, let *p* be a positive prime and *a* be any positive integer. How many of the following statements are NOT true?
I) If $a \equiv -1 \pmod{p}$ and $p \geq 3$, then $a \equiv 0 \pmod{2}$
II) If $a \equiv -1 \pmod{p}$ then $a^p \equiv -1 \pmod{p}$ for all possible *p*.
III) $(p-1)! \equiv -1 \pmod{p}$ for all possible *p*
IV) If $\gcd(a, p) = 1$ then $a^p \equiv a \pmod{p}$

A) 0 B) 1 C) 2 D) 3 E) NOTA

23) Let *n* be the smallest possible positive integer such that $7056n$ is a perfect cube. Find the sum of the digits of *n*.
A) 3 B) 9 C) 12 D) 15 E) NOTA

24) In base-factorial you express a positive integer, *b*, as $b = a_k a_{k-1} \ldots a_2 a_1$ if $b = a_k \cdot k! + a_{k-1} \cdot (k-1)! + \ldots + a_2 \cdot 2! + a_1 \cdot 1!$ where a_n are integers and $0 \leq a_n < (n+1)!$ for all $1 \leq n \leq k$.
Find 4155 in base-factorial.
A) 498011 B) 537411 C) 523111 D) 543011 E) NOTA

25) Consider the following process, denoted by the function $g(n)$. First, we take the number n and round up to the smallest multiple of $n-1$ greater than or equal to n. Call this new number X. Now we round X up to the smallest multiple of $n-2$ greater than or equal to X. The process continues until rounding up to the smallest multiple of 1. For example $g(6)=12$. If we started with 6, we'd round up to $10=2(5)$, then to $12=3(4)$, then to $12=4(3)$ again, then to $12=6(2)$ again, and finally to $12=12(1)$. Find the value of $g(15)$.

A) 60 B) 78 C) 82 D) 96 E) NOTA

26) David decided to have some fun with math one day (Don't we all??). He decided to write down all integers from 0 to 999 on a sheet of paper. He then began to eliminate numbers using the following criteria. He first eliminated all integers that had a digit of 1 anywhere in the number. Then from the remaining numbers he eliminated all numbers that had a digit of 2 anywhere, but not a 3 (such as 425). This means that if the digit had a 3 (such as 234) he kept it. How many numbers did he end up crossing out?

A) 440 B) 446 C) 456 D) 485 E) NOTA

27) Given the following recursion relation:

$a_n=(n+1)a_{n-1}$ for $n\geq 1$ $\quad a_1=1$

What is an explicit formula for a_n?

A) $a_n=\frac{n!}{2}$ B) $a_n=n!$ C) $a_n=\frac{(n+1)!}{2}$ D) $a_n=(n+1)!$ E) NOTA

28) Find the time that a clock reads 2007 minutes after 20:07 in military time.

A) 3:27 B) 5:34 C) 8:54 D) 23:40 E) NOTA

29) Given that x is a positive integer less than 100, find the sum of all possible values of x such that $28x\equiv 2\pmod{54}$.

A) 58 B) 96 C) 144 D) 170 E) NOTA

30) Find the least common multiple of 56 and 72.

A) 168 B) 252 C) 504 D) 1,008 E) NOTA