

1. A 2—Region is bounded by the parabola & the 2 lines  $y = \pm \frac{3}{2}x$ . Area =  $2 \int_0^2 \left( \frac{3}{8}x^2 + \frac{3}{2}x - \frac{3}{2}x \right) dx$
- B  $\frac{1}{2}$ —The graphs intersect at  $x = 0, 1$  and  $-1$ . Area =  $\int_{-1}^0 (x^3 - x) dx + \int_0^1 (x - x^3) dx = \frac{1}{2}$ .
- C 3—The  $x$ -intercepts of the parabola are  $x = 0$  and  $x = 2k$ . Since the area is in the 4<sup>th</sup> quadrant, it is found by  $\int_0^{2k} (-x^2 + 2kx) dx = 36$  and solving gives  $k = 3$ .
- D 8—The area =  $\int_2^k \frac{dx}{x} = \ln 4$  and solving gives  $k = 8$ .
2. A  $\frac{2\pi}{3}$ —Area =  $\frac{1}{2} \pi \left( \frac{1}{2} y \right)^2 = \frac{\pi}{8} y^2 \Rightarrow$  Volume =  $\int_0^4 \frac{\pi}{8} \left( \frac{4-x}{2} \right)^2 dx = \frac{2\pi}{3}$ .
- B  $36\sqrt{3}$ —Area =  $\sqrt{3}y^2 = \sqrt{3}(9-x^2) \Rightarrow$  Volume =  $\sqrt{3} \int_{-3}^3 (9-x^2) dx = 36\sqrt{3}$ .
- C  $\frac{\pi}{3}$ —Volume =  $2 \int_0^{\frac{1}{2}} \frac{1}{2} \pi y^2 dx = \int_0^{\frac{1}{2}} \pi(1-4x^2) dx = \frac{\pi}{3}$ .
- D  $\frac{283}{5}$ —Volume =  $\int_0^2 (3(x-2)^2)^2 dx = \frac{283}{5}$ .
3. A 4--  $f'(x) = 6x^2 - 18x + 12 = 0 @ x = 1, 2$ .  $f(-1) = -24$ ,  $f(1) = 4$  &  $f(2) = 3 \Rightarrow y = 4$  is the maximum.
- B 2 & -2--  $f'(x) = 4x^3 - 16x = 0 @ x = 0, 2, -2$ . Testing these values on a sign chart shows the answer.
- C 1--  $f'(x) = 24x^5 - 24x^2 = 0 @ x = 0$  &  $1$ . Testing these values on a sign chart shows the answer.
- D 2 -  $\ln 4$ --  $f'(x) = e^x - 2 = 0 @ x = \ln 2$  (which is a min). So,  $f(\ln 2)$  gives the answer.
4. A  $2\ln 105$ —The areas of the rectangles are  $2\ln 3 + 2\ln 5 + 2\ln 7 = 2\ln 105$ .
- B 5—Only 2 rectangles on  $[0, 4]$  can be inscribed under  $f(x) = (x-3)^2$ . Each rectangle has  $w = 1$  and the ht. of the 1<sup>st</sup> one is 4 and the height of the 2<sup>nd</sup> one is 1. So,  $1(4 + 1) = 5$  is the answer.
- C  $\frac{89}{20}$ — $T = \frac{1}{2} \cdot 1 [0 + 2(1.1) + 2(1.4) + 2(1.2) + 1.5] = \frac{8.9}{2} = \frac{89}{20}$ .
- D 26—The areas of the rectangles are  $2 \cdot 1 + 2 \cdot 2 + 2 \cdot 10 = 2 + 4 + 20 = 26$ .
5. A  $\frac{-10,000\pi}{30}$ —Use the info to find  $k = 10,000 \cdot 40^2$ .  $L = \frac{10,000 \cdot 40^2}{\left( \frac{40}{\cos \theta} \right)^2} = 10,000 \cos^2 \theta$ . Differentiating
- and using  $\theta = \frac{\pi}{4}$  and  $\frac{d\theta}{dt} = \frac{\pi}{30}$  gives  $\frac{dL}{dt} = 2 \cdot 10,000 \cos \frac{\pi}{4} \left( -\sin \frac{\pi}{4} \right) \left( \frac{\pi}{30} \right) = \frac{-1,000\pi}{3}$ .
- B  $800\pi$ — $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(10)^2 \cdot 2 = 800\pi$ .
- C 0.6—Area =  $A(w) = \int_0^w (24x^2 - 12x^3) dx \Rightarrow \frac{dA}{dt} = (24w^2 - 12w^3) \frac{dw}{dt} = (24 \cdot 1 - 12 \cdot 1)(0.05) = 0.6$
- D 8— $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow 12 = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{3}{\pi r^2}$ .
- And  $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$ . At  $V = 36\pi$ ,  $r = 3$ , so  $\frac{dS}{dt} = 8\pi(3) \left( \frac{3}{\pi \cdot 9} \right) = 8$ .

6. A  $t < 1$  or  $t > 3$  --  $v(t) = 4t(t-3)^2 \Rightarrow v'(t) = 4(t-3)(3t-3)$  and  $v'(t) > 0$  when  $t < 1$  or  $t > 3$ .  
 B  $t = 0, 2, \& 4$ —The avg. veloc. is change in position  $\div$  by change in time. For avg. veloc. to = 0, then the change in distance must = 0. Set the position function = 0 and solve to get the answer.
- C  $\frac{1}{e-1}$  -- Avg. Veloc. = change in position  $\div$  by change in time =  $\frac{\ln e - \ln 1}{e-1} = \frac{1}{e-1}$ .
- D  $\frac{7}{2}$  --  $v(t) = \frac{1}{2} \cos t - 2 \sin(2t) \Rightarrow a(t) = -\frac{1}{2} \sin t - 4 \cos(2t) \Rightarrow a\left(\frac{\pi}{2}\right) = \frac{7}{2}$ .
7. A 0 --  $y = (\sin x)^x \Rightarrow \ln y = x \ln(\sin x) \Rightarrow \frac{1}{y} y' = \frac{x \cos x}{\sin x} + \ln(\sin x) \Rightarrow y' \left(\frac{\pi}{2}\right) = 0$ .
- B  $\frac{-1}{y^3}$  --  $y' = \frac{-x}{y} \Rightarrow y'' = \frac{y(-1) - (-x)(y')}{y^2} = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2} = \frac{-x^2 - y^2}{y^3} = \frac{-1}{y^3}$ .
- C 20 --  $w = u \circ v \Rightarrow w' = u'(v) \cdot v' \Rightarrow w'(0) = u'(v(0)) \cdot v'(0) = u'(2) \cdot 5 = 20$ .
- D 6 --  $y = x^4 + x^2 + 1 \Rightarrow dy = (4x^3 + 2x)dx = (4+2) \cdot 1 = 6$ .
8. A  $-1 \leq x < 5$  --  $R = \lim_{n \rightarrow \infty} \left| \frac{(x-2)^{n+1}}{3^{n+1}(n+1)} \cdot \frac{3^n \cdot n}{(x-2)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{n(x-2)}{3(n+1)} \right| = \left| \frac{x-2}{3} \right| \Rightarrow -1 < x < 5$ . Check endpts (-1 works)
- B 5 --  $R = \lim_{n \rightarrow \infty} \left| \frac{(n+2)x^{n+1}}{5^{n+1}(n^2)} \cdot \frac{5^n \cdot (n-1)^2}{(n+1)x^n} \right| = \left| \frac{x}{5} \right| < 1 \Rightarrow |x| < 5$ .
- C  $\frac{-5^{22}\sqrt{2}}{2(22!)}$  --  $f(0) = \frac{\sqrt{2}}{2} \Rightarrow f'(0) = \frac{5\sqrt{2}}{2} \Rightarrow f''(0) = \frac{-25\sqrt{2}}{2} \Rightarrow f'''(0) = \frac{-5^3\sqrt{2}}{2}$ . Following the pattern yields the answer.
- D  $\frac{2}{9}$  --  $\lim_{x \rightarrow 0} \frac{f(x) - \frac{1}{3}}{x} = \lim_{x \rightarrow 0} \left( \frac{2}{3^2} + \frac{3x}{3^3} + \frac{4x^2}{3^4} + \dots \right) = \frac{2}{9}$ .
9. A  $\frac{3}{2}$  --  $\int_0^1 x^{-1/3} dx = \lim_{a \rightarrow 0^+} \int_a^1 x^{-1/3} dx = \lim_{a \rightarrow 0^+} \frac{3}{2} x^{2/3} \Big|_a^1 = \frac{3}{2}$ .
- B  $\pi$  --  $\int_0^\infty \frac{dx}{\sqrt{x}(x+1)} = \lim_{a \rightarrow 0^+} \int_a^1 \frac{dx}{\sqrt{x}(x+1)} + \lim_{b \rightarrow \infty} \int_1^b \frac{dx}{\sqrt{x}(x+1)} = \lim_{a \rightarrow 0^+} 2 \tan^{-1} \sqrt{x} \Big|_a^1 + \lim_{b \rightarrow \infty} 2 \tan^{-1} \sqrt{x} \Big|_1^b = \pi$ .
- C  $\ln 2$  --  $\int_{-\infty}^0 \frac{e^x}{e^x+1} dx = \lim_{a \rightarrow -\infty} \int_a^0 \frac{e^x}{e^x+1} dx = \lim_{a \rightarrow -\infty} \ln(e^x+1) \Big|_a^0 = \ln 2$ .
- D div. -- Integral =  $\lim_{a \rightarrow -\infty} \int_a^0 \frac{e^x}{e^x+1} dx + \lim_{b \rightarrow \infty} \int_0^b \frac{e^x}{e^x+1} dx = \lim_{a \rightarrow -\infty} \ln(e^x+1) \Big|_a^0 + \lim_{b \rightarrow \infty} \ln(e^x+1) \Big|_0^b = \ln(e^\infty - 1) \Rightarrow \text{div.}$
10. A  $\sqrt{5}$  -- Separating variables gives  $\int y dy = \int \frac{\ln x}{x} dx \Rightarrow \frac{1}{2} y^2 = \frac{1}{2} \ln^2 x + c \Rightarrow c = 2 \Rightarrow y(e) = \sqrt{5}$ .
- B  $e^e$  -- Separating variables gives  $\int \frac{dy}{y} = dx \Rightarrow \ln|y| = x + c \Rightarrow c = 0 \Rightarrow y(e) = e^e$ .
- C  $\frac{10}{3}$  -- Separating variables gives  $\int dy = \int x^2 dx \Rightarrow y = \frac{x^3}{3} + c \Rightarrow c = 3 \Rightarrow y(1) = \frac{10}{3}$ .
- D  $e^3$  -- Separating variables gives  $\int \frac{\ln y}{y} dy = \int \frac{dx}{x} \Rightarrow \frac{1}{2} \ln^2 y = \ln|x| + c \Rightarrow c = \frac{1}{2} \Rightarrow y(e^4) = e^3$ .