1. $r=\sqrt{3}, \quad \pi r^2 = 3\pi \quad B$

2. $\frac{x^2}{\frac{1}{4}}+\frac{y^2}{\frac{1}{4}}=1, \quad ab\pi=\frac{1}{2}\bullet\frac{1}{2}\pi=\frac{\pi}{4} \quad C$

3. The asymptotes are $y=\pm\frac{2}{3}x$. $c=\sqrt{a^2+b^2}=\sqrt{9+4}=\sqrt{13}$. So the region is the triangle bounded by the asymptotes and the line $x=\sqrt{13}$. So we have

$A=\frac{1}{2}bh=\frac{1}{2}\left(2\bullet\frac{2\sqrt{13}}{3}\right)\left(\sqrt{13}\right)=\frac{26}{3} \quad A$

4. $\frac{1}{4p}=\frac{1}{32}, \; p=8$. So the directrix is x = -8. The axis of symmetry is the x-axis. So the region is a triangle with $A=\frac{1}{2}bh=\frac{1}{2}(16)(16)=128 \quad C$

5. Completing the square yields

$x^2-4x+4+3\left(y^2+8y+16\right)=-46+4+48$

$(x-2)^2+3(y+4)^2=6$

$\frac{(x-2)^2}{6}+\frac{(y+4)^2}{2}=1$

So $X=\frac{c}{a}, \; Y=2a, \quad XY=2c=2\sqrt{a^2-b^2}=2\sqrt{6-2}=4 \quad D$.

6. The equation of our parabola can be $y=ax^2+b$ where $a<0$. Since the tunnel is 32 feet wide, the parabola passes through the points (16, 0) and (-16, 0). So we have $0=256a+b \quad \Rightarrow b=-256a$. Four feet from the center being 60 feet tall gives us the point (4, 60), and using the equation $60=16a+b$. $60=16a-256a \Rightarrow a=-\frac{1}{4} \Rightarrow b=64$.

Our equation is $y=-\frac{1}{4}x^2+64$. We want where x = 12, so $-\frac{1}{4}(12)^2+64=28 \quad C$

7. The center of the ellipse is (-3, -1). The distance from the center to the focus gives us c, so c = 4. The ellipse being tangent to the y-axis gives us b = 3. And $a^2=b^2+c^2$, so a = 5. So our equation is $\frac{(x+3)^2}{9}+\frac{(y+1)^2}{25}=1$. Expanding gives us

$25\left(x^2+6x+9\right)+9\left(y^2+2y+1\right)=225$

$25x^2+150x+225+9y^2+18y+9-225=0$

$25x^2+9y^2+150x+18y+9=0$

So the answer is D

8. Completing the square gives

$x^2-10x+25+y^2-8y+16=-37+25+16$

$\left(x-5\right)^2+\left(y-4\right)^2=4$

We want the distance from (2, 0) to (5, 4) minus the radius of the circle, or

$\sqrt{\left(5-2\right)^2+4^2}-2=3 \quad A$

9. With the two vertices given, the length of that side is 4. That gives us 16 – 4 = 12 units left for the triangle. But this is the sum of the distances between two fixed points being a constant, so it is an ellipse. (2, 0) and (-2, 0) are the foci. So c = 2, and 2a = 12, so a = 6. So $36=b^2+4 \Rightarrow b=4\sqrt{2}$. So our equation is $\frac{x^2}{36}+\frac{y^2}{32}=1$, of which the area is

$6\bullet 4\sqrt{2}\pi=24\sqrt{2}\pi \quad A$.

10. Using the points as x and y in the equation of the circle, we have

$1+1+\mu+\alpha+\theta=0$

$9+16+3\mu+4\alpha+\theta=0$

$4+36-2\mu+6\alpha+\theta=0$

Adding these equations gives $2\mu+11\alpha+3\theta=-67 \quad B$

11. $\tan 2\theta=\frac{B}{A-C}=\frac{4}{10-6}=1 \Rightarrow 2\theta=\frac{\pi}{4}=\frac{\pi}{8} \quad C$

12. $r=\frac{abc}{4A}=\frac{(4)(4)(6)}{4\sqrt{7(3)(3)(1)}}=\frac{24}{\sqrt{63}}$. $\pi r^2=\pi\frac{24\bullet 24}{3\bullet 3\bullet 7}=\frac{64\pi}{7} \quad D$

13. Not being a circle, the points must be collinear.

$\frac{4-2k}{3k-1}=\frac{6k-4}{5-3k} \Rightarrow 20-12k-10k+6k^2=18k^2-12k-6k+4$

$12k^2+4k-16=0$

$3k^2+k-4=0$

$(3k+4)(k-1)=0$

$k=-{}^{4}\!/_{3},1 \quad C$

14. Completing the square on the parabola yields $y=\left(x-3\right)^2$. $\frac{1}{4p}=1,\ 4p=1$. The eccentricity of any parabola is always 1, and the eccentricity of any circle is always zero. So 1 + 1 + 0 = 2. B

15. E. A point is when the plane is horizontal straight through the center where the cones meet. A hyperbola is when the plane is vertical through both cones. A parabola is where the plane slices one of the cones on an angle coming out of the base. An ellipse is almost horizontal through one cone, but at an angle so it becomes an ellipse.

16. A unit vector is a vector of length 1, so we divide by the magnitude. The magnitude of the given vector is 3, so we divide the given vector's components by 3. B

17. If the vectors are parallel, then they are scalar multiples of each other. D

18. If the vectors are perpendicular, then the dot product is zero. $17g51+18g6-a=0$ C

19. $2-2-1=-1 \quad D$

20. $\langle 3-5,-10+1,10+1\rangle = \langle -2,-9,9\rangle \quad \left\|\vec{a}\right\| = \sqrt{4+81+121} = \sqrt{206} \quad B$

21. $\dfrac{-4-2-16+1}{\sqrt{4+4+16}} = \dfrac{21}{\sqrt{24}} = \dfrac{21\sqrt{24}}{24} = \dfrac{7\sqrt{6}}{4} \quad B$

22. Works the same way by completing the square.

$$x^2-8x+16+y^2+18y+81+z^2-20z+100=-189+16+81+100$$

$$(x-4)^2+(y+9)^2+(z-10)^2=8$$

$$r=\sqrt{8};\ \frac{4}{3}\pi r^3=\frac{4}{3}\pi 8\sqrt{8}=\frac{64\pi\sqrt{2}}{3} \quad C$$

23. The direction vector of the line is $\langle 6+6,12+3\rangle=\langle 12,15\rangle$. So the point will be

$$(-6,-3)+\frac{1}{3}\langle 12,15\rangle=(-2,2) \quad D$$

24. The normal vector gives the coefficient of x, y, and z. So using the point given:

$$-29x+26y-19z+D=0$$

$$(-29)(-1)+26(-2)-19(-3)+D=0$$

$$D=-34 \Rightarrow -29x+26y-19z-34=0 \quad C$$

25. We have a triangle with side lengths $\sqrt{34}$, $\sqrt{101}$, and $\sqrt{36+169}=\sqrt{205}$. Using the law of cosines, we have $101=205+34-2\sqrt{205 \cdot 34}\cos\theta$, or $\dfrac{69}{\sqrt{6970}}=\cos\theta$ B

26. A

27. Using Cartesian to polar conversions:

$$\left(x^2+y^2-ax\right)^2=a^2\left(x^2+y^2\right)$$

$$\left(r^2-ar\cos\theta\right)^2=a^2r^2$$

$$r^2-ar\cos\theta=ar$$

$$r-a\cos\theta=a$$

$$r=a+a\cos\theta$$

B

28. D

$$x=2^t;\ \ y=2^{-t}$$

$\ln x=t\ln 2;\quad \ln y=-t\ln 2$ Add both equations

$$\ln x+\ln y=0$$

$$\ln(xy)=0\Rightarrow xy=1$$

29. We need to have it in the form $r=\dfrac{a}{1+e\cos\theta}$, where e is the eccentricity. We divide the numerator and denominator by b of the original equation to get $r=\dfrac{\frac{a}{b}}{1+\frac{c}{b}\cos\theta}$. The eccentricity of this conic is $\frac{c}{b}$, and since $b>c>0 \Rightarrow 1>\frac{c}{b}>0$, the graph is an ellipse. B

30. Using polar to Cartesian conversions, we have

$$5\sqrt{x^2+y^2}=6-y$$
$$25\left(x^2+y^2\right)=y^2-12y+36$$
$$25x^2+24y^2+12y=36$$
$$25x^2+24\left(y^2+\frac{1}{2}y+\frac{1}{16}\right)=36+\frac{3}{2}$$
$$25x^2+24\left(y+\frac{1}{4}\right)^2=\frac{75}{2}$$
$$\frac{2}{3}x^2+\frac{16}{25}\left(y+\frac{1}{4}\right)^2=1$$
$$\frac{x^2}{\frac{3}{2}}+\frac{\left(y+\frac{1}{4}\right)^2}{\frac{25}{16}}=1$$
$$A=ab\pi=\sqrt{\frac{3}{2}}\bullet\frac{5}{4}\pi=\frac{5\pi\sqrt{6}}{8}\quad D$$