E. NOTA means "None of the Above"
For this test, assume ellipses are non-circular. All curves, lines, points, and shapes lie in the Cartesian Plane.

1. What is the area bounded by the graph of $x^2 + y^2 = 3$?

A) $\sqrt{3}\pi$ B) 3π C) 9π D) -1 E) NOTA

2. What is the area bounded by the graph of $4x^2 + 4y^2 = 1$?

A) 4π B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{16}$ E) NOTA

3. What is the area bounded by the asymptotes of $\frac{x^2}{9} - \frac{y^2}{4} = 1$ and the line $x = c$, where $(c, 0)$ is a focus of the given hyperbola (use $c > 0$)?

A) $\frac{26}{3}$ B) $\frac{13}{3}$ C) $\frac{39}{2}$ D) 39 E) NOTA

4. For the parabola $x = \frac{1}{32}y^2$, let l_1 be its axis of symmetry and let l_2 be its directrix. What is the area bounded by l_1, l_2, and the line $x + y = 8$?

A) 300 B) 256 C) 128 D) 64 E) NOTA

5. Given the equation $x^2 + 3y^2 - 4x + 24y + 46 = 0$, if X is the eccentricity and Y is the length of the major axis, give XY.

A) 1 B) 2 C) 4 D) 6 E) NOTA

6. Mrs. Sowers is traveling to Nationals and passes through a tunnel. The tunnel has cross sections perpendicular to the ground in the shape of congruent parabolas. The tunnel is 32 feet wide, and at a point 4 feet from the center, the tunnel is 60 feet tall. How tall is the tunnel at a point 12 feet from the center (in feet)?

A) 33.75 B) 30 C) 28 D) 27.5 E) NOTA

7. Which of the following is the equation of the ellipse tangent to the y-axis with foci (-3, 3) and (-3, -5)?

A) $25x^2+9y^2+50x+35y-80=0$ B) $9x^2+25y^2+54x+50y-119=0$
C) $9x^2+25y^2+45x+75y-201=0$ D) $25x^2+9y^2+150x+18y+9=0$
E) NOTA

8. What is the shortest distance between the point (2, 0) and $x^2+y^2-10x-8y+37=0$?

A) 3 B) 2.5 C) 2 D) 1 E) NOTA

9. Two vertices of a triangle are (2, 0) and (-2, 0). Given that the perimeter of the triangle is 16, find the equation that contains the locus of points that could be the third vertex. Give the area bounded by the graph of the equation of this locus.

A) $24\sqrt{2}\pi$ B) 24π C) $16\sqrt{2}\pi$ D) 16π E) NOTA

10. The circle that passes through the points (1, 1), (3, 4), and (-2, 6) has equation $x^2+y^2+\mu x+\alpha y+\theta=0$. What is $2\mu+11\alpha+3\theta=0$?

A) -68 B) -67 C) -66 D) -65 E) NOTA

11. What is the acute angle needed to rotate $10x^2+4xy+6y^2+5x+\sqrt{3}y+1=0$ in order to eliminate the xy term?

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{8}$ D) $\frac{\pi}{16}$ E) NOTA

12. What is the area of the circle circumscribed about a triangle with edge lengths of 4, 4, and 6?

A) 9π B) $\frac{9\pi}{7}$ C) $\frac{16\pi}{7}$ D) $\frac{64\pi}{7}$ E) NOTA

13. Find all k so that $(1,2k)$, $(3k,4)$, and $(5,6k)$ cannot determine 3 points on a circle.

A) $2, -\frac{2}{3}$ B) $\frac{1}{3}, \frac{3}{5}$ C) $1, -\frac{4}{3}$ D) $\frac{2}{3}, \frac{6}{5}$ E) NOTA

14. Let A equal the length of the latus rectum of $x^2-6x-y+9=0$. Let B equal the eccentricity of the given parabola. Let C equal the eccentricity of $x^2+y^2=1$. What is $A+B+C$?

A) 1 B) 2 C) 3 D) 4 E) NOTA

15. Given a double-napped cone and a plane, which of the following **cannot** be their intersection (given that the nappes are infinite)?

A) Point B) Hyperbola C) Parabola D) Ellipse E) NOTA

16. Give a unit vector in the same direction of the vector $\langle 1,2,2 \rangle$.

A) $\left\langle \frac{1}{9}, \frac{2}{9}, \frac{2}{9} \right\rangle$

B) $\left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \right\rangle$

C) $\left\langle \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{\sqrt{5}}{5} \right\rangle$

D) $\langle 1,1,1 \rangle$

E) NOTA

17. If the vectors $17\vec{i} + 6\vec{j} - \vec{k}$ and $51\vec{i} + 18\vec{j} + a\vec{k}$ are parallel, what is a?

A) -6 B) -5 C) -4 D) -3 E) NOTA

18. If the vectors $17\vec{i} + 6\vec{j} - \vec{k}$ and $51\vec{i} + 18\vec{j} + a\vec{k}$ are perpendicular, what is a?

A) 965 B) 970 C) 975 D) 980 E) NOTA

19. $\left(\vec{i} - 2\vec{j} + \vec{k}\right) \cdot \left(2\vec{i} + \vec{j} - \vec{k}\right) =$

A) 2 B) 1 C) 0 D) -1 E) NOTA

20. $\left(3\vec{i} - 10\vec{j} + 10\vec{k}\right) - \left(5\vec{i} - \vec{j} - \vec{k}\right) = \vec{a}$. Give the magnitude of $\vec{a}$.

A) $\sqrt{246}$ B) $\sqrt{206}$ C) $\sqrt{216}$ D) $\sqrt{236}$ E) NOTA

21. What is the shortest distance between (-2, -1, -4) and $2x + 2y + 4z + 1 = 0$?

A) $\frac{11\sqrt{2}}{2}$ B) $\frac{7\sqrt{6}}{4}$ C) $\frac{22\sqrt{3}}{6}$ D) $\frac{11\sqrt{3}}{3}$ E) NOTA

22. Give the volume of the sphere $x^2 + y^2 + z^2 - 8x + 18y - 20z + 189 = 0$.

A) $\frac{16\pi}{3}$ B) $\frac{32\pi}{3}$ C) $\frac{64\pi\sqrt{2}}{3}$ D) $\frac{48\pi\sqrt{2}}{3}$ E) NOTA

23. Given the line segment between the points A(-6, -3) and B(6, 12), what is the point on this line segment $\frac{1}{3}$ of the way from A to B?

A) (1, 5.75) B) (0, 4.5) C) (-1, 3.25) D) (-2, 2) E) NOTA

24. Give the equation of a plane with normal vector $-29\vec{i} + 26\vec{j} - 19\vec{k}$ containing the point (-1, -2, -3).

A) $x + 14y + 37z + 140 = 0$
B) $28x + y + 3z + 38 = 0$
C) $-29x + 26y - 19z - 34 = 0$
D) $22x + 20y - 20z + 2 = 0$
E) NOTA

25. If $\vec{u} = 5\vec{i} + 3\vec{j}$ and $\vec{v} = \vec{i} + 10\vec{j}$, give the secant of the angle between $\vec{u}$ and $\vec{u} + \vec{v}$.

A) $\frac{\sqrt{641}}{3}$ B) $\frac{\sqrt{6970}}{69}$ C) $\frac{\sqrt{6970}}{34}$ D) $\frac{\sqrt{641}}{69}$ E) NOTA

26. What is the graph of the polar equation $r^2 = a\cos(2\theta)$?

A) Lemniscate B) Cardioid C) Parabola D) Hyperbola E) NOTA

27. For $a \neq 0$, what is the best term for the graph of $\left(x^2 + y^2 - ax\right)^2 = a^2\left(x^2 + y^2\right)$?

A) Lemniscate B) Cardioid C) Parabola D) Hyperbola E) NOTA

28. The graph of the parametric equation $x = 2^t$; $y = 2^{-t}$ is part of a graph of a…

A) Lemniscate B) Cardioid C) Parabola D) Hyperbola E) NOTA

29. What is graph of the polar equation $r = \frac{a}{b + c\cos(-\theta)}$, where $a > b > c > 0$, and a, b, c are natural numbers?

A) Circle B) Ellipse C) Parabola D) Hyperbola E) NOTA

30. What is the area bounded by the graph of the polar equation $5r - 6 = -r\sin\theta$?

A) $\frac{5\pi\sqrt{3}}{2}$ B) $\frac{5\pi\sqrt{3}}{4}$ C) $\frac{5\pi\sqrt{6}}{2}$ D) $\frac{5\pi\sqrt{6}}{8}$ E) NOTA