

NOTA means "None of the Above." For this test, assume that ellipses are non-circular.

1. Find the distance between the points (1, 3) and (18, 3).

- A) 3 B) 17 C) 0 D) $\sqrt{17}$ E) NOTA

2. Let the slope of the line $3x + 4y = 5$ equal A, and let the y-intercept be (0, B). $A + B =$

- A) 0.5 B) 0.75 C) $1.\bar{3}$ D) 2.3 E) NOTA

3. Give the equation of the line that passes through the points (260, 221) and (400, 81).

- A) $2x + 3y = 600$ B) $3x - y = 962$ C) $x + y = 481$
D) $x - 2y = 50$ E) NOTA

4. Find the distance between the x- and y-intercepts of $Ax + By + C = 0$, for $A \neq B \neq 0$.

- A) $\left| \frac{C}{AB} \right| \sqrt{A^2 + B^2}$ B) C^2 C) 1 D) $A^2 + B^2$ E) NOTA

The next three questions concern points $A(-5, 10)$ and $B(1, 4)$, and the line l that passes through them.

5. Give the equation of the line perpendicular to l that passes through the midpoint of \overline{AB} .

- A) $y = x + 5$ B) $y = 2x + 5$ C) $y = x$ D) $y = x + 9$ E) NOTA

6. What is the acute angle that l makes with the x-axis?

- A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{7}$ E) NOTA

7. Choose the point on line segment AB three-quarters of the way from A to B.

- A) $\left(-\frac{3}{4}, \frac{23}{4}\right)$ B) $\left(-\frac{3}{5}, \frac{28}{5}\right)$ C) $\left(-\frac{1}{2}, \frac{11}{2}\right)$ D) $\left(-\frac{1}{4}, \frac{21}{4}\right)$ E) NOTA

The next four questions concern the parabola $x^2 + 6x - 4y + 13 = 0$ and the circle $x^2 + y^2 - 6x + 12y + 29 = 0$.

8. What are the coordinates of the focus of the parabola?

- A) (-3, 5) B) (3, 5) C) (3, 3) D) (-3, 2) E) NOTA

9. What is the radius of the circle?

- A) 4 B) 8 C) 12 D) 16 E) NOTA

10. Give the equation of the directrix of the parabola.

- A) $y = 0$ B) $y = 1$ C) $y = 2$ D) $y = 3$ E) NOTA

11. A line segment is drawn starting at the vertex of the parabola and ending tangent to the circle. What is its length?

- A) $\sqrt{61}$ B) $\sqrt{67}$ C) $\sqrt{69}$ D) $\sqrt{71}$ E) NOTA

12. What is the equation of the graph $x^2 + y^2 + 18x - 18y + 162 = 0$?

- A) line B) circle C) point D) plane E) NOTA

13. $\int_{-10}^{13} ||x-1|-9| dx =$

- A) $\frac{175}{2}$ B) 81 C) 86 D) $\frac{171}{2}$ E) NOTA

14. For $a, b, c > 0$, given that $(0, 0)$, $(a, 0)$, and (c, b) are the coordinates of a parallelogram, which of the following could be the fourth vertex?

- I. $(a+c, b)$ II. $(a+c, a+b)$ III. $(a, b+c)$ IV. $(a, -b)$ V. $(a-c, -b)$

- A) IV B) II, IV C) I, V D) IV, V E) NOTA

15. Give the cross product of the vectors $3\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$ and $4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$.

- A) $\mathbf{i} + 14\mathbf{j} + 37\mathbf{k}$ B) $28\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ C) $-29\mathbf{i} + 26\mathbf{j} - 19\mathbf{k}$
D) $22\mathbf{i} + 20\mathbf{j} - 20\mathbf{k}$ E) NOTA

16. Consider the polygon formed when the points $(-5, 3)$, $(7, 11)$, $(11, 4)$, and $(-1, -4)$ are connected in order. Which of the following terms is the *most specific* name of the given polygon?
- A) Quadrilateral B) Trapezoid C) Rhombus D) Parallelogram E) NOTA
17. The plane $-29x + 26y - 19z - 34 = 0$ intersects another plane with equation $x + z = 0$ in a line. Give the direction vector of this line. (The cross product of the normal vectors of the planes give the direction vector of the line).
- A) $26\mathbf{i} - 48\mathbf{j} + 26\mathbf{k}$ B) $26\mathbf{i} + 10\mathbf{j} - 26\mathbf{k}$ C) $19\mathbf{i} + 26\mathbf{j} + 4\mathbf{k}$
D) $\mathbf{i} + \mathbf{k}$ E) NOTA
18. Give the distance between the polar points $\left(1, \frac{\pi}{4}\right)$ and $(1, \pi)$.
- A) $\sqrt{2 + \sqrt{2}}$ B) $\sqrt{2}$ C) $\sqrt{5}$ D) $\frac{3\pi}{4}$ E) NOTA
19. Give the area of the region given by the intersection of the interiors of the polar graphs $r = a \cos \theta$ and $r = a \sin \theta$.
- A) $\frac{a^2}{64}(\pi - 8)$ B) $\frac{a^2}{4}(\pi - 8)$ C) $a^2(\pi - 8)$ D) $\frac{a^2}{8}(\pi - 2)$ E) NOTA
20. Give the coordinates of the point of intersection of the asymptotes of the hyperbola:
 $13x^2 - 7y^2 + 26x + 14y - 85 = 0$.
- A) $(-1, 1)$ B) $(1, 1)$ C) $(1, -1)$ D) $(-1, -1)$ E) NOTA
21. Give the equation that contains the locus of points twice as far from $(-1, 1)$ as from $(4, 5)$ in the xy -plane.
- A) $x^2 + y^2 - 34x - 18y + 80 = 0$
B) $x^2 + y^2 + 12x + 6y - 37 = 0$
C) $3x^2 + 3y^2 - 34x - 38y + 162 = 0$
D) $3x^2 + 3y^2 + 18x + 2y - 25 = 0$
E) NOTA

22. Give the area of the region between the graph of $|x| + |y| = 4$ and the circle inscribed within $|x| + |y| = 4$.
- A) $32 - 8\pi$ B) $16 - 8\pi$ C) $16 - 4\pi$ D) $32 - 4\pi$ E) NOTA
23. An equilateral triangle and a circle are drawn that don't overlap. They have a combined area of $\sqrt{3} + \pi$. If the side of the triangle and radius of the circle are natural numbers, what is the combined perimeter of the triangle and the circle?
- A) $3 + 2\pi$ B) $6 + 2\pi$ C) $12 + \pi$ D) $12 + 2\pi$ E) NOTA
24. Give the cosecant of the angle between the vectors $\langle 1, 1, 1 \rangle$ and $\langle -1, 1, 1 \rangle$.
- A) $\frac{3\sqrt{2}}{2}$ B) $\frac{\sqrt{2}}{3}$ C) $\frac{3\sqrt{2}}{4}$ D) $\frac{3\sqrt{2}}{8}$ E) NOTA
25. What is the graph of the equation $r = \frac{2}{1 - \sin \theta}$ in the polar coordinate system?
- A) Circle
B) Ellipse
C) Parabola
D) Hyperbola
E) NOTA
26. A trapezoid has vertices with coordinates $(4\sqrt{3}, 0)$, $(0, 4)$, $(-10, 4)$, $(-10 - 4\sqrt{3}, 0)$. A line segment with one endpoint $(0, 4)$ and length 2 is rotated 360 degrees. What is the area swept by the line segment outside of the trapezoid?
- A) $\frac{7\pi}{3}$ B) $\frac{7\pi}{6}$ C) $\frac{7\pi}{9}$ D) $\frac{7\pi}{12}$ E) NOTA
27. For $0 < \theta < \frac{\pi}{2}$, what is the area of the triangle with vertices $(\cos \theta, 0)$, $(0, \sin \theta)$, and $(0, 0)$?
- A) $\sin 2\theta$ B) $2 \sin 2\theta$ C) $\frac{1}{2} \sin 2\theta$ D) $\frac{1}{4} \sin 2\theta$ E) NOTA

28. Consider the equation $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ where $a > b$. What are coordinates of the foci of this graph?

- A) $(h \pm \sqrt{a^2 - b^2}, k)$ B) $(h \pm \sqrt{a^2 + b^2}, k)$ C) $(h, k \pm \sqrt{a^2 - b^2})$
D) $(h, k \pm \sqrt{a^2 + b^2})$ E) NOTA

29. Consider the convex polygonal region formed when the points $(-1, 8)$, $(3, 10)$, $(5, 4)$, $(1, 1)$, and $(-4, 3)$ are connected in order. Give the area of the region.

- A) 42.5 B) 43 C) 46.5 D) 49.5 E) NOTA

30. Pappus' Theorem states that the volume of a region in the xy -plane when rotated around a line is $2\pi Ad$, where A is the area of the region and d is the distance between the centroid of the region and the line of rotation. Give the volume of the solid formed when the region bounded by the graph of $16x^2 + 4y^2 + 32x + 8y - 44 = 0$ is rotated around the line $5x + 5y = 98$.

- A) $\frac{864\pi^2\sqrt{2}}{5}$ B) $\frac{1,728\pi^2\sqrt{2}}{5}$ C) $\frac{1061\pi\sqrt{2}}{2}$
D) $\frac{4543\pi\sqrt{2}}{2}$ E) NOTA