NOTA means “None of the Above.” For this test, assume that ellipses are non-circular.

1. Find the distance between the points (1, 3) and (18, 3).
   
   A) 3    B) 17    C) 0    D) \( \sqrt{17} \)    E) NOTA

2. Let the slope of the line \( 3x + 4y = 5 \) equal A, and let the y-intercept be (0, B). \( A + B = \)
   
   A) 0.5    B) 0.75    C) 1.3    D) 2.3    E) NOTA

3. Give the equation of the line that passes through the points (260, 221) and (400, 81).
   
   A) \( 2x + 3y = 600 \)    B) \( 3x - y = 962 \)    C) \( x + y = 481 \)    D) \( x - 2y = 50 \)    E) NOTA

4. Find the distance between the x- and y-intercepts of \( Ax + By + C = 0 \), for \( A \neq B \neq 0 \).
   
   A) \( \left| \frac{C}{AB} \right| \sqrt{A^2 + B^2} \)    B) \( C^2 \)    C) 1    D) \( A^2 + B^2 \)    E) NOTA

The next three questions concern points \( A(-5,10) \) and \( B(1,4) \), and the line \( l \) that passes through them.

5. Give the equation of the line perpendicular to \( l \) that passes through the midpoint of \( AB \).
   
   A) \( y = x + 5 \)    B) \( y = 2x + 5 \)    C) \( y = x \)    D) \( y = x + 9 \)    E) NOTA

6. What is the acute angle that \( l \) makes with the x-axis?
   
   A) \( \frac{\pi}{3} \)    B) \( \frac{\pi}{4} \)    C) \( \frac{\pi}{6} \)    D) \( \frac{\pi}{7} \)    E) NOTA

7. Choose the point on line segment AB three-quarters of the way from A to B.
   
   A) \( \left( \frac{3}{4}, \frac{23}{4} \right) \)    B) \( \left( \frac{3}{5}, \frac{28}{5} \right) \)    C) \( \left( \frac{1}{2}, \frac{11}{2} \right) \)    D) \( \left( \frac{1}{4}, \frac{21}{4} \right) \)    E) NOTA
The next four questions concern the parabola \( x^2 + 6x - 4y + 13 = 0 \) and the circle \( x^2 + y^2 - 6x + 12y + 29 = 0 \).

8. What are the coordinates of the focus of the parabola?
   A) (-3, 5)  B) (3, 5)  C) (3, 3)  D) (-3, 2)  E) NOTA

9. What is the radius of the circle?
   A) 4  B) 8  C) 12  D) 16  E) NOTA

10. Give the equation of the directrix of the parabola.
    A) \( y = 0 \)  B) \( y = 1 \)  C) \( y = 2 \)  D) \( y = 3 \)  E) NOTA

11. A line segment is drawn starting at the vertex of the parabola and ending tangent to the circle. What is its length?
    A) \( \sqrt{61} \)  B) \( \sqrt{67} \)  C) \( \sqrt{69} \)  D) \( \sqrt{71} \)  E) NOTA

12. What is the equation of the graph \( x^2 + y^2 + 18x - 18y + 162 = 0 \)?
    A) line  B) circle  C) point  D) plane  E) NOTA

13. \( \int_{-10}^{13} |x-1| - 9 \, dx = \)
    A) \( \frac{175}{2} \)  B) 81  C) 86  D) \( \frac{171}{2} \)  E) NOTA

14. For \( a, b, c > 0 \), given that (0, 0), (\( a, 0 \)), and (\( c, b \)) are the coordinates of a parallelogram, which of the following could be the fourth vertex?
   I. (\( a+c, b \))  II. (\( a+c, a+b \))  III. (\( a, b+c \))  IV. (\( a-b \))  V. (\( a-c, -b \))
   A) IV  B) II, IV  C) I, V  D) IV, V  E) NOTA

15. Give the cross product of the vectors \( 3i + 7j + 5k \) and \( 4i + 3j - 2k \).
    A) \( i + 14j + 37k \)  B) \( 28i + j + 3k \)  C) \( -29i + 26j - 19k \)
    D) \( 22i + 20j - 20k \)  E) NOTA
16. Consider the polygon formed when the points (-5, 3), (7, 11), (11, 4), and (-1, -4) are connected in order. Which of the following terms is the most specific name of the given polygon?

A) Quadrilateral    B) Trapezoid    C) Rhombus    D) Parallelogram    E) NOTA

17. The plane \(-29x + 26y - 19z - 34 = 0\) intersects another plane with equation \(x + z = 0\) in a line. Give the direction vector of this line. (The cross product of the normal vectors of the planes give the direction vector of the line).

A) \(26i - 48j + 26k\)    B) \(26i + 10j - 26k\)    C) \(19i + 26j + 4k\)
D) \(i + k\)    E) NOTA

18. Give the distance between the polar points \(1, \frac{\pi}{4}\) and \((1, \pi)\).

A) \(\sqrt{2} + \sqrt{2}\)    B) \(\sqrt{2}\)    C) \(\sqrt{5}\)    D) \(\frac{3\pi}{4}\)    E) NOTA

19. Give the area of the region given by the intersection of the interiors of the polar graphs \(r = a \cos \theta\) and \(r = a \sin \theta\).

A) \(\frac{a^2}{64}(\pi - 8)\)    B) \(\frac{a^2}{4}(\pi - 8)\)    C) \(a^2(\pi - 8)\)    D) \(\frac{a^2}{8}(\pi - 2)\)    E) NOTA

20. Give the coordinates of the point of intersection of the asymptotes of the hyperbola: \(13x^2 - 7y^2 + 26x + 14y - 85 = 0\).

A) \((-1, 1)\)    B) \((1, 1)\)    C) \((1, -1)\)    D) \((-1, -1)\)    E) NOTA

21. Give the equation that contains the locus of points twice as far from \((-1, 1)\) as from \((4, 5)\) in the xy-plane.

A) \(x^2 + y^2 - 34x - 18y + 80 = 0\)    B) \(x^2 + y^2 + 12x + 6y - 37 = 0\)    C) \(3x^2 + 3y^2 - 34x - 38y + 162 = 0\)
D) \(3x^2 + 3y^2 + 18x + 2y - 25 = 0\)    E) NOTA
22. Give the area of the region between the graph of $|x| + |y| = 4$ and the circle inscribed within $|x| + |y| = 4$.

A) $32 - 8\pi$   B) $16 - 8\pi$   C) $16 - 4\pi$   D) $32 - 4\pi$   E) NOTA

23. An equilateral triangle and a circle are drawn that don’t overlap. They have a combined area of $\sqrt{3} + \pi$. If the side of the triangle and radius of the circle are natural numbers, what is the combined perimeter of the triangle and the circle?

A) $3 + 2\pi$   B) $6 + 2\pi$   C) $12 + \pi$   D) $12 + 2\pi$   E) NOTA

24. Give the cosecant of the angle between the vectors $\langle 1,1,1 \rangle$ and $\langle -1,1,1 \rangle$.

A) $\frac{3\sqrt{2}}{2}$   B) $\frac{\sqrt{2}}{3}$   C) $\frac{3\sqrt{2}}{4}$   D) $\frac{3\sqrt{2}}{8}$   E) NOTA

25. What is the graph of the equation $r = \frac{2}{1 - \sin \theta}$ in the polar coordinate system?

A) Circle   B) Ellipse   C) Parabola   D) Hyperbola   E) NOTA

26. A trapezoid has vertices with coordinates $\langle 4\sqrt{3}, 0 \rangle$, $(0, 4)$, $(-10, 4)$, $(-10 - 4\sqrt{3}, 0)$.

A line segment with one endpoint $(0, 4)$ and length 2 is rotated 360 degrees. What is the area swept by the line segment outside of the trapezoid?

A) $\frac{7\pi}{3}$   B) $\frac{7\pi}{6}$   C) $\frac{7\pi}{9}$   D) $\frac{7\pi}{12}$   E) NOTA

27. For $0 < \theta < \frac{\pi}{2}$, what is the area of the triangle with vertices $(\cos \theta, 0)$, $(0, \sin \theta)$, and $(0, 0)$?

A) $\sin 2\theta$   B) $2\sin 2\theta$   C) $\frac{1}{2}\sin 2\theta$   D) $\frac{1}{4}\sin 2\theta$   E) NOTA
28. Consider the equation \( \left( \frac{x-h}{a} \right)^2 + \left( \frac{y-k}{b} \right)^2 = 1 \) where \( a > b \). What are coordinates of the foci of this graph?

A) \( (h \pm \sqrt{a^2 - b^2}, k) \)  
B) \( (h \pm \sqrt{a^2 + b^2}, k) \)  
C) \( (h, k \pm \sqrt{a^2 - b^2}) \)  
D) \( (h, k \pm \sqrt{a^2 + b^2}) \)  
E) NOTA

29. Consider the convex polygonal region formed when the points (-1, 8), (3, 10), (5, 4), (1, 1), and (-4, 3) are connected in order. Give the area of the region.

A) 42.5  
B) 43  
C) 46.5  
D) 49.5  
E) NOTA

30. Pappus' Theorem states that the volume of a region in the xy-plane when rotated around a line is \( 2\pi Ad \), where \( A \) is the area of the region and \( d \) is the distance between the centroid of the region and the line of rotation. Give the volume of the solid formed when the region bounded by the graph of \( 16x^2 + 4y^2 + 32x + 8y - 44 = 0 \) is rotated around the line \( 5x + 5y = 98 \).

A) \( \frac{864\pi^2 \sqrt{2}}{5} \)  
B) \( \frac{1,728\pi^2 \sqrt{2}}{5} \)  
C) \( \frac{1061\pi \sqrt{2}}{2} \)  
D) \( \frac{4543\pi \sqrt{2}}{2} \)  
E) NOTA