Algebra Theta Solutions
2007 Mu Alpha Theta National Convention

1. Answer: A 4

$g(x) = \ln(x)$, so $g^{-1}(x) = e^x$. $f(x) = e^x$, so $f^{-1}(x) = \ln(x)$. Therefore, $f^{-1}(g^{-1}(x)) = \ln(e^x) = x$. Therefore, $f^{-1}(g^{-1}(4)) = 4$.

2. Answer: A 100

$F = \frac{kS}{J}$, and F + S + J = X. 200 + S + 30 = 380, so there are 150 sophomores in the first example. This means that $200 = \frac{150k}{30}$, so the constant of variability k = 40. That means that when there are 50 juniors and 230 seniors, $F = \frac{40S}{50}$, and 230 = F + S + 50. Thus, $180 - S = \frac{4S}{5}$, and solving for S finds that there are 100 sophomores.

3. Answer: A $\frac{25}{4}$

Expanded, the sequence contains the terms 5, 1, $\frac{1}{5}, \frac{1}{25}, \ldots$ and is thus a geometric series with common ratio $\frac{1}{5}$. The total value of the series is therefore $\frac{5}{1-\frac{1}{5}} = \frac{25}{4}$.

4. Answer: B. 4

$g(2) = f(2 + 2) = f(4) = 4^2 - 2(4) - 4 = 4$
$f(g(2)) = f(4) = 4^2 - 2(4) - 4 = 4$

5. Answer: C. 8

Absolute values cannot be negative, and thus neither $|2x + y + 6|$ nor $|2x - y - 14|$ is negative. As the two values add to zero, both must therefore equal zero. Therefore, 2x + y + 6 = 0 and 2x – y – 14 = 0. Solving the system yields values of 2 for x and -10 for y. The absolute value of (2 + -10) is 8.

6. Answer: C. 101

X + (X + 1) + (X + 2) + (X + 3) + (X + 4) = 1000
5X + 10 = 1000
5X = 990
X = 198

The largest number is therefore 198 + 4, or 202. 202 = 2 * 101, so the largest prime factor of the largest of the five integers is 101.

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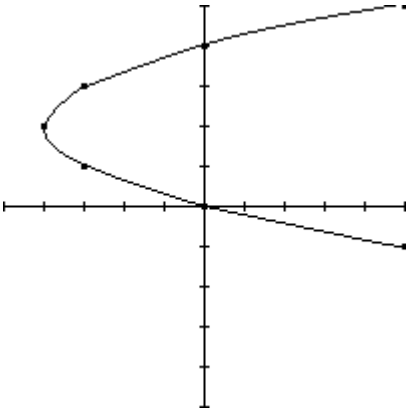
7. Answer: C. Quadrant III

The graph of $x = y^2 - 4y$ resembles the following:



It passes through Quadrant I at the point (5,5) and others, Quadrant II at the point (-3,1) and others, and Quadrant IV at the point (5,-1) and others. Quadrant III is the only quadrant through which the graph never passes.

8. Answer: B. 4

Using the change-of-base technique for logarithms:

$$K = \frac{\ln 3}{\ln 2} * \frac{\ln 4}{\ln 3} * \frac{\ln 5}{\ln 4} * \frac{\ln 6}{\ln 5} * \frac{\ln 7}{\ln 6} * \frac{\ln 8}{\ln 7} * \frac{\ln 9}{\ln 8} = \frac{\ln 9}{\ln 2} = \log_2 9$$

The expression $\log_3 4^K$ (which can be rewritten as $K\log_3 4$) therefore equals

$$\log_2 9 * \log_3 4 = \frac{\ln 9}{\ln 2} * \frac{\ln 4}{\ln 3} = \frac{\ln 9}{\ln 3} * \frac{\ln 4}{\ln 2} = \log_3 9 * \log_2 4 = 2 * 2 = 4.$$

9. Answer: E 11

If X students like both Chinese and Japanese food, then (18 – X) like Chinese alone, and (14 – X) like Japanese alone. All students must like either Chinese, Japanese, both, or neither, so therefore (18 – X) + (14 – X) + X + 6 = 27. Thus, X = 11.

10. Answer: A $x^4 + x^2 + 1$

$$\frac{(x^3+1)(x^3-1)}{(x^2-1)} = \frac{(x+1)(x^2-x+1)(x-1)(x^2+x+1)}{(x+1)(x-1)} = (x^2-x+1)(x^2+x+1) = x^4+x^2+1$$

11. Answer: C 8

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If B = 2D and (B – 5) = 3(D – 5), then Barbara is currently twenty years old, and her sister is ten. (Five years ago she was fifteen and Darla was five.) Since B = D + 10, B is equal to 6D only when B = 12 and D = 2. Since Barbara is twenty now, it has been eight years since she was 12 (6 times her sister's age).

12. Answer: D 3

Combining the left side of the equation into a single term gives: $\frac{(Ax - 4A + Bx + 3B)}{(x^2 - x - 12)} = \frac{(6x - 3)}{(x^2 - x - 12)}$. Thus, Ax – 4A + Bx + 3B = 6x – 3. Like terms must add up to like terms, so Ax + Bx = 6x and -4A + 3B = -3. Solving the system for A and B finds that both are equal to 3. Therefore, their average is likewise 3.

13. Answer: D $5 + i$

Expanded, $(1+ i)(2 - 3i)$ equals $2 - 3i + 2i + 3$, or $5 - i$. The conjugate of $(5 - i)$ is $(5 + i)$.

14. Answer: B 14

As Point B is 300 miles directly east of Point A, and Point C is 400 miles directly north of Point B, Points A and C are 500 miles apart. Therefore, Bill Wynn, traveling at 125 miles per hour, can get the van from C to A in 4 hours. Mr. Wynn drove from A to B (300 miles) at 50 mph, so he drove for 6 hours, and his wife drove from B to C (400 miles) at 100 mph, so she drove for 4 hours. Therefore, the entire trip took 6 + 4 + 4 = 14 hours.

15. Answer: B $x^2 - 2x - 8$

$$\frac{2(x+2)(x-3)}{(x+4)} * \frac{(x+4)(x-4)}{2(x-3)} = (x+2)(x-4) = x^2 - 2x - 8$$

16. Answer: C $x^2 - x$

Where defined, $f(g(x)) = (x - 2)!$. Therefore, $f(x)$ divided by $f(g(x))$ is

$$\frac{x!}{(x-2)!} = \frac{x(x-1)(x-2)(x-3)(x-4)...(x-n)...(1)}{(x-2)(x-3)(x-4)...(x-n)...(1)} = x(x-1) = x^2 - x$$

17. Answer: A 10

Theatrics aside, this question is merely asking for the number of combinations possible for 5 items taken 2 at a time. 5C2 equals $\frac{5!}{(2!)((5-2)!)} = 10$

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18. Answer: B 9

The base-seven number 1234_7 equals $1(7^3) + 2(7^2) + 3(7^1) + 4(7^0) = 343 + 98 + 21 + 4 = 466$ in base-ten. As this equals the base-n number 567_n, $5n^2 + 6n + 7 = 466$, so $5n^2 + 6n - 459 = 0$. Solving for n finds that n equals either 9 or -10.2. As n is a whole number, the correct answer is base-9, for n = 9.

19. Answer: D 15

Starting with its first bounce of 2 meters, the ball bounces in a geometric sequence of common ratio $\frac{2}{3}$. Therefore, its total distance traveled while bouncing is $\frac{2}{(1-\frac{2}{3})} = 6$ meters. However, it also travels an equal distance when falling back to the ground between bounces. Therefore, the ball travels a total of 12 meters while bouncing before it finally stops. Additionally, it originally fell 3 meters before its first bounce. Therefore, the total distance traveled by the ball is 15 meters.

20. Answer: C 62

$$z(2) = \frac{(2^5 - 2(2^4) + 3(2^3) + 4(2^2) - 6(2) - 180)}{(2-3)} = 152$$

$$z(1) = \frac{(1^5 - 2(1^4) + 3(1^3) + 4(1^2) - 6(1) - 180)}{(1-3)} = 90$$

$\frac{152}{90} = 1\frac{31}{45}$, or 1 remainder 62.

21. Answer: D $25W^2 - 50W + 61$

$W = \frac{X}{5} + 1$, therefore $X = 5(W - 1) = 5W - 5$. X also equals 3Y, so $Y = \frac{X}{3} = \frac{(5W-5)}{3}$.

$$Z = Y^2 + 4 = \left[\frac{(5W-5)}{3}\right]^2 + 4 = \frac{25W^2 - 50W + 25}{9} + 4.$$

9Z therefore equals $25W^2 - 50W + 25 + 36$, or $25W^2 - 50W + 61$.

22. Answer: E NOTA

The domain of the function does not include values for which $x^2 - 4x = 0$, because such values would necessitate division by zero, which is undefined. Therefore, neither 0 nor 4 are included in the function's domain. Also, the domain does not include any values for which $x^2 - 5x + 4 < 0$, since such values would yield an imaginary or complex range. As

the value of $x^2 - 5x + 4$ is negative between its x-intercepts of (1,0) and (4,0), no values for x on the interval (1,4) are included in the domain of the original function. Therefore, neither 2 nor 3 is in the function's domain, and thus none of the given choices is included in the domain. Therefore, E.

23. Answer: D. 52
Solving the system yields values of 76, -22, and -2 for x, y, and z respectively. Thus, x + y + z = 76 – 22 – 2 = 52.

24. Answer: C $124\sqrt{2}$
T = 2B, and T + B = 372. Therefore, 3B = 372, B = 124, and T = 248. The geometric mean of T and B is therefore $\sqrt{248 * 124}$, or $124\sqrt{2}$ pounds.

25. Answer: A 27
$abc = 48$ and $bcd = 96$. Therefore, $ab^2c^2d = 4608$. $ad = 72$, so $b^2c^2 = 64$. b and c are both positive, so $bc = 8$. And as $c - b = 7$, $b = 1$ and $c = 8$. $abc = 48$ and $bc = 8$, so $a = 6$. $bcd = 96$ and $bc = 8$, so $d = 12$. Therefore, $a + b + c + d = 6 + 1 + 8 + 12 = 27$.

26. Answer: B 12
Sreya wants her smoothie to be 60% banana. Therefore, she wants the ounces of banana divided by the total ounces of smoothie to be 60%. Her 16-ounce smoothie is 75% banana; therefore, her starting smoothie mixture has 12 ounces of banana in it. She then wants to add x ounces of the other smoothie mixture, of which 40% is banana. She will keep adding this second mixture until $\frac{(12 + .4x)}{(16 + x)} = .6$. Solving for x finds that 12 ounces of the 40%-banana smoothie will be added.

27. Answer: D $\frac{33}{B}$
As $A \neq \frac{12}{7}$ (in which case the system would have infinite solutions in the event that B equaled $\frac{77}{4}$), the ratio of 3:A does not equal the ratio of 7:4. Therefore, if the ratio of 3:A equals the ratio of B:11, the system will have no solution. Therefore, $\frac{3}{A} = \frac{B}{11}$, and therefore $A = \frac{33}{B}$.

28. Answer: A Between Jordan and Steve

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Tommy is twice as tall as Steve, and Steve is half of Jordan's height. Therefore, Tommy and Jordan have the same height. Rachel is the shortest present and Eddie the third-shortest, therefore the friends from shortest to tallest are Rachel, Steve, Eddie, and Tommy and Jordan. Therefore, Eddie's height falls between Jordan's and Steve's.

29. Answer: A 277

If x = 1.79797979797979…, then 100x = 179.797979797979… Therefore, 100x – x = 179.797979797979… – 1.79797979797979… = 99x = 178. Therefore, $x = \frac{99}{178}$, and thus the sum of its numerator and denominator is 99 + 178 = 277.

30. Answer: C GH

As $G \neq 0$ and $H \neq 0$, $\frac{H-G}{G^{-1}-H^{-1}} = \frac{H-G}{(\frac{1}{G}-\frac{1}{H})}$. And as $G \neq H$, the expression thus equals $\frac{H-G}{(\frac{H}{GH}-\frac{G}{GH})}$, or $\frac{H-G}{(\frac{H-G}{GH})}$. This can be rewritten as $(H-G)\times\frac{GH}{(H-G)} = GH$.