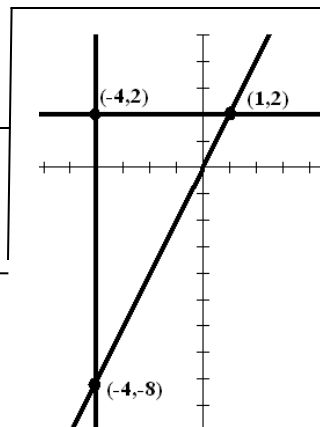


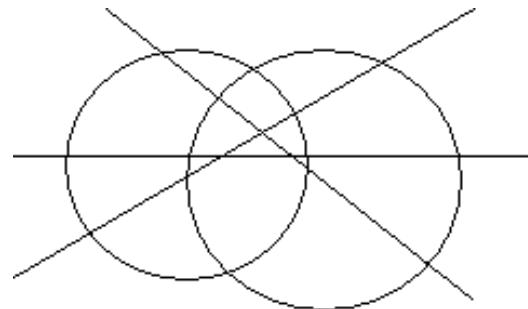
**THETA GEMINI SOLUTIONS**  
**Mu Alpha Theta National Convention**  
**Tampa, Florida 2007**



1. (D)  $|2 - 2i| = \sqrt{(2)^2 + (-2)^2} = 2\sqrt{2}$ , which is greater than 2.  $1 < \ln(\pi) < 2$ , because  $e^1 < \pi < e^2$ . Finally, any nonzero number to the zero power equals 1. So  $z < y < x$ .

2. (C) The area of the triangle (the graph is shown to the right) is  $\frac{1}{2}(5)(10) = 25$ . The area in Quadrant III (the lower-left quadrant) is  $\frac{1}{2}(4)(8) = 16$ . Then  $\frac{16}{25} = \frac{64}{100} = 64\%$ .

3. (D) The circles can share two intersection points (2); each line can intersect each circle twice ( $3 \times 2$ ); and the three lines can share three intersection points (3), but we must divide by two to prevent counting intersection points twice (3). So the maximum total is  $2 + 12 + 3 = 17$ .



4. (E) The sum of the coefficients of a polynomial function  $f(x)$  can be found by plugging 1 into the function. For this function, 1 is a root, so  $f(1) = 0$ . This problem was inspired by the function  $f(x) = x^4 - 9x^2 - 4x + 12$ .

5. (B) There is a  $\frac{1}{3}$  probability that Player X takes the top-middle square, which will result in no winner. There is a  $\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{6}$  probability that Player X takes the top-left square, then Player O takes the middle-left square, which will result in no winner. There is a  $\left(\frac{1}{3}\right)\left(\frac{1}{2}\right) = \frac{1}{6}$  probability that Player X takes the middle-left square, then Player O takes the top-left square, which will result in no winner.  $\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$ .

6. (B) The conic is a hyperbola, since the eccentricity is greater than 1. The distance from the center to the vertex,  $a$ , is 3. Since eccentricity is  $\frac{c}{a}$ , solve  $\frac{c}{3} = 2$  to find that  $c = 6$ . In a hyperbola,  $c^2 = a^2 + b^2$ , and then  $6^2 = 3^2 + b^2$ . Solving and choosing the positive answer, find that  $b = 3\sqrt{3}$ , and the length of the conjugate axis of a hyperbola is  $2b$ .

7. (A) Using divisibility rules, an integer is divisible by 8 if and only if the last 3 digits are divisible by 8, so  $x$  is not. An integer is divisible by 9 if and only if the sum of the digits is divisible by 9, so  $x$  is not. A number is divisible by 12 if and only if it is divisible by 4 (see if the last two digits are divisible by 4) and 3 (see if the sum of the digits is divisible by 3), so  $x$  is not. Finally, a number is divisible by 15 if and only if it is divisible by 5 (see if the last digit is a 5 or a 0) and 3, so  $x$  is divisible by 15.

8. (D) Simply add all the equations together to get  $4x + 4y + 4z = 12$ ; then divide by 4.

9. (A) Solving  $x^2 + (2x)^2 = 15^2 \rightarrow 5x^2 = 225 \rightarrow x^2 = 45$ , and choosing the positive value, reveals  $x = 3\sqrt{5}$ .

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$$\begin{array}{r} 100x = 71.46\cancel{46} \\ - x = -0.71\cancel{46} \\ \hline 99x = 70.75 \end{array}$$

10. (C) Let  $x = 0.714646\dots$ ; then  $100x = 71.4646\dots$ . Subtracting (shown to the right), find that  $99x = 70.75$ , so  $x = \frac{7075}{9900}$ .

11. (E) The total investment was \$1000, and Emily contributed \$450 of that. Solving  $\frac{450}{1000} = \frac{x}{300}$ , she would be entitled to \$135 if the profit were split proportionally. However, she is only getting \$100. So the answer is \$35.

12. (D) Using the hint given in the problem and a little insight, find that every term can be expressed in the manner suggested. In fact the sequence is  $\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{4}\right), \left(\frac{1}{3} - \frac{1}{4} + \frac{1}{5}\right), \left(\frac{1}{4} - \frac{1}{5} + \frac{1}{6}\right), \left(\frac{1}{5} - \frac{1}{6} + \frac{1}{7}\right), \dots$  so the term in question is  $\left(\frac{1}{8} - \frac{1}{9} + \frac{1}{10}\right)$ , and  $|8+9-10| = 7$ .

13. (A) Letting  $x = 2 + 3\sqrt{2+3\sqrt{2+\dots}}$ , then  $x-2 = 3\sqrt{x} \rightarrow x^2 - 4x + 4 = 9x \rightarrow x^2 - 13x + 4 = 0$ . Using the quadratic formula and choosing the positive answer,  $x = \frac{13 + \sqrt{13^2 - 16}}{2} = \frac{13 + 3\sqrt{17}}{2}$ .

14. (A) There are  ${}_5C_3 = \frac{5!}{3!(5-3)!} = 10$  possible combinations of three characters. There are  $3! = 6$  ways to arrange three distinct characters. So multiplying these, there are 60 possible expletives that are 3 characters long. Similarly, there are  $({}_5C_4)(4!) = \frac{5!}{4!(5-4)!}(4!) = 120$  possible 4-character expletives, and  $({}_5C_5)(5!) = \frac{5!}{5!(5-5)!}(5!) = 120$  possible 5-character expletives. In all, there are 300 possible expletives.

15. (C) Trying choices revealed by the Rational Root Theorem, it is not too tough to discover that 1 is a root. Using synthetic division (shown to the right), the depressed polynomial equation is  $4x^2 + 4x - 3 = 0 \rightarrow (2x+3)(2x-1) = 0$ , so the other roots are  $-\frac{3}{2}$  and  $\frac{1}{2}$ . And then,  $\left|3\left(-\frac{3}{2}\right) - 5\left(\frac{1}{2}\right) + 3(1)\right| = 4$ .

$$\begin{array}{r|rrrr} 1 & 4 & 0 & -7 & 3 \\ & & 4 & 4 & -3 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

16. (B) The probability that Omar will win is  $\frac{3}{3+7} = \frac{3}{10}$ . The probability that Andrew will win is  $\frac{1}{1+3} = \frac{1}{4}$ . The probability that Omar will win is then  $1 - \left(\frac{3}{10} + \frac{1}{4}\right) = \frac{9}{20}$ . So the odds are  $(9):(20-9) = 9:11$ .

17. (E) Let  $y$  be the degree measure of arc  $\widehat{EN}$ . Solving  $\frac{4\pi}{32\pi} = \frac{y}{360}$ , find that  $y = 45$ . By a well-known theorem,  $2x = 100 - y$ . Solving,  $x = \frac{55}{2} = 27.5$ .

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18. (B)  $(\sqrt{-2})(\sqrt{-8}) = (i\sqrt{2})(i\sqrt{8}) = (-1)(\sqrt{16}) = -4$ .

19. (B) The interior angles of a pentagon add up to  $180(5 - 2)^\circ = 540^\circ$ . Adding up the angle measures given, find that  $54x + 135 = 540 \rightarrow x = 7.5$ . Substituting, find that the unknown angles measure 80, 68, 164, and 138 degrees. The largest acute angle measures 80 degrees.

20. (A) The slope of the line given is  $\frac{3}{4}$ , so the slope of the perpendicular line is  $-\frac{4}{3}$ . Using the slope formula,

$$\frac{-4}{3} = \frac{(2k+1)-(0)}{(0)-(3-k)} \rightarrow \frac{-4}{3} = \frac{2k+1}{k-3} \rightarrow 12 - 4k = 6k + 3 \rightarrow k = \frac{9}{10}$$

21. (C) Note that  $\log(x-1) + \log(x+2) = \log(x^2 + x - 2)$ . Moving the equation into exponential form,  $\log_{10}(x^2 + x - 2) = 1 \rightarrow x^2 + x - 2 = 10^1 \rightarrow x^2 + x - 12 = 0 \rightarrow (x+4)(x-3) = 0$ , and so  $x = -4, 3$ . However, the first value is not in the domain implied by the problem (it gives a negative argument to a log when you plug it back into the original equation.) So the only solution is 3.

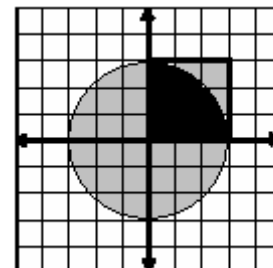
22. (A) First we are taking the inverse of the function, so after flipping over the line  $y = x$ , the domain and range switch. So the domain is  $[3, 10]$  and the range is  $[-2, 5]$ . Then, when flipping over the  $x$ -axis, the domain remains the same, but the bounds on the range reverse, and so do their signs. So  $g$  has domain  $[3, 10]$  and range  $[-5, 2]$ .

23. (A) We need a number such that  $\sqrt{x} - 1 = 4$ , and that number is 25. Plugging 25 into the given function for  $x$ ,  $f(4) = \frac{5}{25} + 7 = 7.2$ .

24. (B) First, put anyone but Thierry and George in the first seat (3 possibilities.) Thierry and George are still in the group, along with two others. Now, put anyone but Thierry in seat two. (3 possibilities.) Now there are  $3! = 6$  ways to fill the last 3 seats.  $(3)(3)(6) = 54$ .

25. (C) Sets  $C$  and  $Q$  are represented by a circle and the square, respectively, in the complex plane. The area of the square that is overlapped by the circle is  $\frac{1}{4}\pi r^2 \approx \left(\frac{1}{4}\right)\left(\frac{22}{7}\right)(9) = \frac{99}{14}$ . The area of the square is 9. The probability that a point selected at random from the square is also in the sector is the ratio of their areas.

Evaluating  $\frac{\frac{99}{14}}{\frac{9}{1}} = \left(\frac{99}{14}\right)\left(\frac{1}{9}\right) = \frac{11}{14} \approx 0.8$ .



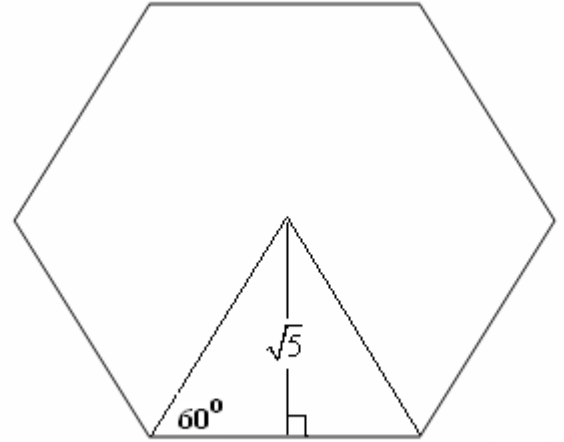
**26. (C)** Let the diagonals of the rhombus be  $2x$  and  $2y$ , and then  $x + y = 16$ . The area of the rhombus is half the product of the diagonals, so  $k - 4 = 2xy$ . One side of the rhombus is the hypotenuse of a triangle whose legs are half of each diagonal. So  $x^2 + y^2 = k$ . Squaring the first equation,  $x^2 + y^2 + 2xy = 256$ . Substituting the second and third equations,  $k + (k - 4) = 256$ , and then  $k = 130$ .

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$$\begin{array}{r} 0 \text{ r}2 \\ 8 \overline{) 2} \text{ r}1 \\ 8 \overline{) 17} \text{ r}6 \\ 8 \overline{) 142} \end{array}$$

27. (B) Converting to base-10, the value is  $2(5^0)+3(5^1)+0(5^2)+1(5^3)=142$ . Then use stair-step division and read the remainders downward, as shown.

28. (A) A hexagon is made of six equilateral triangles. Each of these can be split into two 30-60-90 triangles as shown in the diagram. Each side of the hexagon is  $2\left(\frac{\sqrt{5}}{\sqrt{3}}\right)$ . The area of the hexagon is the sum of the areas of the six triangles, or  $6\left(\frac{1}{2}\right)2\left(\frac{\sqrt{5}}{\sqrt{3}}\right)(\sqrt{5}) = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ . The volume of the prism is the area of the base times the height, or  $(10\sqrt{3})(2\sqrt{6}) = 60\sqrt{2}$ . So  $xy = 120$ .



29. (D) Right now, there are  $(80)(.35) = 28$  gallons of alcohol. The ratio of alcohol to total liquid needs to be  $\frac{1}{5}$ . After adding  $x$  gallons of water, the ratio of alcohol to total liquid is  $\frac{28}{80+x}$ . Solving, find that  $\frac{28}{80+x} = \frac{1}{5} \rightarrow 140 = 80 + x \rightarrow x = 60$ .

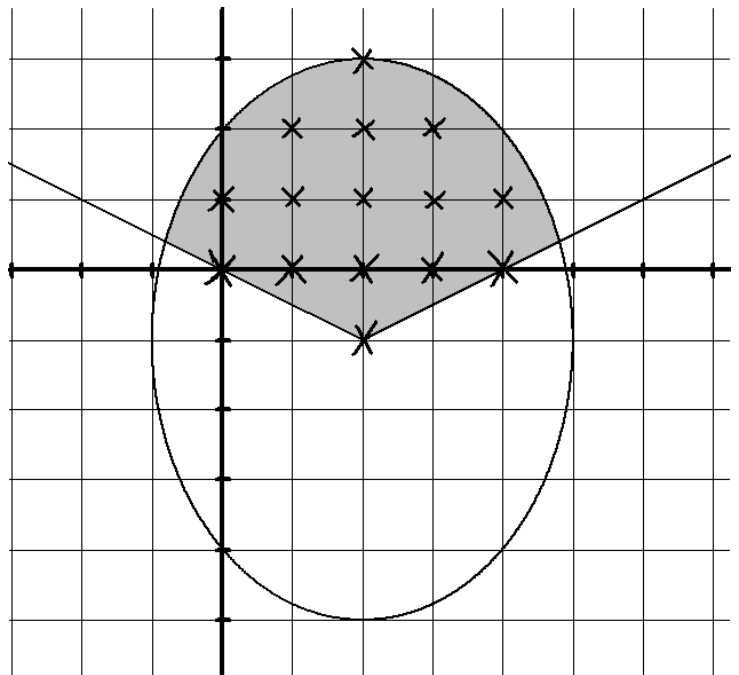
30. (B) Completing the square, the first inequality defines the interior of the ellipse

$$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{16} \leq 1.$$

The ellipse has a vertical major axis of length 8 and a horizontal minor axis of length 2; its center is  $(2, -1)$ . The second inequality describes the region above a “happy V” whose branches have slopes of  $\pm \frac{1}{2}$  and whose corner is at  $(2, -1)$ .

When graphing the intersection, it may necessary to test the points “very close” (especially where  $y = 2$ ) to the ellipse by checking them in the actual equation.

There are 15 lattice points in the solution set, as marked with X’s in the graph to the right.



1. D
2. C
3. D
4. E
5. B

6. B
7. A
8. D
9. A
10. C

11. E
12. D
13. A
14. A
15. C

16. B
17. E
18. B
19. B
20. A

21. C
22. A
23. A
24. B
25. C

26. C
27. B
28. A
29. D
30. B