

Logarithms/Radicals/Exponents Topic Test
2007 Mu Alpha Theta National Convention

The abbreviation NOTA denotes
"None of These Answers."

1. If $29^{x+2} = 16^x$ then give the value of $10 - x$.

- A. 3 B. 4
C. 6 D. 7 E. NOTA

2. If $\frac{9^{2x} + 3^{4x}}{3^{-x}} = 6$ then $x =$

- A. $\frac{1}{5}$ B. $\frac{1}{2}$
C. $\frac{6}{7}$ D. $\frac{7}{6}$ E. NOTA

3. If $4x^{\frac{2}{3}} - x^{\frac{1}{3}} = 4 - x$ has real solutions

$q, r,$ and $s,$ then $\left| \frac{q99}{8} \right| =$

- A. 16 B. 8
C. 4 D. 1 E. NOTA

4. If $x \neq 0,$ $29x^{\frac{2}{3}} = 3x,$ and $x = \frac{a}{b}$ for a and b relatively prime positive integers, then $a + 2b =$

- A. 24 B. 35
C. 43 D. 62 E. NOTA

5. If $\sum_{n=1}^3 (\sqrt{3})^n = p\sqrt{3} + q$ for p and q rational, then give the value of $p + q$.

- A. 8 B. 11
C. 15 D. 16 E. NOTA

6. If $\sqrt{x} + \sqrt{y} = 6$ and $xy = 4$ then for $x > 0, y > 0$ give the value of $x + y$.

- A. 2 B. 28
C. 32 D. 34 E. NOTA

7. If $\left(\frac{x+1}{x-1}\right)^3 = \frac{1}{8}$ for $x \in \text{Reals},$ then give the value of $|x| + 1$.

- A. 2 B. 3
C. 4 D. 5 E. NOTA

8. If $f(x) = \sqrt{x-1}$ has domain $[a, \infty)$ and $g(x) = -\sqrt{1-x}$ has domain $(-\infty, b]$ then give the value of $\frac{a}{b} + ab$.

- A. -2 B. -1
C. 1 D. 2 E. NOTA

9. If $\sqrt{x - \sqrt[3]{2 - \sqrt{x - \sqrt[3]{2 - \dots}}} = 4$ then $x = 2^n - 2^m$. Give the value of $699n$.

- A. 4 B. 8
C. 12 D. 16 E. NOTA

10. For $f(x) = (4x)^{\frac{1}{x}}$, how many integral values of x , such that $0 < x < 10$ result in integral values for $f(x)$?

- A. 0 B. 1
C. 2 D. 4 E. NOTA

11. For functions f and $g,$ $f(x) = \sqrt[5]{x^2}.$ $f(g(x)) = g(f(x)) = x$ for all $x \geq 0$. What is the value of $g(2)$?

- A. 32 B. $4\sqrt{2}$
C. $\sqrt[5]{4}$ D. $\frac{1}{\sqrt[3]{4}}$ E. NOTA

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12. If $g(x) = \frac{x}{\sqrt{x}}$ then what is the least positive value of x such that $g(x)$ is an integer greater than 20 ?
- A. 5 B. 399
C. 401 D. 441 E. NOTA
13. If $(x+2)(x^2 - 2x + 4) = 9$ and $(y-3)(y^2 + 3y + 9) = 10$ then give the value of $(xy)^3$.
- A. 38^3 B. 37
C. 10 D. 6 E. NOTA
14. $\left(\frac{2x\sqrt[4]{x}}{x^{-2}}\right)^{-2} = ax^n$ for $x > 0$. Give the value of $4n$.
- A. $\frac{13}{2}$ B. $\frac{2}{13}$
C. $-\frac{2}{13}$ D. $-\frac{13}{2}$ E. NOTA
15. If $\left(\frac{a^{\frac{1}{3}}}{a^2} g^{\frac{1}{2}}\right)^{-1}$ is a positive integer greater than 1, then what is the least possible positive value of a ?
- A. 8 B. 2^6
C. 2^7 D. 2^{13} E. NOTA
16. $f(x) = (8)^{\frac{x-1}{x}}$. Give the value of $f(f(3)-2)$.
- A. $\sqrt[3]{2}$ B. $\sqrt{2}$
C. $2\sqrt[3]{2}$ D. $2\sqrt{2}$ E. NOTA
17. If $a, r,$ and s are members of the set $\{-3, -\frac{1}{2}, 0, \frac{1}{2}, 2\}$ and are not necessarily distinct, then the greatest possible value of $(a)^{rs}$ is what? (Do not consider 0^0 .)
- A. 1 B. 3^4
C. 2^9 D. 3^9 E. NOTA
18. For positive integers x and m , $\sqrt[n]{(x+1)^m} = 36$. If $n = 17$ and $1 < m < 19$ then $x = 6^k - 1$. Find the largest possible value of k .
- A. 16 B. 17
C. 29 D. 34 E. NOTA
19. $\sqrt[4]{4} =$
- A. $\sqrt[3]{3}$ B. $\sqrt{2}$
C. $2\sqrt{2}$ D. $2\sqrt[4]{4}$ E. NOTA
20. Which is NOT a factor of $(12^4 - 1)$?
- A. 145 B. 143
C. 13 D. 11 E. NOTA
21. $\sqrt{13 + \sqrt{88}} = \sqrt{a} + \sqrt{b}$ and $a > b$. For positive integers a and b , give the value of $\sqrt{a-b}$.
- A. 2 B. 3
C. 9 D. 11 E. NOTA
22. The area of a rectangle is $x^3 + 7^3$ square cm. The length of the rectangle is $7 + x$ cm. What is the width in cm of the rectangle?
- A. $7 - x$ B. $x^2 + 14x + 49$
C. $x^2 - 49$ D. $x^2 + 7x + 49$ E. NOTA

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23. $a = (1 + 2 + 3 + 4 + \dots + 10)^2$
 $b = (1^3 + 2^3 + 3^3 + \dots + 10^3)$
 $a - b = ?$
- A. -1 B. 0
 C. 1 D. 900 E. NOTA

24. $\sum_{n=0}^{12} \binom{12}{n} = a^b$ for positive integers
 a and b . Give the largest possible
 value for b .
- A. 24 B. 13
 C. 12 D. 11 E. NOTA

25. If $\log_2(\log_2(\log_2 x)) = 3$ then
 how many distinct positive prime
 integer factors does x have?
- A. 1 B. 2^8
 C. $2^8 + 1$ D. 2^{257} E. NOTA

26. For $xy \neq 0$, $2^{\frac{x}{y}} = 2^{\frac{1}{3}} 8^x$. Which is an
 expression for y in terms of x ?
- A. $\frac{x}{3-x}$ B. $\frac{x^2}{x+3}$
 C. $\frac{x^2}{4}$ D. $\frac{x^2-4}{x}$ E. NOTA

27. If $x \neq 0$ and $9^x - 3^{x+1} = -2$ then
 what is the value of 3^{-x} ?
- A. $\frac{1}{9}$ B. $\frac{1}{3}$
 C. $\frac{4}{9}$ D. $\frac{1}{2}$ E. NOTA

28. $x^{-1} + \frac{1}{4} = y^{-1}$ and $2x^{-1} + \frac{1}{3} = 5y^{-1}$
 for $xy \neq 0$, and $|x|$ can be written
 as $\frac{a}{b}$ for a and b relatively prime
 positive integers. Give the value
 of $\frac{a+2}{b}$.
- A. $\frac{38}{11}$ B. 2
 C. $\frac{7}{12}$ D. $\frac{5}{23}$ E. NOTA

29. For $i = \sqrt{-1}$ find $\sum_{n=1}^3 (1-i)^n$.
- A. $-\frac{1}{2} + \frac{3}{2}i$ B. $\frac{7}{2} - \frac{3}{2}i$
 C. $\frac{1}{2} - \frac{9}{2}i$ D. $\frac{5}{2} - \frac{3}{2}i$ E. NOTA

30. If x and y are randomly chosen
 from the set $\{-4, -3, -1, 2, 3, 5\}$,
 and are not equal to each other, then
 what is the probability that $16^{\frac{x}{y}}$ is
 rational?
- A. $\frac{5}{6}$ B. $\frac{3}{5}$
 C. $\frac{17}{30}$ D. $\frac{8}{15}$ E. NOTA