For all questions, answer choice (E) NOTA means that none of the given answers is correct. i is defined as $\sqrt{-1}$.
Good luck and have fun!

PART I: Mr. Jensen's Math Class

Mr. Jensen has planned a fun class activity for today! To begin, solve the first five equations.

1. First, solve for x, where $\log_{10}(x-8)+\log_{10}(x+5)=\log_{10}(x-1)+1$. Let $A=x$ (for the sixth question). The answer to this question (number 1) is the sum of digits of A.

   A. 2 B. 5 C. 6 D. 10 E. NOTA

2. Evaluate $\ln|\sin\theta|+\ln|\cos\theta|+\ln|\cos 2\theta|+\ln|\cos 4\theta|$ at $\theta=\frac{5\pi}{48}$, and let this quantity be τ. Let the answer to this question be $B=64e^{\tau}$.

   A. 4 B. 8 C. 16 D. 32 E. NOTA

3. Let c be the sum of solutions to the equation $2\cos\theta\tan^2\theta=1+\cos\theta$ for $\theta\in[0,2\pi]$. Then the answer to this question is $C=c/\pi$.

   A. 1 B. 2 C. 3 D. 4 E. NOTA

4. Consider the conic $x^2+4y^2-8x+8y+4=0$. Let R be the point on the ellipse with the maximal y-coordinate, and S be an endpoint of a latus rectum with maximal y-coordinate such that line RS has negative slope. Line RS can be expressed as $x+ny+p=0$. Compute $D=|1+n+p|$.

   A. 3 B. $3+2\sqrt{3}$ C. $3+4\sqrt{3}$ D. $5+4\sqrt{3}$ E. NOTA

5. The p be the product of the non-rational roots of $2x^6+7x^5+14x^4+19x^3+32x^2+37x-30$. For the sixth question, let E be $|p|$. For this question, select p, not $|p|$.

   A. -15 B. -5 C. 5 D. 15 E. NOTA

6. Take A, B, C, D, and E as in the previous questions. Mr. Jensen has a wheel with sectors labeled A, B, C, D, and E, where the probability of the wheel landing on A is $\frac{A}{A+B+C+D+E}$, and likewise for the other four sectors. If you spin the wheel twice (assuming the first spin is independent of the second), in which of the following ranges is the numerator of the probability (as a fully reduced fraction) that you land on A and E in some order?

   A. $(0,5]$ B. $(5,10]$ C. $(10,15]$ D. $(15,20]$ E. NOTA

PART 2: All's Fair in Love and War

Daniel is trying to win Catherine's heart. Catherine wants to ensure that Daniel is a man of mathematical skill, so she puts him through some assessments.

7. Daniel is going to give Catherine some roses to impress her. He finds out that her favorite positive integer is the number of Gaussian integers z such that $|z| \leq 7$, and would like to give her the same number of roses. How many roses should Daniel give Catherine? (Note: A Gaussian integer is a complex number $a + bi$ such that a, b are integers.)

A. 30 B. 45 C. 120 D. 149 E. NOTA

8. Catherine is thinking of an angle $\theta \in \left[\frac{\pi}{2}, 2\pi\right]$. She tells Daniel that $\csc\theta = \frac{17}{8}$ and wants him to find $\cot\frac{\theta}{2}$. Help Daniel out! What answer should he tell her?

A. -4 B. $-1/4$ C. $1/4$ D. 4 E. NOTA

9. Impressed with his trig skills, Catherine thinks of another angle $\tau \in [0, \pi]$. She tells Daniel that $\sec\tau = -\frac{5}{4}$. Using θ from the previous question, what's $\tan(\tau + \theta)$?

A. $-77/36$ B. $-11/12$ C. $-17/36$ D. $13/16$ E. NOTA

10. Catherine now wants to make sure that Daniel's love for her isn't imaginary, so she tests his complex number skills and asks him to rotate the point $(-4,5)$ by $390°$ clockwise. If the answer is in the form $\left(a + b\sqrt{3}, c + d\sqrt{3}\right)$, what is $abcd$?

A. -25 B. $-25/2$ C. $25/2$ D. 25 E. NOTA

11. Catherine then asks Daniel to solve for a complex number w in the third quadrant of the Argand plane satisfying $w^6 = 4 - 4i\sqrt{3}$. He should obtain a number in the form of $a(cis(b°))$, where a, b are positive real numbers and $0 \leq b \leq 360$. What is a^2b?

A. 400 B. 420 C. 440 D. 460 E. NOTA

12. Daniel has passed all of Catherine's tests! As a final assessment, Catherine has a set of complex numbers of the form $a + bi$ for integers $a, b \in \{-1,0,1\}$, from which Daniel will choose two, z and w, randomly and with replacement. Catherine will go on a date with Daniel if $\frac{z}{w}$ is also in the set. What is the probability that Catherine will go on a date with Daniel?

A. $21/81$ B. $25/81$ C. $56/81$ D. $60/81$ E. NOTA

PART 3: Shenanigans in Space

Look at the stars!

13. The planet Leontief has an elliptical orbit with the Sun at one focus. If the distance between the Sun and the center of the elliptical orbit is 5 Leonmeters, and the length of the latus rectum is 12 Leonmeters, what's the eccentricity of Leontief's elliptical orbit?

A. $\frac{\sqrt{61}-6}{5}$ B. $\frac{\sqrt{34}-3}{5}$ C. $\frac{\sqrt{34}+3}{5}$ D. $\frac{\sqrt{61}+6}{5}$ E. NOTA

14. Greg Mankiw is on Planet Macro, where acceleration due to gravity is 4 meters per second squared. Assume that the ground is flat. He launches a ball from ground level at an angle of thirty degrees and an initial velocity of 24 meters per second. What is the maximum height the ball will reach?

A. 9 B. 18 C. 36 D. 72 E. NOTA

15. John Nash has discovered a new constellation! John realizes that the stars in the constellation are all on the graph of $r = \frac{3}{2-\sin\theta}$. What is the area of this shape?

A. $\pi\sqrt{3}$ B. $2\pi\sqrt{3}$ C. $4\pi\sqrt{3}$ D. $8\pi\sqrt{3}$ E. NOTA

16. Allais, A, and Ellsberg, E, are two planets orbiting a Sun, S. Let θ be the measure of the angle ASE, where $\theta \in [0°, 180°]$. If $5\sin\theta > \cos 2\theta + 2$, Allais and Ellsberg are said to be *en paradox*. Assuming that at any given moment, the measure of θ is equally likely to be any value in $[0°, 180°]$, what is the probability that Allais and Ellsberg are *en paradox* at midnight on January 1st, 2020?

A. $1/3$ B. $5/12$ C. $7/12$ D. $2/3$ E. NOTA

17. The cow that usually jumps over the moon has decided to land on Earth today and start swinging on a swing set. He starts in a vertical, stationary position, then swings back and forth, trying to build height. Suppose the length of the swing is nine feet, and that his own size is negligible. He starts by swinging forward, traveling $\frac{\pi}{2}$ feet along the arc, then backwards $\frac{3\pi}{2}$ feet along the arc, then forwards $\frac{5\pi}{2}$ feet along the arc, and so on. He jumps off the swing when the swing reaches $60°$ from vertical for the first time. What is the total distance (along the arc) that the cow travels on the swing?

A. $21\pi/2$ B. 15π C. 18π D. 21π E. NOTA

18. Consider the parabola $8x = y^2 - 6y + 17$. Mr. Lu is positioned at the vertex of the parabola, and the Frazership is traveling along the latus rectum. What is the ratio between the minimum and maximum distances between Mr. Lu and the Frazership?

A. $\frac{\sqrt{5}}{10}$ B. $\frac{\sqrt{10}}{10}$ C. $\frac{\sqrt{5}}{5}$ D. $\frac{\sqrt{2}}{2}$ E. NOTA

PART 4: Party Preparations

Help Ed Glaeser and his friends throw a party for his economics class!

19. Walras has a decorative cone that has radius 40 and slant height 60. Let segment AB be a diameter of the base, and let C be the vertex of the cone. A Giffenbug is at the midpoint of segment AC. In which of the following ranges is the shortest distance (on the surface of the cone) the Giffenbug can crawl to get to point B?

A. $[40, 55)$ B. $[55, 70)$ C. $[70, 85)$ D. $[85, 100)$ E. NOTA

20. Shephard the Sheep has a decorative tetrahedron, ABCD, that he would like to alter. Tetrahedron ABCD has equilateral base ABC with side length 6, and $AD = BD = 5$. The volume of this tetrahedron is $\frac{36\sqrt{3}}{5}$. Find the new volume of this tetrahedron if the dihedral angle between face ABD and the base is doubled.

A. $\frac{144\sqrt{3}}{25}$ B. $\frac{216\sqrt{3}}{25}$ C. $\frac{288\sqrt{3}}{25}$ D. $\frac{324\sqrt{3}}{25}$. E. NOTA

21. Consider the triangle formed by the points $(0, 0)$, $(3, 0)$, and $(0, 4)$. Cournot the Cat wants to form a cone of maximal volume by rotating this triangle about the x or y-axis. What is the volume of this cone?

A. 12π B. 16π C. 36π D. 48π E. NOTA

22. Stackelberg and Bertrand are painting pretty shapes on the wall. Stackelberg draws the following series of graphs: $r = \sin(2k\theta)$ for $k = 1, 2, 3, 4, 5$. Bertrand draws the following series of graphs: $r = 3\cos((2j + 1)\theta)$ for $j = 1, 2, 3, 4, 5$. None of the graphs are overlapping. How many petals are drawn in total?

A. 65 B. 95 C. 100 D. 130 E. NOTA

23. Brian Wheaton is also trying to decorate the wall. His centerpiece will be a shape formed by the solutions of $x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$ on the Argand plane. Find the area of the shape formed, so that Brian can buy a sufficient amount of paint.

A. $\frac{3\sqrt{2}}{2}$ B. $\frac{6\sqrt{2}+1}{4}$ C. $\frac{3\sqrt{2}+1}{2}$ D. $2\sqrt{2}$ E. NOTA

24. Ed needs to find out how many people he should invite to the party. He is told that the number of people he should invite is equal to the number of real values of x such that $\frac{36}{2+xi}$ is a Glaesian integer. How many people should he invite? (Note: A Glaesian integer is a complex number $a + bi$ such that a is an integer.)

A. 8 B. 12 C. 16 D. 24 E. NOTA

PART 5: Casino Party

You're in Las Vegas – it's time to go to a casino!

25. Mr. Lu shuffles a standard deck of 52 cards and starts dealing them one by one. What is the probability that exactly one King, one Queen, and one Jack (in any order) are dealt before the first Ace?

A. $3/16$ B. $1/4$ C. $7/16$ D. $15/32$ E. NOTA

26. Mr. Frazer plays a series of three games, entitled A_1, A_2, and A_3. He has a probability of 0.6 of winning the first game, A_1. If he wins game A_i, he wins game A_{i+1} with probability 0.8. However, if he loses game A_i, he gets discouraged and wins game A_{i+1} with probability 0.4. Given that Mr. Frazer wins A_1, what's the probability that he wins A_3?

A. 0.08 B. 0.68 C. 0.72 D. 0.76 E. NOTA

27. In poker, a full house (three of one kind, two of another kind) is more desirable than a flush (five of a suit). Let X be the number of possible full house hands, and Y be the number of possible flush hands in a standard 52-card deck. What is X/Y? (Include royal flushes, which consist of a 10, Jack, Queen, King, and Ace of a suit, as well as straight flushes, which consist of five consecutive cards of the same suit, when counting flushes.)

A. $8/11$ B. $26/33$ C. $33/26$ D. $11/8$ E. NOTA

28. Chris Rycroft has a deck of 4 cards, consisting of the numbers 1 through 4. You choose 3 at random and with replacement, and you win if the numbers on the three cards you choose form the side lengths of a nondegenerate triangle. What is the probability that you win?

A. $7/16$ B. $15/32$ C. $17/32$ D. $5/8$ E. NOTA

29. Honty Mall invites you to play his game! He has three doors, numbered Door 1, Door 2, and Door 3. For each door, there is a goat behind the door with probability $\frac{2}{3}$, and a car behind the door with probability $\frac{1}{3}$. Assume that the contents behind each door are independent. You pick a door, and Honty then opens Door 2. You can then stay with your current choice of door or switch to the unopened door. Your strategy is to pick Door 1, and to switch to the unopened door, Door 3, with probability $\frac{1}{2}$. Given that Honty opens Door 2 to reveal a goat, what is the probability that you win a car?

A. $2/9$ B. $1/3$ C. $4/9$ D. $5/9$ E. NOTA

30. The rapper B. Tshishi is coincidentally at the same casino! He wants impress the casino's guests, so he claims to know the number of zeros at the end of 1011!, when expanded completely. He confides to you that he needs your help, however – what is the correct number of zeros?

A. 202 B. 250 C. 251 D. 310 E. NOTA