

- Given the system $\begin{cases} 5x + 9y = 7 \\ 2x + 5y = 5 \end{cases}$, compute $x - y$.
A. -3 B. -2 C. 2 D. 3 E. NOTA
- Compute $\cos\left(\frac{17\pi}{3}\right)\tan\left(\frac{14\pi}{3}\right) + \sin\left(-\frac{11\pi}{6}\right)\cot\left(\frac{7\pi}{2}\right) + \csc\left(\frac{13\pi}{6}\right)\sec\left(-\frac{23\pi}{6}\right)$.
A. $-\frac{11\sqrt{3}}{6}$ B. $-\frac{5\sqrt{3}}{6}$ C. $\frac{5\sqrt{3}}{6}$ D. $\frac{11\sqrt{3}}{6}$ E. NOTA
- Compute $\log_4 343 \cdot \log_{81} 32 \cdot \log_{625} 243 \cdot \log_{49} 125$.
A. 1 B. $\frac{225}{64}$ C. $\frac{11}{2}$ D. $\frac{168}{5}$ E. NOTA
- Evaluate $\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{\sqrt{2x+3} - 3}$.
A. 3 B. 6 C. 9 D. 12 E. NOTA
- Compute the area of the hexagon when the 6 roots of $x^6 = 8$ are connected in the complex plane.
A. $\frac{3\sqrt{3}}{2}$ B. $3\sqrt{3}$ C. $6\sqrt{3}$ D. $96\sqrt{3}$ E. NOTA
- Which of the following is not an asymptote of $f(x) = \frac{e^{2x} - e^x + 2}{e^{2x} - 5e^x + 6}$?
A. $x = \ln 2$ B. $x = \ln 3$ C. $y = 1$ D. $y = \frac{1}{3}$ E. NOTA
- Which of the following is a factor of $a^2 - b^2 + 6a + 4b + 5$?
A. $a + b + 5$ B. $a + b - 5$ C. $a - b + 5$ D. $a - b - 5$ E. NOTA
- Compute $\sin\left(\tan^{-1}\left(-\frac{3}{4}\right) + \cot^{-1}\left(-\frac{5}{12}\right)\right)$.
A. $-\frac{56}{65}$ B. $-\frac{51}{65}$ C. $\frac{51}{65}$ D. $\frac{56}{65}$ E. NOTA

9. Find sum of the squares of the roots of $f(x) = 2x^3 - 4x^2 + 5x - 2$.
A. 0 B. 4 C. 8 D. 9 E. NOTA
10. The constant term in the expansion of $\left(\frac{x^2}{2} + \frac{2}{x}\right)^{12}$ can be written as $a \cdot 2^b$, where a is an odd integer and b is an integer. Compute the value of $a + b$.
A. 487 B. 491 C. 495 D. 499 E. NOTA
11. Let \vec{u} the projection of $\langle 2, 3, 1 \rangle$ onto $\langle 1, 2, 4 \rangle$. Compute $\|\vec{u}\|$.
A. $\frac{4}{7}$ B. $\frac{6}{7}$ C. $\frac{4\sqrt{21}}{7}$ D. $\frac{6\sqrt{14}}{7}$ E. NOTA
12. The sum of the reciprocals of the positive integral factors of 600 can be expressed as $\frac{a}{b}$ for relatively prime natural numbers a and b . Compute the value of $a + b$.
A. 26 B. 31 C. 36 D. 41 E. NOTA
13. Find the sum of the solutions over $[0, 2\pi)$ for $2 \sin 2x + 2 \sin x + 2 \cos x = -1$.
A. 2π B. 3π C. 5π D. 7π E. NOTA
14. Let A, B, C be 3×3 matrices, with $\det(A) = 2$, $\det(B) = 3$, and $\det(C) = 5$. Compute the determinant of the matrix $(2A)^{-1}B^TC^2$.
A. $\frac{25}{12}$ B. $\frac{75}{16}$ C. $\frac{75}{4}$ D. 75 E. NOTA
15. Consider $\triangle ABC$, where $m\angle A = 30^\circ$ and $AB = 6$. For how many integral lengths of \overline{BC} are there two possible shapes for $\triangle ABC$?
A. 1 B. 2 C. 3 D. 4 E. NOTA
16. Let $f(x) = 3x - 2$, $g(x) = f(x) + 3$, $h(x) = f(x + 3)$. Compute the shortest distance between a point on g and a point on h .
A. $\frac{3\sqrt{10}}{5}$ B. $3\sqrt{2}$ C. $\frac{9\sqrt{10}}{5}$ D. 6 E. NOTA

17. Let a_0, a_1, a_2, \dots be a geometric sequence. If $\sum_{n=0}^{\infty} a_{2n} = 3$ and $\sum_{n=0}^{\infty} a_{3n} = 2$, compute $\sum_{n=0}^{\infty} a_{4n}$.
- A. 1 B. $\frac{4}{3}$ C. $\frac{3}{2}$ D. $\frac{5}{2}$ E. NOTA
18. An ellipse has its minor axis lying on the line $2x - y = 3$, with one endpoint at $(-2, -7)$. If one of its foci is at $(6, -1)$, compute the length of its latus rectum.
- A. 8 B. 12 C. 16 D. 24 E. NOTA
19. Each of two beakers contains 100mL of 20% acid solution. Some amount of pure acid is added to the first beaker, and some amount of water is evaporated from the second beaker. Both actions result in doubling the concentration. Which of the following is the closest to the amount of acid solution in the end? (That is, in both beakers combined, measured in mL.)
- A. 176 B. 186 C. 196 D. 206 E. 216
20. How many points do the polar graphs $r = 10 \cos 10\theta$ and $r = 5 \sin \theta$ share?
- A. 20 B. 21 C. 22 D. 23 E. NOTA
21. Four dice are rolled. Given that no more than one of them rolled a 6, what is the probability that none of them rolled a 6?
- A. $\frac{5}{9}$ B. $\frac{3}{4}$ C. $\frac{5}{6}$ D. $\frac{625}{1296}$ E. NOTA
22. Given that no complex number z can satisfy all three of the following equations simultaneously, how many complex numbers z satisfy two of the three equations?
- $$|z - 3| = 4$$
- $$|z + 2| + |z - 6| = 10$$
- $$|z + 3| = |z + 3i|$$
- A. 5 B. 6 C. 7 D. 8 E. NOTA
23. Find the area of a triangle with side lengths $\sqrt{5}$, $\sqrt{41}$, and $2\sqrt{13}$.
- A. 1 B. $\frac{13}{2}$ C. 7 D. 13 E. NOTA

24. In $\triangle ABC$, $AB = 12$, $BC = 8$, and $\tan B = -\frac{\sqrt{5}}{2}$. Let P be the centroid of $\triangle ABC$. Compute the length PC .
- A. $\frac{2\sqrt{41}}{3}$ B. $\frac{4\sqrt{14}}{3}$ C. $\frac{4\sqrt{41}}{3}$ D. $\frac{8\sqrt{14}}{3}$ E. NOTA
25. Given the polar graph $r = 1 + 2 \cos \theta$, if a value of θ is randomly selected from the interval $[0, 2\pi)$, what is the probability that the point is on the inner loop of the limaçon?
- A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. NOTA
26. In $\triangle ABC$, $\tan A = \frac{8}{7}$. Which of the following is closest to the value of $\sin 5A$?
- A. -0.8 B. -0.4 C. 0 D. 0.4 E. 0.8
27. Trapezoid $ABCD$ with bases $AB = 2$ and $CD = 18$ has both an inscribed circle and a circumscribed circle. Let O be the center of its circumscribed circle. Compute the distance from O to \overline{AB} .
- A. $\frac{11}{3}$ B. $\frac{19}{3}$ C. 9 D. $\frac{29}{3}$ E. NOTA
28. How many 4-digit integers contain at least three distinct digits, but no 0 at all?
- A. 3024 B. 4536 C. 6048 D. 7560 E. NOTA
29. Find the amplitude of $f(x) = 3 \sin\left(x - \frac{\pi}{4}\right) + 4 \cos\left(x + \frac{\pi}{4}\right)$.
- A. 1 B. 5 C. 7 D. 12 E. NOTA
30. You are given 3 irregularly shaped containers. When full, they contain 3, 7, and 10 liters of liquid. Initially, the 10-liter container is full of water. You can transfer water from one container to another, but you have no additional ways to measure. Therefore, precision is only guaranteed if either the container transferred from is empty, or the container transferred to is full. What is the minimum number of transfers needed so that each of the 7-liter and 10-liter containers contains 5 liters of water? Assume no water is spilled in the process.
- A. 8 B. 9 C. 10 D. 11 E. NOTA