1. A Multiply the second equation by -2, and add to the first equation to get that $x - y = -3$.

2. C $\left(\frac{1}{2}\right)(-\sqrt{3}) + \left(\frac{1}{2}\right)(0) + (2)\left(\frac{2\sqrt{3}}{3}\right) = -\frac{\sqrt{3}}{2} + \frac{4\sqrt{3}}{3} = \frac{5\sqrt{3}}{6}$

3. B $\frac{3\ln 7}{2\ln 2} \cdot \frac{5\ln 2}{4\ln 3} \cdot \frac{5\ln 3}{4\ln 5} \cdot \frac{3\ln 5}{2\ln 7} = \frac{225}{64}$

4. B $\lim_{x\to 3} \frac{(x-1)(x-3)(\sqrt{2x+3}+3)}{(\sqrt{2x+3}-3)(\sqrt{2x+3}+3)} = \lim_{x\to 3} \frac{(x-1)(x-3)(\sqrt{2x+3}+3)}{2x+3-9} = \lim_{x\to 3} \frac{(x-1)(\sqrt{2x+3}+3)}{2} = 6$

5. B The hexagon has a radius of $\sqrt[6]{8} = \sqrt{2}$. So the area is $6 \cdot \frac{1}{2} \cdot (\sqrt{2})^2 \sin 60° = 3\sqrt{3}$.

6. E $f(x) = \frac{e^{2x}-e^x+2}{(e^x-2)(e^x-3)}$. So $x = \ln 2$ and $x = \ln 3$ are both vertical asymptotes. Note that $\lim_{x\to\infty} f(x) = 1$ and $\lim_{x\to-\infty} f(x) = \frac{1}{3}$, so the graph has 2 horizontal asymptotes.

7. C Completing the square on the a terms and the b terms to get $a^2 + 6a + 9 - (b^2 - 4b + 4) = (a+3)^2 - (b-2)^2 = (a+b+1)(a-b+5)$

8. E Let $x = \tan^{-1}\left(-\frac{3}{4}\right)$ and $y = \cot^{-1}\left(-\frac{5}{12}\right)$, then x is in quadrant 4 and y is in quadrant 2. Thus $\sin(x+y) = \sin x \cos y + \cos x \sin y =$ $\left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) = \frac{63}{65}$

9. E Let r, s, t be the 3 roots. Then $r^2 + s^2 + t^2 = (r+s+t)^2 - 2(rs+st+tr) =$ $\left(\frac{4}{2}\right)^2 - 2\left(\frac{5}{2}\right) = -1$

10. D The constant term is $\binom{12}{4}\left(\frac{x^2}{2}\right)^4\left(\frac{2}{x}\right)^8 = \frac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2\cdot 1} \cdot \frac{1}{2^4} \cdot 2^8 = 495 \cdot 2^4$. So $a + b =$ 499.

11. C Let θ be the angle between the two vectors. $\| < 2,3,1 > \| = \sqrt{14}$, and the magnitude of the projection is $\sqrt{14}\cos\theta = \sqrt{14} \cdot \frac{<2,3,1>\cdot<1,2,4>}{\sqrt{14}\cdot\sqrt{21}} = \frac{12}{\sqrt{21}} = \frac{4\sqrt{21}}{7}$.

12. D $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{300} + \frac{1}{600} = \frac{600+300+\cdots+3+2+1}{600}$, so the sum of the reciprocals is simply sum of the positive integral factors over 600. $600 = 2^3 \cdot 3 \cdot 5^2$, so the desired sum is $\frac{(2^3+2^2+2+1)(3+1)(5^2+5+1)}{600} = \frac{15\cdot 4\cdot 31}{600} = \frac{31}{10}$. $31 + 10 = 41$.

13. C Using double angle, $4\sin x\cos x + 2\sin x + 2\cos x + 1 = 0$. Factoring the left side, $(2\sin x + 1)(2\cos x + 1) = 0$. So $\sin x = -\frac{1}{2}$ or $\cos x = -\frac{1}{2}$. Therefore $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{2\pi}{3}, \frac{4\pi}{3}$, which add to 5π.

14. B $\det(2A) = 2^3 \det(A) = 16$, so $\det((2A)^{-1}) = \frac{1}{16}$. $\det(B^T) = \det(B) = 3$. $\det(C^2) = (\det(C))^2 = 25$. Therefore, $\det((2A)^{-1}B^TC^2) = \frac{75}{16}$.

15. B BC must be long enough to reach side AC, so $BC \geq 6\sin 30° = 3$. However, when $BC = 3$, there is only one shape (the right triangle). Once $BC \geq AC$, there is also only one shape, as BC can no longer be located on either side of the height from B to AC. So the only possibilities are 4 and 5.

16. A $g(x) = 3x + 1$, $h(x)$ goes through $(-3, -2)$. We can compute the distance as $\left|\frac{3(-3)-(-2)+1}{\sqrt{3^2+1^2}}\right| = \frac{6}{\sqrt{10}} = \frac{3\sqrt{10}}{5}$.

17. E The given information is that $\frac{a_0}{1-r^2} = 3$ and $\frac{a_0}{1-r^3} = 2$. Then $a_0 = 3(1-r^2)$ from the first equation. Substitute into the second equation to get $\frac{3(1-r)(1+r)}{(1-r)(1+r+r^2)} = 2$. Continue to solve, $3 + 3r = 2 + 2r + 2r^2$, $2r^2 - r - 1 = 0$, $(2r+1)(r-1) = 0$, or $r = -\frac{1}{2}, 1$. Since the infinite series converges, $r \neq 1$. Thus $\frac{a_0}{1-\frac{1}{4}} = 3$, or $a_0 = \frac{9}{4}$.

Then $\frac{a_0}{1-r^4} = \frac{\frac{9}{4}}{\left(1-\frac{1}{16}\right)} = \frac{9}{4} \cdot \frac{16}{15} = \frac{12}{5}$.

18. C The distance from the focus to the end point of the minor axis is $a = \sqrt{8^2 + 6^2} = 10$ The distance from the focus to the minor axis is $c = \frac{|12+1-3|}{\sqrt{4+1}} = \frac{10}{\sqrt{5}} = \sqrt{20}$.

Then $b^2 = a^2 - c^2 = 100 - 20 = 80$, and the length of the latus rectum is $\frac{2b^2}{a} = 16$

19. B Both beakers start with 20mL of acid and 80mL of water. In the final solutions, the ratio acid make up 40% of the solution, and the water make up 60% of the solution. For both beakers, we will look at the part that does not change. The final volume of the first beaker is $80\left(\frac{5}{3}\right) = \frac{400}{3} \approx 133$. The final volume of the second beaker is $20\left(\frac{5}{2}\right) = 50$. So the total volume is approximately 183mL.

20. B $r = 5\sin\theta$ is a circle of diameter 5 with a diameter along the positive y-axis. $r = 10\cos 10\theta$ is a 20-petal rose of length 10. There is a petal on each side of each axis, and 4 petals within each quadrant.
Clearly, the two graphs share the origin. In addition, the petals along the positive and negative x-axis intersect the circle once each, while the petals in quadrants 1 and 2, as well as the one along the positive y-axis intersect the circle twice each. This makes up a total of 21 points of intersection.

21. A There are $5^4 = 625$ ways to roll four dice without a 6. For there to be exactly one 6, there are $4 \cdot 5^3 \cdot 1 = 500$ ways. So the desired probability is $\frac{625}{625+500} = \frac{5}{9}$.

22. C The locus of points satisfying $|z-3|=4$ is a circle of radius 4 centered at $(3,0)$. The locus of points satisfying $|z+2|+|z-6|=10$ is an ellipse with major axis 10 with foci at $(-2,0)$ and $(6,0)$. The locus of points satisfying $|z+3|=|z+3i|$ are all points equidistance to $(-3,0)$ and $(0,-3)$, or the line $y=x$.
Sketching the graphs of the three shapes, there are 7 points in common. The line intersect the circle and the ellipse twice each, while the circle and ellipse are tangent at $(7,0)$ and intersect at two other points.

23. C Call x the angle opposite of the side $\sqrt{41}$. Then we can compute $\cos x$ through law of cosines: $5+52-2\sqrt{5}\sqrt{52}\cos x=41$, so $\cos x=\frac{8}{\sqrt{260}}$. Then $\sin x=\frac{14}{\sqrt{260}}$. Thus the area is $\frac{1}{2}\cdot\sqrt{5}\cdot\sqrt{52}\cdot\frac{14}{\sqrt{260}}=7$.

24. C The centroid is the intersection of the medians, two thirds of the way from the vertex to the midpoint. Let M be the midpoint of AB, then we can compute MC by applying law of cosine on ΔMBC, with $MB=6$ and $\cos B=-\frac{2}{3}$. So
$MC^2=6^2+8^2-2\cdot6\cdot8\cdot\left(-\frac{2}{3}\right)=164$, and $PC=\frac{2}{3}MC=\frac{2}{3}\cdot2\sqrt{41}=\frac{4\sqrt{41}}{3}$.

25. C While tracing through a looped limacon, the graph is on the looped portion when r is negative. $1+2\cos\theta<0$ when $\cos\theta<-\frac{1}{2}$, or $\frac{2\pi}{3}<\theta<\frac{4\pi}{3}$, for a probability of $\frac{1}{3}$.

26. A While computing $\sin 5A$ is possible, it is not realistic. Instead, we need to make a reasonable approximation. A is an angle in a triangle, and since $\tan A>0$, A must be acute. Further, $\frac{8}{7}$ is between $\tan\frac{\pi}{4}=1$ and $\tan\frac{\pi}{3}=\sqrt{3}$, so $5A$ is between $\frac{5\pi}{4}$ and $\frac{5\pi}{3}$. This gives a range of $\sin\frac{3\pi}{2}=-1$ to $\sin\frac{5\pi}{4}=-\frac{\sqrt{2}}{2}\approx-0.7$ for $\sin 5A$.

27. D Let E, F, and G be the point of tangency of the inscribed circle on sides AB, BC, and CD respectively. Clearly, E and G must be midpoints of AB and CD. Then $BE=BF=1$ and $CF=CG=9$, or $BC=10$.
Let H be on CD such that $BH\perp CD$. Then $BHGE$ is a rectangle. Thus,
$CH=CG-GH=CG-BE=8$. By Pythagorean theorem $BH^2=CB^2-CH^2$. In other words, $BH=6$, which is the distance between AB and CD.

Next, we will inspect the circumcenter. O is necessarily on line EG, as EG is the perpendicular bisector of both chords AB and CD.
Let $OE=x$, then OG can be either $x-6$, if AB and CD are on the same side of O, or $6-x$, if AB and CD are on opposite sides.
In ΔOEB, $OB^2=OE^2+EB^2=x^2+1^2$.
In ΔOGC, $OC^2=OG^2+GC^2=(x-6)^2+9^2$. Note that the two cases of OG yield the same equation, as the side length is squared.
Both OB and OC are radii, so $x^2+1=x^2-12x+36+81$,
Salving to get $12x=116$, or $x=\frac{29}{3}$.

28. C There are $9 \cdot 8 \cdot 7 \cdot 6 = 3024$ 4-digit integers without 0 and 4 distinct digits. For ones with exactly 3 distinct digits, one of the digits must occur twice, so there are 9 ways to select that digit, and $\binom{8}{2} = 28$ ways to select the other 2 digits. Once the digits are selected, there are $\frac{4!}{2} = 12$ ways to arrange the 4 selected digits. This makes for a total of $3024 + 9 \cdot 28 \cdot 12 = 6048$ integers that satisfy the given condition.

29. A The sine graph is shifted $\frac{\pi}{4}$ to the right, and the cosine graph is shifted $\frac{\pi}{4}$ to the left, taking the two graphs perfectly out of phase with each other. So the amplitude is simply $4 - 3 = 1$.

30. B This can be accomplished in 9 transfers. The algorithm is to prioritize transfer in the following manner until the goal is accomplished.

1. Transfer from 3 to 10 if 3 is full.
2. Transfer from 7 to 3 if 7 has water and 3 is not full.
3. Transfer from 10 to 7 if 7 is empty.

The 3 and 7 liter containers are interchangeable within this algorithm. However, the flipped version takes one extra transfer to complete the task.

The chart below shows the volume of water within each container after the Nth transfer.

N	0	1	2	3	4	5	6	7	8	9
3	0	0	3	0	3	0	1	1	3	0
7	0	7	4	4	1	1	0	7	5	5
10	10	3	3	6	6	9	9	2	2	5