

- 1) **B** – $\log_3 3x = 3 \rightarrow 3^3 = 27 \rightarrow 3x = 27 \rightarrow x = 9$.
- 2) **C** – $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ is one of the definitions of e .
- 3) **A** – $e^{i\theta} = \cos \theta + i \sin \theta$
- 4) **A** – The PageRank Score works on a logarithmic (base 10) scale. Therefore, a difference of k in the PageRank between two pages means the page with the higher score receives 10 times more visits than the other page. Thus, in this case, the difference of $3k$ in PageRank means legendsonly.com receives $10^3 = 1000$ times more visits than penguinonly.com. So, the answer is $16500 \times 1000 = 16,500,000$.
- 5) **B** – $\sqrt{1406 - \sqrt{1406 - \sqrt{1406} \dots}} = x \rightarrow \sqrt{1406 - x} = x \rightarrow 1406 - x = x^2 \rightarrow x^2 + x - 1406 \rightarrow (x + 38)(x - 37) \rightarrow$ The answer must be positive, so $x = 37$.
- 6) **D** – Convert all the logs to base 10, giving: $\frac{\log 2 + \log 4 + \log 6 + \log 8 + \log 12 + \log 2}{\log 48} =$
 $\frac{\log 110,592}{\log 48} = 3$.
- 7) **D** – $\frac{x^2(y^{\frac{1}{2}})^3 z^5}{x^{-5} y^{\frac{-1}{2}} (z^{10})^{\frac{1}{2}}} = x^7 y^2$
- 8) **C** – $\begin{vmatrix} 2 & 3 & 2 \\ 2 & 3 & 1 \\ 4 & 9 & 1 \end{vmatrix} = 6 \rightarrow \log_{\sqrt{6}} 6 = x \rightarrow x = 2$
- 9) **B** – The x-coordinate of the maximum is found with the formula $\frac{-b}{2a}$ given quadratic equation $ax^2 + bx + c$. So, $\frac{-b}{2a} = \frac{-9}{-4}$. Plug it into $v(t)$ to get $\frac{49}{8}$.
- 10) **B** – $v(t)$ factors as $-(2t - 1)(t - 4)$, so its roots are $\frac{1}{2}$ and 4. She reaches maximum velocity at $t = \frac{9}{4}$, so she has been running for $\frac{7}{4}$, or 1.75, seconds.

- 11) **E** – Make the substitution $\log x = y$. Equation now is $-17y + y^2 + 74 = 2 \rightarrow y^2 - 17y + 72 \rightarrow (y - 8)(y - 9) \rightarrow y = 8, 9 \rightarrow \log x = 10^8, \log x = 10^9 \rightarrow 10^9 + 10^8 = 1100000000$.
- 12) **B** – By inspection, the solutions to the equation should be in the form 4001^a , where a is any value. Plugging in 4001^a for x , you get $\sqrt[3]{4001} \cdot 4001^{a^2} = 4001^{8a} \rightarrow 4001^{a^2 + \frac{1}{3}} = 4001^{8a} \rightarrow a^2 + \frac{1}{3} = 8a \rightarrow a^2 - 8a + \frac{1}{3} = 0$. The variable a can take on multiple values, and the product of the solutions to this equation will be the sum of the values a as a power of 4001, so the answer is 4001^8 .
- 13) **B** – $\log_2(\log_3(\log_7(\log_{15} C))) = 13 \rightarrow \log_3(\log_7(\log_{15} C)) = 2^{13} \rightarrow \log_7(\log_{15} C) = 3^{2^{13}} \rightarrow \log_{15} C = 7^{3^{2^{13}}} \rightarrow C = 15^{7^{3^{2^{13}}}}$. Since the prime factorization of $15 = 3 * 5$, the answer is 2.
- 14) **A** – $\sin(2x) = 2 \sin(x) \cos(x) \rightarrow \sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$, $\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$ in Euler's form. Therefore, $\sin(2x) = 2 * \frac{e^{ix} - e^{-ix}}{2i} * \frac{e^{ix} + e^{-ix}}{2} = \frac{e^{2ix} - e^{-2ix}}{2i}$.
- 15) **A** - The only real solutions are 2 and 4 (this can be seen graphically), so the sum of the solutions is 6.
- 16) **B** - $\frac{\log 625}{\log 11} \cdot \frac{\log 243}{\log 7} \cdot \frac{\log 14641}{\log 5} \cdot \frac{\log 16807}{\log 3} = \frac{\log 625}{\log 5} \cdot \frac{\log 243}{\log 3} \cdot \frac{\log 14641}{\log 11} \cdot \frac{\log 16807}{\log 7} = 4 \cdot 5 \cdot 4 \cdot 5 = 400$.
- 17) **C** - $\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3} = \lim_{x \rightarrow 3} \frac{(\sqrt{x} - \sqrt{3})(\sqrt{x} + \sqrt{3})}{(x - 3)(\sqrt{x} + \sqrt{3})} = \lim_{x \rightarrow 3} \frac{1}{(\sqrt{x} + \sqrt{3})} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$
- 18) **E** - $f(x) = \sqrt{\frac{\cos^2(x) - \sin^2(x)}{1 - \tan^2(x)}} = \sqrt{\frac{\cos(2x)}{1 - \tan^2(x)}}$. Plugging in $\frac{3\pi}{8}$ for x , we get $\frac{\sqrt{2 - \sqrt{2}}}{2}$.

19) **D**—For $f(x)$ to be defined, we should have $-1 \leq \log_2\left(\frac{x^2}{2}\right) \leq 1 \rightarrow \frac{1}{2} \leq \frac{x^2}{2} \leq 2 \rightarrow$

$$1 \leq x^2 \leq 4 \rightarrow [-2,1] \cup [1,2]$$

20) **A**—The range of the function is just the range of $\sin^{-1} x$, which is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

21) **C**—Let $\log_2 a = \log_3 b = \log_6 c = \log_7(a + b + c) = x \rightarrow 2^x = a, 3^x = b, 6^x =$

$$c, 7^x = a + b + c \rightarrow 2^x + 3^x + 6^x = 7^x \rightarrow \left(\frac{2}{7}\right)^x + \left(\frac{3}{7}\right)^x + \left(\frac{6}{7}\right)^x = 1 \rightarrow x = 2 \rightarrow$$

$$\log_{4 \cdot 9 \cdot 36} 6 = \frac{1}{4}$$

22) **C**—The domain of $\frac{1}{\log(1-x)}$ is $(-\infty, 0)$ or $(0,1)$ and the domain of $\sqrt{x+2}$ is $[-2, \infty)$. The

intersection of these is $[-2,0) \cup (0,1)$.

23) **B** - $\log_3(x-5) - \log_{27}(79x-185) \rightarrow \frac{\log(x-3)}{\log 3} - \frac{\log(79x-185)}{3\log 3} \rightarrow$

$$\frac{3\log(x-3) - \log(79x-185)}{3\log 3} \rightarrow 3\log(x-3) - \log(79x-185) = 0 \rightarrow \frac{(x-3)^3}{79x-185} = 1 \rightarrow x^3 -$$

$$15x^2 - 4x + 60 = 0 \rightarrow (x-2)(x+2)(x-15) \rightarrow x = 15. X \text{ cannot equal } 2 \text{ or } -2$$

because then the $\log(x-3)$ will be undefined.

24) **B**—The hypotenuse of the triangle is e^x , and the side opposite the angle is $2 \ln x$. Thus,

the side adjacent to the angle is $\sqrt{e^{2x} - 4(\ln x)^2}$. The tangent of an angle is opposite over

adjacent, so you get $\frac{2 \ln x}{\sqrt{e^{2x} - 4(\ln x)^2}}$.

25) **A** - $y = 2^{x(x-1)} \rightarrow x^2 - x - \log_2 y = 0$. Using the quadratic formula, you get

$$\frac{1 \pm \sqrt{1 + 4 \log_2 y}}{2}. f(x) \geq 1, \text{ so } x = \frac{1}{2} \left(1 + \sqrt{1 + 4 \log_2 y}\right) \rightarrow f^{-1}(x) = \frac{1}{2} \left(1 + \sqrt{1 + 4 \log_2 y}\right).$$

26) **D** - $f(x) = \log(x + \sqrt{x^2 + 1})$. $f(-x) = \log(-x + \sqrt{x^2 + 1}) = \log \frac{-x^2 + x^2 + 1}{x + \sqrt{x^2 + 1}} =$

$$-\log(x + \sqrt{x^2 + 1}) = -f(x) \rightarrow f(-x) = -f(x), \text{ so, the function is odd.}$$

27) **E** – Plugging in $\frac{\pi}{2}$ for $f'(x)$ gives us -1. So, using point-slope form for the equation of the

line, we get $y - 1 = -1 \left(x - \frac{\pi}{2} \right) \rightarrow y = -x + 1 + \frac{\pi}{2} \rightarrow$ y-intercept is $1 + \frac{\pi}{2}$.

28) **C** - $x \log_x 10 = x \cdot \frac{\log 10}{\log x} = \frac{x}{\log x}$. So going back to the original equation, and dividing

both sides by $\log x$, we have $\frac{2x}{\log x} = 1 - \frac{\ln x}{\log x} = 1 - \frac{\log x}{\log e} \cdot \frac{1}{\log x} = 1 - \frac{\log 10}{\log e} = 1 - \ln 10$.

Therefore, $\frac{x}{\log x} = \frac{1}{2} \left(1 + \ln \frac{1}{10} \right)$, and $\frac{a}{b} = 5$.

29) **C** – The measured amplitude of the earthquake is the amplitude of $150 \sin x + 360 \cos x$,

which is $\sqrt{150^2 + 360^2} = 390$. So, $magnitude = \log \frac{A}{A_0} \rightarrow \log \frac{390}{1.95} = \log 200 =$

$\log 2 + \log 100 = \log(2) + 2 = 2.301$

30) **D** – The domain of $1 - x$ is all reals except 1; the domain of $\sqrt{4 - x^2}$ is $[-2, 2]$; the

domain of $\ln x$ is $(1, \infty)$; and the domain of the sine function is all reals. The intersection

of all these is $(-2, 1)$.