1. C $r = 5\cos\Theta - 8\sin\Theta$
$r^2 = 5\,\text{r}\cos\Theta - 8\,\text{r}\sin\Theta$
$x^2 + y^2 = 5x - 8y$
$x^2 + 5x + \frac{25}{4} + y^2 - 8y + 16 = \frac{25}{4} + 16$
$\left(x - \frac{5}{2}\right)^2 + (y-4)^2 = \frac{89}{4}$
$(x-h)^2 + (y-k)^2 = radius^2$
Area of a circle = $\pi(radius)^2 = \frac{89\pi}{4}$ (c)

2. C $\sin\left(\frac{\pi x^2}{3}\right) = 1 \rightarrow \frac{\pi x^2}{3} = \frac{\pi}{2}$ since $-2 \leq x \leq 2$ Which gives (x,y) = $(\pm\frac{\sqrt{6}}{2}, \pm\frac{4-\sqrt{6}}{2})$ (b)

3. D $\cos\theta = \frac{a\cdot b}{|a||b|} = \frac{<2,4,5> \cdot <3,2,7>}{|<2,4,5>||<3,2,7>|} = \frac{49}{3\sqrt{5}*\sqrt{62}} = \frac{49\sqrt{310}}{930}$ (d)

4. B $\frac{\cos 25^\circ \cos 65^\circ \cos 50^\circ \cos 100^\circ \cos 200^\circ}{\sin 40^\circ} = \frac{\cos 25^\circ \sin 25^\circ \cos 65^\circ \cos 50^\circ \cos 100^\circ \cos 200^\circ}{\sin 40^\circ \sin 25^\circ} =$
$\frac{\sin 50^\circ \cos 65^\circ \cos 50^\circ \cos 100^\circ \cos 200^\circ}{2\sin 40^\circ \sin 25^\circ} = \frac{\cos 65^\circ \sin 100^\circ \cos 100^\circ \cos 200^\circ}{4\sin 4\ ^\circ \sin 25^\circ} =$
$\frac{\cos 65^\circ \sin 200^\circ \cos 200^\circ}{8\sin 40^\circ \sin 25^\circ} = \frac{\cos 65^\circ \sin 40^\circ}{16\sin 40^\circ \sin 25^\circ} = \frac{1}{16}$ (b)

5. C Call tan x = a and tan y = b. Then we have $\frac{1}{a} + \frac{1}{b} = 7 \rightarrow \frac{a+b}{7} = ab \rightarrow ab = 6/7$

Then the tangent sum formula gives $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan(x)tan(y)} = \frac{a+b}{1-ab} = 42$ (c)

6. B The following function factors to $f(x) = (sin^2(x) + cos^2(x))(3 + \cos^2(x))$
$= 3 + \cos^2(x)$
Since the range of $\cos^2(x)$ is [0,1[, the range of the functions is [3,4] (b)

7. A From the known fact that the inradius is the area over the semiperimeter we have
$\frac{qrsin\,P}{p+q+r} = \frac{q+r-p}{2}$
This gives $2qrsin(P) = q^2 + r^2 + 2qr - p^2$
By law of cosines we have$q^2 + r^2 + 2qrcos\,P = p^2$ or $q^2 + r^2 + 2qr - p^2 = 2qr(1+\cos P)$
From this we have that $\sin P = 1 + \cos P \rightarrow \sin P - \cos P = 1$
Squaring gives $1 - \sin 2P = 1$
Solving gives $\angle P = \pi/2$ (A)

8. B If $0 < x \leq \pi$, then we know that $\sin x \geq 0$ and $x^3 + x^2 + 4x > 0$. If the domain is $\pi < x \leq 2\pi$, then clearly $x^3 + x^2 + 4x > 2$ since the terms will all be positive. Thus there is no solution for $x \neq 0$. The only solution that is possible is x = 0, so only 1 solution (b)

9. A $\sin(105°) = \sin(60+45) = \sin(60°)\cos(45°) + \sin(45°)\cos(60°) = \frac{\sqrt{6}+\sqrt{2}}{4}$ (a)

10. B $A = \cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}$

Multiplying both sides by $\sin\frac{\pi}{7}$ we get $\sin\frac{\pi}{7}A = \sin\frac{\pi}{7}\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}$
Where $\sin\frac{2\pi}{7} = \frac{1}{2}\sin\frac{\pi}{7}\cos\frac{\pi}{7}$.

Using double-angle for the sin functions we'll eventually get $A = \frac{\sin\frac{8\pi}{7}}{8\sin\frac{\pi}{7}} = -\frac{1}{8}$ (b)

11. A $\tan(\arccos(\sin(-\frac{\pi}{6}))) = \tan\left(\arccos\left(-\frac{1}{2}\right)\right) = \tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$ (a)

12. D From the well-known fact $\sin(x-y) = \sin(x)\cos(y) - \sin(y)\cos(x)$

we get $x - y = \sin^{-1}(\sin(x)\cos(y) - \sin(y)\cos(x))$

Call $\sin(x) = \frac{1}{\sqrt{n}}$ so then we know $\cos(x) = \sqrt{1-\frac{1}{n}} = \frac{\sqrt{n-1}}{\sqrt{n}}$

Call $\cos(y) = \frac{\sqrt{n}}{\sqrt{n+1}}$ therefore $\sin(y) = \sqrt{1-\frac{n}{n+1}} = \frac{1}{\sqrt{n+1}}$

Thus, we have

$x - y = \sin^{-1}(\frac{1}{\sqrt{n}}\cdot\frac{\sqrt{n}}{\sqrt{n+1}} - \frac{1}{\sqrt{n+1}}\cdot\frac{\sqrt{n-1}}{\sqrt{n}}) = \sin^{-1}(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n+1}})$

$$x = \sin^{-1}\frac{1}{\sqrt{n}}$$

$$y = \sin^{-1}\frac{1}{\sqrt{n+1}}$$

$\sum_{n=1}^{\infty}\sin^{-1}(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n+1}}) = \sum_{n=1}^{\infty}\sin^{-1}(\frac{1}{\sqrt{n}}) - \sum_{n=1}^{\infty}\sin^{-1}(\frac{1}{\sqrt{n+1}})$ which telescopes to answer $\sin^{-1}1 = \frac{\pi}{2}$ (d)

13. B By DeMoivre's we have $\sum_{k=1}^{2019} cis(2\pi k)$ Therefore our total sum is 2019. (b)

14. B $\prod_{n=1}^{89}(\tan n°\cos 1° + \sin 1°) = \prod_{n=1}^{89}\frac{\sin n°\cos 1° + \cos n°\sin 1°}{\cos n°} = \prod_{n=1}^{89}\frac{\sin(n°+1°)}{\cos n°} = \frac{\sin 2°\sin 3°\sin 4°\ldots\sin 90°}{\cos 1°\cos 2°\cos 3°\ldots\cos 89°} = \frac{\cos 88°\cos 87°\cos 86°\ldots\cos 1°\sin 90°}{\cos 1°\cos 2°\cos 3°\ldots\cos 89°} = \frac{\sin 90°}{\sin 1°} = \csc 1°$ (b)

15. E An odd function is defined as when

$$f(x) = -f(x)$$

When testing this for all the functions, this satisfies for I, II, and IV (e)

16. C This expression is equivalent to $\cos\left(x+\frac{3\pi}{2}\right) = \sin x$. Given the domain is in quadrant IV, the value of $\sin x = -3/5$ (c)

17. A By product-to-sum we have

$\frac{\frac{1}{2}(\cos(68)+\cos(60)) - \frac{1}{2}(\cos(112)+\cos(60))}{\frac{1}{2}(\cos(112)+\cos(30)) - \frac{1}{2}(\cos(68)+\cos(30))} = \frac{\frac{1}{2}(\cos(68)-\cos(112))}{\frac{1}{2}(\cos(112)-\cos(68))} = -1$

18. A Factoring the left hand side we get

$$(\sin x + \cos x)(sin^2x - \sin x \cos x + \ cos^2x)$$

$= (\sin x + \cos x)(-\sin x \ cos\ x + 1)$

Moving everything in the equation to one side we get

$$(\sin x + \cos x)(-\sin x \ cos\ x + 1) - \frac{1}{2}(\sin x + \cos x) = 0$$

Factoring we get $(\sin x + \cos x)(-\sin x\ cosx + \frac{1}{2}) = 0$

We then have the equations

$\sin x + \cos x = 0$ and$(-\sin x\ cosx + \frac{1}{2}) = 0$

From the first equation we get the solutions $\frac{3\pi}{4}, \frac{7\pi}{4}$ and second we get $\frac{\pi}{4}, \frac{5\pi}{4}$

The sum is therefore 4π (a)

19. B . By using our knowledge of the unit circle we have $\cos 135° + \sin\frac{7\pi}{6} - \cot 300° - \sec\frac{11\pi}{6} + + \csc 45° - \tan\frac{7\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{1}{2} + \frac{\sqrt{3}}{3} - \frac{2\sqrt{3}}{3} + \sqrt{2} + 1 = \frac{1}{2} + \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3}$ (b)

20. D We can rewrite the expression as $9xsin\ x + \frac{4}{xsin\ x}$

Then by AM-GM, we have that $9xsin\ x + \frac{4}{xsin\ x} \geq 12$. There for our minimum is 12 (d)

21. B $\sin(x + y) = \sin x \cos y + \cos x \sin y = \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) = -\frac{16}{65}$. (b)

22. B Let $S_1 = \cos\left(\frac{2\pi}{2019}\right) + \cos\left(\frac{4\pi}{2019}\right) + \cos\left(\frac{6\pi}{2019}\right) + \cos\left(\frac{8\pi}{2019}\right) \ldots + \cos\left(\frac{2018\pi}{2019}\right)$ and let $S_2 = \cos\left(\frac{-2\pi}{2019}\right) + \cos\left(\frac{-4\pi}{2019}\right) + \cos\left(\frac{-6\pi}{2019}\right) + \cos\left(\frac{-8\pi}{2019}\right) \ldots + \cos\left(\frac{-2018}{2019}\right)$.

Clearly, $S_1 = S_2$ since cosine is an odd function. Further, $1 + S_1 + S_2$ is equivalent to the real part of the sum of the 2019 roots of unity, which is 0. Therefore, $S_1 = S_2 = -\frac{1}{2}$. (b)

23. C . $r = 7(\cos^2 9\Theta - \ \sin^2 9\Theta) = 7\cos(18\theta)$

In $r = a\cos(n\theta)$ if n is an even number, the number of petals is 2n, therefore the answer is 36 (c)

24. C Let O be the airport, A be the point where Steve must turn to catch up to the helicopter, and B be the point where he catches up to the helicopter. Then $OA + AB = 4800$, since Steve has 4 hours of fuel, and $OB = 1200$, as the helicopter also would fly for 4 hours. As numbers are quite large, we will call $OB = a, OA = b$, which makes $AB = 4a - b$. We need to solve for b. $\angle O = 120°$. By law of cosine, we have $a^2 + b^2 - 2ab\cos 120° = (4a - b)^2$ Expanding, $a^2 + b^2 + ab = 16a^2 - 8ab + b^2$, or $a^2 + ab = 16a^2 - 8ab$, since $a = 1200$, $a + b = 16a - 8b$, and $9b = 15a$, or $b = \frac{5}{3}(1200) = 2000$.

25. D The sum telescopes to $\lim_{n\to\infty}(\tan^{-1}(1) - \ \tan^{-1}(n+1))$ which clearly computes to $\frac{\pi}{4} - \ \frac{\pi}{2} = \ - \ \frac{\pi}{4}$ (d)

26. C $\cos(cos(x + \ \pi)) = \cos(-cos(x)) = \cos(\cos(x))$ so the period is π (c)

27. B $\lim_{x\to 0}\frac{\sin x}{\sin 2x} = \lim_{x\to 0}\frac{\sin x}{2\cos \text{x} \sin x} = \lim_{x\to 0}\frac{1}{2\cos x} = \frac{1}{2}$ (b)

28. E $x = n\pi, n \in Z \to 2 < n\pi < 19 \to \frac{2}{\pi} < n < \frac{19}{\pi} \to 0 < n < 7$

So we have 6 solutions. (e)

29. C First find the period of both of the sin and cos functions, which are ½ and 2/5, respectively. The period of the entire function will be the lcm of the two periods, which is 2. (c)

30. E $\tan 3x = \frac{\sin 3x}{\cos 3x} = \frac{3\sin x - 4sin^3(x)}{4cos^3(x) - 3\cos x} = \tan x \cdot \frac{3 - 4sin^2(x)}{4cos^2(x) - 3} = \tan x \cdot$

$\frac{3\left(sin^2(x)+cos^2(x)\right)-4sin^2(x)}{4cos^2(x)-3\left(sin^2(x)+cos^2(x)\right)} = \tan x \cdot \frac{3cos^2(x)-sin^2(x)}{cos^2(x)-3sin^2(x)} = \tan x \cdot \frac{3-\frac{sin^2(x)}{cos^2(x)}}{1-3\frac{sin^2(x)}{cos^2(x)}} =$

$\tan x \cdot \frac{3 - tan^2x}{1-3tan^2x} = \frac{3ta \quad tan^3x}{1-3tan^2x}$ (e)