

Answers:

1. 38
2. $y = -\frac{2}{3}$
3. $\frac{5}{108}$
4. \$40.40
5. -120
6. 18π
7. $-2a + 2b - 4c$
8. 120
9. $(-\infty, 1) \cup [2, \infty)$
10. 100
11. 1150
12. (3, 9)
13. $8\sqrt{3}\pi$
14. 672
15. $\frac{\sqrt{14}}{7}$
16. $[3, \infty)$
17. False
18. (13, -11)
19. 73
20. 2
21. 2,037,172
22. $y = 2x + 7$
23. 29.75
24. $-\frac{20}{21}$
25. $\frac{11}{2}$

Solutions:

1. By Vieta's formula, the sum of the solutions is $-\frac{-76}{2} = 38$.
2. Since the numerator and the denominator both have degree 2, the horizontal asymptote is at the ratio of the leading coefficients: $y = -\frac{2}{3}$.
3. The only ways to roll a sum of 16 are: 6-6-4 (3 ways), 6-5-5 (3 ways), 6-6-5 (3 ways), and 6-6-6 (1 way). Since there are $6^3 = 216$ different possible rolls, the probability is $\frac{10}{216} = \frac{5}{108}$.
4. $A = 1000\left(1 + \frac{.04}{2}\right)^{21} = 1000(1.02)^{21} = 1000(1.0404) = 1040.40$. The interest is \$40.40.
5.
$$\begin{vmatrix} -1 & 2 & -3 \\ 4 & 5 & 6 \\ -7 & 8 & -9 \end{vmatrix} = 45 - 84 - 96 - 105 + 48 + 72 = -120$$
6. $0 = 27x^2 + 12y^2 + 108x - 72y - 108 = 27(x+2)^2 + 12(y-3)^2 - 324$
 $\Rightarrow \frac{(x+2)^2}{12} + \frac{(y-3)^2}{27} = 1$, so the enclosed area is $\pi(2\sqrt{3})(3\sqrt{3}) = 18\pi$.
7. $(2a - 3b - 2c) - (4a - 5b + 2c) = -2a + 2b - 4c$
8. BUBBLE has six letters, three of which are B, so the number is $\frac{6!}{3!} = 120$
9. The function has a horizontal asymptote at $y = 1$ that it never intersects, and vertical asymptotes at $x = 2$ and $x = -2$. These asymptotes completely divide the graph, and since the graph has x-intercepts at $(2\sqrt{2}, 0)$ and $(-2\sqrt{2}, 0)$, part of the range consists of $(-\infty, 1)$. For the interval $(-2, 2)$, since there are no x-intercepts, the graph goes up toward both vertical asymptotes. The point $(0, 2)$ is on the graph, and that is the lowest point on that portion of the graph since as x increases in the interval $(0, 2)$, y increases as well, making the range on this portion $[2, \infty)$, making the range $(-\infty, 1) \cup [2, \infty)$.

10. Since Todd's average over three tests was an 88, the sum of those scores was $3 \cdot 88 = 264$. Since Todd's average over all four tests was a 91, the sum of all four scores was $4 \cdot 91 = 364$. Therefore, Todd's fourth test score was $364 - 264 = 100$.
11. Since the common difference is 3 and $82 = 10 + 24 \cdot 3$, this series has 25 terms. Therefore, the sum of the series is $\frac{25}{2}(10 + 82) = 1150$.
12. Written in vertex form, $-4y + 18 = 2x^2 - 12x \Rightarrow -4y = 2(x - 3)^2 - 36 \Rightarrow y = -\frac{1}{2}(x - 3)^2 + 9$, so the vertex is at $(3, 9)$.
13. $0 = 3x^2 + 4y^2 - 12x + 8y - 32 = 3(x^2 - 4x + 4) + 4(y^2 + 2y + 1) - 32 - 12 - 4 \Rightarrow 3(x - 2)^2 + 4(y + 1)^2 = 48 \Rightarrow \frac{(x - 2)^2}{16} + \frac{(y + 1)^2}{12} = 1$, so the area is $\pi \cdot 4 \cdot 2\sqrt{3} = 8\sqrt{3}\pi$.
14. The constant would be $\binom{9}{3}(2x^2)^3\left(-\frac{1}{x}\right)^6 = 84 \cdot 2^3 \cdot (-1)^6 = 672$.
15. Drawing a right triangle with hypotenuse length 3, the leg adjacent to θ has length $\sqrt{7}$, so the leg opposite to θ has length $\sqrt{2}$ (according to Pythagorean Theorem). Therefore, $\tan \theta = \frac{\sqrt{2}}{\sqrt{7}} = \frac{\sqrt{14}}{7}$.
16. Since $(f \circ g)(x)$ is a polynomial, the domain of the composite function will be the same as the domain of its inner function g . This domain is $[3, \infty)$.
17. A complex number is one of the form $a + bi$, where a and b are integers. Allowing $b = 0$ means that all real numbers are complex numbers, which is contradictory to the statement, making the given statement False.
18. Multiplying the first equation by -4 , the second equation by 3, then adding the two resulting equations yields $x = 13$. Plugging this into either given equation yields $y = -11$, so the solution is the ordered pair $(13, -11)$.
19. $|-48 + 55i| = \sqrt{(-48)^2 + 55^2} = \sqrt{2304 + 3025} = \sqrt{5329} = 73$

20. Let $f(x) = -3x^8 + 3x^4 + 3x^2 + 3$, which is written in standard form. Since this polynomial has exactly one sign change, it has exactly one positive root (which would be a solution to the given equation) by Descartes' Rule of Signs. Further, $f(x) = f(-x)$, so this polynomial also has exactly one negative root, also a solution to the equation. Since 0 is not also a root, this equation has exactly 2 real solutions.

21. By definition, $a_n - a_{n-1} = (n-1)$, $a_{n-1} - a_{n-2} = (n-2)$, $a_{n-2} - a_{n-3} = (n-3)$, ..., $a_2 - a_1 = 1$. Summing all of these equations telescopes the left hand side and yields the equation

$$a_n - a_1 = 1 + 2 + \dots + (n-1) = \frac{(n-1)n}{2} \Rightarrow a_n = a_1 + \frac{(n-1)n}{2} = 1 + \frac{(n-1)n}{2}. \text{ Therefore,}$$

$$a_{2019} = 1 + \frac{2018 \cdot 2019}{2} = 2,037,172.$$

22. The non-vertical asymptote will be the slant asymptote. Since $\frac{2x^2 + x - 3}{x - 3}$

$$= 2x + 7 + \frac{18}{x - 3}, \text{ the non-vertical asymptote is } y = 2x + 7.$$

23. Since this is a quadratic function that opens downward, the maximum will occur at the vertex. The x-coordinate is $x = -\frac{-15}{2(-3)} = -\frac{5}{2}$, so the maximum value is $f\left(-\frac{5}{2}\right)$

$$= -3\left(-\frac{5}{2}\right)^2 - 15\left(-\frac{5}{2}\right) + 11 = -\frac{75}{4} + \frac{75}{2} + 11 = \frac{119}{4} = 29.75.$$

24. The common ratio r satisfies $\frac{45}{64} = -\frac{5}{3}r^3 \Rightarrow r^3 = -\frac{27}{64} \Rightarrow r = -\frac{3}{4}$, so the sum of the infinite

$$\text{geometric series is } \frac{-\frac{5}{3}}{1 - \left(-\frac{3}{4}\right)} = \frac{-\frac{5}{3}}{\frac{7}{4}} = -\frac{20}{21}.$$

25. The area is the absolute value of $\frac{1}{2} \begin{vmatrix} 2 & -4 & 1 \\ -1 & -3 & 1 \\ -5 & 2 & 1 \end{vmatrix} = \frac{1}{2}(-6 + 20 - 2 - 15 - 4 - 4) = -\frac{11}{2}$, which

$$\text{is } \frac{11}{2}.$$