What is the volume of a cylinder with radius 5 and height 21?

Answer: ________________________

Round 1 2 3 4 5

What is the volume of a cylinder with radius 5 and height 21?

Answer: ________________________

Round 1 2 3 4 5
The apothem of an \( n \)-gon measures 21. As \( n \) approaches infinity, what does the area of the polygon approach?

Answer: ________________________

Round 1 2 3 4 5

The apothem of an \( n \)-gon measures 21. As \( n \) approaches infinity, what does the area of the polygon approach?

Answer: ________________________

Round 1 2 3 4 5
What is the maximum number of regions into which 3 planes can split up space?

Answer: ________________________

Round 1 2 3 4 5

What is the maximum number of regions into which 3 planes can split up space?

Answer: ________________________

Round 1 2 3 4 5
Points \( T, S, H, I \) lie on a circle. Chords \( TH \) and \( SI \) are perpendicular and intersect at point \( B \). Let \( TB = 3, IB = 2, \) and \( SB = 6 \). Compute \( HB \).

**Answer:** ________________________

Round 1 2 3 4 5

Points \( T, S, H, I \) lie on a circle. Chords \( TH \) and \( SI \) are perpendicular and intersect at point \( B \). Let \( TB = 3, IB = 2, \) and \( SB = 6 \). Compute \( HB \).

**Answer:** ________________________

Round 1 2 3 4 5
Let triangle $KIM$ have side lengths $KI = 10$ and $IM = 25$, and $m\angle KIM = 150^\circ$. What is the area of triangle $KIM$?

Answer: ________________________

Round 1 2 3 4 5
What is the area of a triangle with vertices at coordinates (1,1), (5,6), and (0,4)?

Answer: ________________________

Round    1    2    3    4    5

What is the area of a triangle with vertices at coordinates (1,1), (5,6), and (0,4)?

Answer: ________________________

Round    1    2    3    4    5
Aaron the ant is on the coordinate plane! He starts at the point (1,2), and he wants to go to grad school, which is at the point (5,5). In one step, Aaron can either move one unit up or one unit right (he cannot move diagonally). In how many ways can Aaron get from (1,2) to (5,5)?

Answer: ________________________
Round 1 2 3 4 5

#7 Geometry – Hustle
MA National Convention 2019

Aaron the ant is on the coordinate plane! He starts at the point (1,2), and he wants to go to grad school, which is at the point (5,5). In one step, Aaron can either move one unit up or one unit right (he cannot move diagonally). In how many ways can Aaron get from (1,2) to (5,5)?

Answer: ________________________
Round 1 2 3 4 5
In quadrilateral $PACK$, $PA = 4$, $AC = 10$, $CK = 7$, and $KP = 6$. If the length of diagonal $PC$ is an integer, what is the positive difference between the maximum possible length of $PC$ and the minimum possible length of $PC$?

Answer: ________________

Round 1 2 3 4 5
In triangle $FAN$, $FA = 4$, $AN = 7$, and $FN = x$. In interval notation, for what values of $x$ is $FAN$ an acute triangle?

Answer: ____________________

Round 1 2 3 4 5

In triangle $FAN$, $FA = 4$, $AN = 7$, and $FN = x$. In interval notation, for what values of $x$ is $FAN$ an acute triangle?

Answer: ____________________

Round 1 2 3 4 5
Find the maximum possible area of a triangle with sides of length 5, 13, and $k$, where $8 < k < 18$.

Answer: ________________

Round 1 2 3 4 5

Find the maximum possible area of a triangle with sides of length 5, 13, and $k$, where $8 < k < 18$.

Answer: ________________

Round 1 2 3 4 5
Triangle $CAM$ has a vertex on each of the $x$-axis, $y$-axis, and $z$-axis. Let $O$ be the origin $(0,0,0)$. Given that $CA = AM = \sqrt{5}$ and $CM = \sqrt{2}$, what is the volume of the tetrahedron $OCAM$?

Answer: _________________

Round 1 2 3 4 5

Triangle $CAM$ has a vertex on each of the $x$-axis, $y$-axis, and $z$-axis. Let $O$ be the origin $(0,0,0)$. Given that $CA = AM = \sqrt{5}$ and $CM = \sqrt{2}$, what is the volume of the tetrahedron $OCAM$?

Answer: _________________

Round 1 2 3 4 5
The diagonals of an equilateral quadrilateral measure 5 and 12. What is the area of this quadrilateral?

Answer: ________________________

Round 1 2 3 4 5

The diagonals of an equilateral quadrilateral measure 5 and 12. What is the area of this quadrilateral?

Answer: ________________________

Round 1 2 3 4 5
In triangle $SUN$, $SN = 2\sqrt{3}$ and $SU = 4$. $UN$ is of length 2 with probability $1/2$ and $2\sqrt{3}$ with probability $1/2$. Compute the expected value of the area of $SUN$.

Answer: ________________

Round 1 2 3 4 5

In triangle $SUN$, $SN = 2\sqrt{3}$ and $SU = 4$. $UN$ is of length 2 with probability $1/2$ and $2\sqrt{3}$ with probability $1/2$. Compute the expected value of the area of $SUN$.

Answer: ________________

Round 1 2 3 4 5
A circle with center $I$ has radius 4. Point $H$ lies on the circle and point $S$ lies on the ray $IH$ so that $IS = 7$. A line passing through $S$ intersects the circle at points $N$ and $E$. Compute the maximum possible area of triangle $INE$.

Answer: ________________________

Round 1 2 3 4 5

A circle with center $I$ has radius 4. Point $H$ lies on the circle and point $S$ lies on the ray $IH$ so that $IS = 7$. A line passing through $S$ intersects the circle at points $N$ and $E$. Compute the maximum possible area of triangle $INE$.

Answer: ________________________

Round 1 2 3 4 5
Let the *romi ratio* of a right triangle be the ratio of the length of its hypotenuse squared to its area. For right triangles with positive leg lengths, what is the minimum value of the romi ratio?

Answer: ________________

Round 1 2 3 4 5
Apollonius's identity states that for triangle $ABC$, with $D$ the midpoint of side $BC$, $AB^2 + AC^2 = \frac{1}{2} BC^2 + 2AD^2$.

Utilizing this useful fact, let $KIM$ be a triangle such that $KI = 7$, $IM = 8$, and $KM = 9$. Let $N$ be the midpoint of side $IM$. Compute the length of the cevian $KN$.

Answer: ________________

<table>
<thead>
<tr>
<th>Round</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
</table>

Apollonius's identity states that for triangle $ABC$, with $D$ the midpoint of side $BC$, $AB^2 + AC^2 = \frac{1}{2} BC^2 + 2AD^2$.

Utilizing this useful fact, let $KIM$ be a triangle such that $KI = 7$, $IM = 8$, and $KM = 9$. Let $N$ be the midpoint of side $IM$. Compute the length of the cevian $KN$.

Answer: ________________

<table>
<thead>
<tr>
<th>Round</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
</table>
In triangle $STY$, $R$ lies on segment $TY$ so that $RT = RY$. Given that $O$ is the midpoint of segment $SR$, what is the ratio of the area of triangle $ROT$ to the area of triangle $STY$?

Answer: ________________

Round 1 2 3 4 5
In cyclic quadrilateral $KONG$, $KO = 2$, $ON = 9$, and $NG = 6$. Given that $KN$ is the diameter of $KONG$’s circumscribed circle, compute the length of $KG$. 

Answer: ________________

Round 1 2 3 4 5

In cyclic quadrilateral $KONG$, $KO = 2$, $ON = 9$, and $NG = 6$. Given that $KN$ is the diameter of $KONG$’s circumscribed circle, compute the length of $KG$. 

Answer: ________________

Round 1 2 3 4 5
Square $STEF$ has side length 4. A semicircle with diameter $SF$ is constructed inside the square. The tangent to the semicircle from $E$ that is not parallel to $ST$ intersects side $ST$ at $V$. Compute the length of segment $EV$.

Answer: ________________

Round 1 2 3 4 5
In rectangle $HASE$, $HA = 6$ and $AS = 12$. Let $SE$ be extended 2 units past $E$ to point $U$, and let $M$ be the intersection of segments $UA$ and $EH$. Compute the area of triangle $HAM$.

Answer: ________________

Round 1 2 3 4 5
How many of the following constructions are always possible with a straightedge and a compass?

i. Constructing a square with the same area as a given circle.
ii. Bisecting an angle.
iii. Trisecting an angle.
iv. Constructing a 30° angle.
v. Constructing the center of a given circle.

Answer: ________________________
Round 1 2 3 4 5
In logic, when one says $X \iff Y$ is true, which of the following statements must also be true?

a. $X \rightarrow Y$

b. The contrapositive of $X \rightarrow Y$

c. The converse of $X \rightarrow Y$

d. The inverse of $X \rightarrow Y$

(Write the letters of all the true statements in alphabetical order. For example, if only a. and b. must be true, write ab.)
A certain type of proof that often appears in geometry and other branches of mathematics shows the existence of a mathematical object (such as a number or a point) that satisfies given conditions by either explicitly creating or giving a way to create such an object. This type of proof is often referred to as a proof by __________. Fill in the blank.

Answer: ________________

Round 1 2 3 4 5

A certain type of proof that often appears in geometry and other branches of mathematics shows the existence of a mathematical object (such as a number or a point) that satisfies given conditions by either explicitly creating or giving a way to create such an object. This type of proof is often referred to as a proof by __________. Fill in the blank.

Answer: ________________

Round 1 2 3 4 5
How many of the following quadrilaterals must be cyclic?

i. Square
ii. Rectangle
iii. Isosceles Trapezoid
iv. Equiangular Rhombus
v. Kite

Answer: ________________

Round 1 2 3 4 5

How many of the following quadrilaterals must be cyclic?

i. Square
ii. Rectangle
iii. Isosceles Trapezoid
iv. Equiangular Rhombus
v. Kite

Answer: ________________

Round 1 2 3 4 5
Let $STORY$ be a cyclic pentagon such that $ST = TO = OR = 5$, $RY = 6$, and $YS = 8$. Let $X$ be the center of the circumscribed circle of $STORY$. Given that $\tan \angle SRY = \frac{4}{3}$, compute the length of segment $TX$.

Answer: ________________

Round 1 2 3 4 5