Mu ciphering Nationals 2019 solutions

0. $y' = \cos x + 1 \to y' = 2 \to y = 2x$ so 2+0=2

$$x + 2yy' = 0 \to y' = \frac{-x}{2y} \to (2,0),(x,y) \to \frac{y}{x-2} = \frac{-x}{2y}$$

1. $2y^2 + x^2 = 2x \to 2x = 2 \to x = 1 \to y = \frac{\sqrt{2}}{2}$

$$m = \frac{-\sqrt{2}}{2} \to b = \sqrt{2} \to -2$$

2.
$$\lim_{b\to 1^-}\int_{-7}^{b} \frac{dx}{(x-1)^{\frac{2}{3}}} + \lim_{b\to 1^+}\int_{b}^{2} \frac{dx}{(x-1)^{\frac{2}{3}}}$$
$$3\sqrt[3]{x-1}\Big|_{-7}^{b} + 3\sqrt[3]{x-1}\Big|_{b}^{2} = 0 - -6 + 3 - 0 = 9$$

3.
$$y' = 72\cos 3x - 48\sin 6x = 0 \to 3\cos 3x - 4\sin 3x\cos 3x = 0$$
$$\cos 3x(3 - 4\sin 3x) = 0 \to x = \frac{\pi}{6}, \frac{\arcsin\left(\frac{3}{4}\right)}{3}$$
$$24\sin 3x + 8(1 - 2\sin^2 3x) = 24\sin\left(\arcsin\frac{3}{4}\right) + 8 - 16\sin^2(\arcsin\left(\frac{3}{4}\right))$$
$$18 + 8 - 9 = 17$$

4. legs are: $48\sin\theta, 48\cos\theta$

$$A = 2(24)^2 \sin\theta\cos\theta = 576\sin 2\theta$$
$$\frac{dA}{dt} = 2(576\cos 2\theta)\frac{d\theta}{dt} \to \theta = \frac{\pi}{6} \to \frac{d\theta}{dt} = \frac{\pi}{18}$$
$$\frac{dA}{dt} = \frac{576\pi}{18} = 32\pi \to 32$$

5. $L=\int_0^{\frac{\pi}{4}}\tan y\,dy=\ln(\sec x)\Big|_0^{\frac{\pi}{4}}=\ln\sqrt{2}$

$e^{8\ln\sqrt{2}}=16$

6. $\int_0^3 v(t)dt=\frac{t^3}{3}-\frac{3t^2}{2}\Big|_0^3=\frac{-9}{2}\rightarrow\int_3^4 v(t)dt=\frac{t^3}{3}-\frac{3t^2}{2}\Big|_3^4=\frac{11}{6}$

$4-\frac{9}{2}=\frac{-1}{2}\rightarrow\frac{-1}{2}+\frac{11}{6}=\frac{4}{3}$ starting point of 4!!

$P=\operatorname{Re}v-\cos t=L(200-4L)-(200+10(200-4L))$

7. $-4L^2+200L-200-2000+40L=-4L^2+240L-2200$

$\frac{-b}{2a}=\frac{-240}{-8}=30$

$\int_1^2(-6x^2+12x)dx+\int_2^3(6x^2-12x)dx$

8. $-2x^3+6x^2\Big|_1^2=4\rightarrow 2x^3-6x^2\Big|_2^3=8\rightarrow 4+8=12$

$\pi\int_0^2 x\,dx=\frac{x^2}{2}\Big|_0^2=2\pi\rightarrow\pi\int_2^4(-x^2+5x-4)dx$

9. $\frac{-x^3}{2}+\frac{5x^2}{2}-4x\Big|_2^4=\frac{-64}{3}+40-16-\left(\frac{-8}{3}+10-8\right)$

$\pi(\frac{8}{3}+\frac{2}{3}+2)=\frac{16}{3}\pi\rightarrow 16+3=19$

10. $\frac{dy}{dx}=\frac{\frac{2t}{1+t^2}}{2t}=\frac{1}{(1+t^2)}\rightarrow\frac{d^2y}{dx^2}=\frac{\frac{-2t}{(1+t^2)^2}}{2t}=\frac{-1}{(1+t^2)^2}=\frac{-1}{4}$

$$A = \frac{1}{2}\left(\frac{H}{\sqrt{2}}\right)^2 = \frac{H^2}{4} \to \frac{1}{4}\int_0^1 \left(-x^2 + x\right)^2 dx$$

11. $$\frac{1}{4}\int_0^1 \left(x^4 - 2x^3 + x^2\right) dx = \frac{1}{4}\left(\frac{x^5}{5} - \frac{x^4}{2} + \frac{x^3}{3}\right)\Big|_0^1$$

$$\frac{1}{4}\left(\frac{1}{5} - \frac{1}{2} + \frac{1}{3}\right) = \frac{1}{120}$$

12.

$$f(h(x)^3) = f(f^{-1}(6057x - 6057))$$

$$f(h(x)^3) = 6057x - 6057$$

$$3h(x)^2 h'(x) f'\left(h(x)^3\right) = 6057$$

$$h(x)^2 h'(x) f'\left(h(x)^3\right) = 2019$$

Answers:

0. 2
1. -2
2. 9
3. 17
4. 32
5. 16
6. 4
7. 30
8. 12
9. 19
10. $\frac{-1}{4}$
11. $\frac{1}{120}$
12. 2019