1) Find $f'(3)$ if $f(x) = x\cos(2x)$.

(A) $\cos(3) + 3\sin(3)$ (B) $\cos(6) - 3\sin(6)$

(C) $\cos(6) - 6\sin(6)$ (D) $\cos(6) + 6\sin(6)$ (E) NOTA

2) Find the slope of the line normal to $f(x) = e^{2x}$ at $x = \ln(2)$.

(A) 4 (B) $-\frac{1}{4}$

(C) 8 (D) $-\frac{1}{8}$ (E) NOTA

3) Find $\frac{d}{dx}\left[\frac{\sin(\pi x)+1}{x^2+4}\right]\Big|_{x=1}$.

(A) $\frac{\pi}{5} + \frac{2}{25}$ (B) $-\frac{\pi}{5} + \frac{2}{25}$

(C) $\frac{\pi}{5} - \frac{2}{25}$ (D) $-\frac{\pi}{5} - \frac{2}{25}$ (E) NOTA

4) Find $\lim\limits_{x\to 3} \frac{x^3-3x^2-x+3}{x^3+x^2-9x-9}$.

(A) $\frac{1}{3}$ (B) $-\frac{1}{3}$

(C) $-\frac{2}{3}$ (D) $\frac{2}{3}$ (E) NOTA

5) Find the equation of the tangent line to the curve $x^3 - xy^2 + 2y^4 = 8$ at the point $(2,1)$.

(A) $y = -\frac{11}{4}x + \frac{9}{2}$ (B) $y = \frac{11}{4}x - \frac{9}{2}$

(C) $y = -\frac{11}{4}x + \frac{13}{2}$ (D) $y = \frac{11}{4}x - \frac{13}{2}$ (E) NOTA

6) Approximate the area between the curve $y = x^3 + 1$ and the x-axis from $x = 1$ to $x = 3$, using the Trapezoidal Rule with four intervals of equal width.

(A) 16 (B) $\frac{45}{2}$

(C) 29 (D) 45 (E) NOTA

7) Evaluate: $\int_1^2 \left(x^3 + \frac{1}{x^2}\right) dx$

(A) $\frac{7}{2}$ (B) $\frac{3}{4}$

(C) $\frac{17}{4}$ (D) 2 (E) NOTA

8) Find $\int_{-3}^{3} \sqrt{9-x^2}\,dx$.

(A) 9π (B) $\frac{9}{2}\pi$

(C) $\frac{9}{4}\pi$ (D) $\frac{9}{8}\pi$ (E) NOTA

9) Evaluate: $\int_0^1 \frac{x-1}{x^2-2x+5}\,dx$

(A) $\ln\left(\frac{2\sqrt{5}}{5}\right)$ (B) $\ln\left(\frac{4}{5}\right)$

(C) $\ln\left(\frac{\sqrt{5}}{2}\right)$ (D) $\ln\left(\frac{2\sqrt{5}}{2}\right)$ (E) NOTA

10) Evaluate: $\int_1^3 \frac{1}{x^2-2x+5}\,dx$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{8}$ (D) $\frac{\pi}{16}$ (E) NOTA

11) Evaluate: $\int_0^1 \frac{1}{x^2-2x-3}\,dx$

(A) $\frac{\ln(3)}{3}$ (B) $\frac{\ln(3)}{4}$

(C) $-\frac{\ln(3)}{3}$ (D) $-\frac{\ln(3)}{4}$ (E) NOTA

12) Find $\lim_{n\to\infty}\left(\sum_{k=1}^{n} \frac{k}{k^2+n^2}\right)$.

(A) $\ln(2)$ (B) $\frac{1}{2}\ln(2)$

(C) $\ln\left(\frac{5}{2}\right)$ (D) $\frac{1}{2}\ln\left(\frac{5}{2}\right)$ (E) NOTA

13) Find $\lim_{x\to\infty}\left(\sqrt{3x^2-2x+5}-\sqrt{3x^2-7x+11}\right)$.

(A) $\frac{5\sqrt{3}}{6}$ (B) $\frac{5\sqrt{3}}{3}$

(C) $-\frac{5\sqrt{3}}{6}$ (D) $-\frac{5\sqrt{3}}{3}$ (E) NOTA

14) Find $\frac{d}{dx}[x^x]$.

(A) x^x (B) $x^x\ln(x)$

(C) $x^x(\ln(x)+1)$ (D) $x^x(\ln(x)-1)$ (E) NOTA

15) The functions $f(x) = x^2 + 1$ and $g(x) = -x^2$ share a common tangent line of positive slope. What is its equation?

(A) $y = \frac{\sqrt{2}}{2}x + 1$ (B) $y = \frac{\sqrt{2}}{2}x + \frac{1}{2}$

(C) $y = \sqrt{2}x + 1$ (D) $y = \sqrt{2}x + \frac{1}{2}$ (E) NOTA

16) Find the range of $f(x) = \frac{x}{x^6+1}$.

(A) $\left[-\frac{\sqrt[6]{5}}{6}, \frac{\sqrt[6]{5}}{6}\right]$ (B) $\left[-\sqrt[6]{5}, \sqrt[6]{5}\right]$

(C) $\left[-\frac{\sqrt[6]{5^5}}{6}, \frac{\sqrt[6]{5^5}}{6}\right]$ (D) $\left[-\sqrt[6]{5^5}, \sqrt[6]{5^5}\right]$ (E) NOTA

17) Evaluate: $\int_0^{\sqrt{\pi}} e^x(\cos(x^2) - 2x\sin(x^2))dx$

(A) $-e^{\sqrt{\pi}} - 1$ (B) $e^{\sqrt{\pi}} - 1$

(C) $-e^{\sqrt{\pi}} + 1$ (D) $e^{\sqrt{\pi}} + 1$ (E) NOTA

18) Evaluate: $\int_0^{\pi/3} \sec(x)\tan(x) \cdot \ln|\sec(x) + \tan(x)|\, dx$

(A) $2\ln|2+\sqrt{3}| + \sqrt{3}$ (B) $2\ln|2+\sqrt{3}|$

(C) $2\ln|2+\sqrt{3}| - \sqrt{3}$ (D) $\sqrt{3}\ln|2+\sqrt{3}|$ (E) NOTA

19) From the top of a tree 30 meters tall, a monkey is pulling up a bundle of bananas attached to a rope. The bundle of bananas has a weight of 20 Newtons, and the rope has a linear density of 6 Newtons per meter. How much work (in Newton-meters) does the monkey do when pulling the bundle of bananas to the top of the tree?

(A) 600 (B) 780

(C) 3,300 (D) 6,000 (E) NOTA

20) For which of the following functions is the average rate of change of the function equal to the average value of the function over <u>any</u> real interval?

(A) $f(x) = 2019$ (B) $f(x) = \sin(x)$

(C) $f(x) = x^2$ (D) $f(x) = e^x$ (E) NOTA

21) Consider the region between $f(x) = x^r$ $(r \geq 1)$ and the x-axis from $x = 0$ to $x = a$. If, for any real value of a, the y-coordinate of the centroid of this region is equal to the average value of $f(x)$ over the interval $(0, a)$, then what is r?

(A) $1 - \sqrt{2}$ (B) 1

(C) $1 + \sqrt{2}$ (D) 2 (E) NOTA

22) Find $\frac{d^{2019}}{dx^{2019}}[x^3 \cos(x^2)]\Big|_{x=0}$.

(A) $\frac{2019!}{1008!}$ (B) $\frac{2019!}{1009!}$

(C) $\frac{2018!}{1008!}$ (D) $\frac{2018!}{1009!}$ (E) NOTA

23) The finite region in the first quadrant bounded by the x- and y-axes and the curve $y = r^2 - x^2$ is divided into two regions of equal area by the curve $y = ax^2$. Assume r is a non-zero real constant. Find a.

(A) 2 (B) 3

(C) 4 (D) 5 (E) NOTA

24) Consider a matrix of the form $\begin{bmatrix} x & 1 & 0 \\ y^2 & y & 5 \\ x & 1 & y \end{bmatrix}$ with non-negative entries and a determinant of 12. What is the maximum possible trace of such a matrix?

(A) 1 (B) 3

(C) 6 (D) 9 (E) NOTA

25) Evaluate for $k > 0$: $\int_0^{\infty} \frac{dx}{\cosh(x) + k\sin\ (x) + 1}$

(A) $\frac{1}{k}\ln(k)$ (B) $\frac{1}{k}\ln(k+1)$

(C) $\frac{1}{k+1}\ln(k)$ (D) $\frac{1}{k+1}\ln(k+1)$ (E) NOTA

26) Let $f(x)$ be a continuous, differentiable function such that, for any real $a \geq 0$, $\int_1^{\infty} e^{-a}\ f(x)dx = \frac{a^2}{a^6+1}$. Find $\int_1^{\infty} \frac{f(x)}{x} dx$.

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$ (E) NOTA

27) Which of the answer choices is equivalent to the expression below?

$$\frac{d}{d\left(\frac{d}{d\left(\frac{d}{d\left(\frac{d}{d(\ddots)}(x^{2019})\right)}(x^{2019})\right)}(x^{2019})\right)}(x^{2019})$$

(A) $2019x^{2018}$ (B) $2019x^{1009}$

(C) $x^{\frac{2019}{2}}$ (D) $\sqrt{2}x^{\frac{2019}{2}}$ (E) NOTA

28) Let $f(x)$, $g(x)$, and $h(x)$ be differentiable functions with the following properties:

$$f'(x) = f(x) + g(x) + h(x)$$
$$g'(x) = f(x) - 2h(x)$$
$$h'(x) = 2f(x) - g(x) + h(x)$$

Further, $f(0) = 0$, $g(0) = 3$, and $h(0) = -1$. Find $f(\ln(2))$.

(A) $\frac{15}{16}$ (B) $\frac{7}{8}$

(C) $\frac{15}{8}$ (D) $\frac{7}{4}$ (E) NOTA

29) The point P begins at the origin, then moves in the Cartesian plane along the line $y = \frac{1}{2}x$ such that the x-coordinate of P is changing at a rate of $+3$ units per second. Consider the area enclosed by the locus of all points that are exactly $\frac{1}{3}$ as far away from the point P as they are from the line $y = -2x$. At what rate is this area changing when $P = (8, 4)$?

(A) $\frac{81\pi\sqrt{2}}{2}$ (B) $\frac{81\pi\sqrt{2}}{4}$

(C) $\frac{9\pi\sqrt{2}}{2}$ (D) $\frac{9\pi\sqrt{2}}{4}$ (E) NOTA

30) Find $\lim_{n\to\infty}\left(1 + \frac{3}{n}\right)^{5n}$.

(A) e^{15} (B) e^{5}

(C) e^{3} (D) $e^{5/3}$ (E) NOTA