For all questions, answer choice E. "NOTA" stands for "None of the Above", "DNE" stands for "Does Not Exist", and "GLHF!" stands for "Good Luck and Have Fun!"

1. Evaluate $\int_{-2}^{-1} 3x^2\, dx$.

A. -9 B. -7 C. 7 D. 9 E. NOTA

2. Let $f(x)$ be a continuous function with domain of all real numbers. Given $\int_{-1}^{3} f(x)dx = 4$ and $\int_{7}^{3} f(x)dx = -1$, compute the value of $\int_{-1}^{7}\big(1 + f(x)\big)dx$.

A. 5 B. 6 C. 11 D. 13 E. NOTA

3. Find the average value of $f(x) = 4e^x + 2019$ over the interval $[0, 4]$.

A. $e^4 - 1$ B. $4e^4 - 4$ C. $e^4 + 2018$ D. $4e^4 + 2015$ E. NOTA

4. The left-hand Riemann sum approximation for $\int_{2}^{8} \ln x\, dx$ using three equal subdivisions is equal to

A. $2\ln 48$ B. $2\ln 80$ C. $2\ln 192$ D. $2\ln 384$ E. NOTA

5. Let $f(x)$ be a degree two polynomial with nonzero, distinct roots a and b. If $F(x) = \int f(x)dx$ and $F(0) = 0$, then if $F(a) = 0$, what is the value of $\frac{a}{b}$?

A. -1 B. $\frac{1}{2}$ C. 2 D. 3 E. NOTA

6. $\int_{1}^{2} \frac{2x-3}{x^2-2x+2}\, dx =$

A. $\frac{\pi}{4}$ B. $\ln 2 - \frac{\pi}{4}$ C. $\ln 2 - \frac{\pi}{2}$ D. $2\ln 2 - \frac{\pi}{4}$ E. NOTA

7. $\lim\limits_{t\to\infty} \frac{\int_{2019}^{t} x^x dx}{t^t} =$

A. 0 B. 1 C. e D. $\ln 2019$ E. NOTA

8. Every year, the summer integral briefly visits, and the people hear its inspirational and familiar tone. What is $\int_0^{2019} x\,dx$?

A. 2019 B. $\frac{2019^2}{2}$ C. 2019^2 D. Does Not Exist E. NOTA

9. $\int_1^2 (3^x \ln 9)\,dx =$

A. 6 B. $6 \ln 3$ C. 12 D. $6 \ln 9$ E. NOTA

10. Let the population of ducks in Gena′s pond be $P(t)$ for a continuous function P. If the rate of change of the duck population is inversely proportional to the square root of the current population, and $P(0) = 36$ and $P'(0) = 2$, then find $P(3)$.

A. $\frac{225}{4}$ B. $9\sqrt[3]{100}$ C. $18\sqrt[3]{18}$ D. $\frac{441}{4}$ E. NOTA

11. Find the volume created when the region bounded by $y = 1 - x^2$ and $y = 0$ is rotated around the line $y = 0$.

A. $\frac{8\pi}{15}$ B. $\frac{2\pi}{3}$ C. $\frac{16\pi}{15}$ D. $\frac{4\pi}{3}$ E. NOTA

12. Approximate $\int_{\pi}^{2\pi} \frac{\sin}{x}\,dx$ using the first two nonzero terms of the Maclaurin series for $\sin x$ centered around $x = 0$.

A. $-\pi - \pi^3$ B. $\pi - \pi^3$ C. $\pi - \frac{7\pi^3}{9}$ D. $-\pi$ E. NOTA

13. Find the area bounded between the graphs of $y = x^2$ and $y = x + 2$.

A. $\frac{3}{2}$ B. $\frac{5}{2}$ C. $\frac{7}{2}$ D. $\frac{9}{2}$ E. NOTA

14. Find the area bounded by $f(x) = \sqrt{16 - x^2}$ and $g(x)$, where $g(x) = 0$ for $x < 0$ and $g(x) = x$ for $x \geq 0$.

A. 3π B. 4π C. 6π D. 8π E. NOTA

15. A rectangle has vertices at $(-4, 16), (4, 16), (-4, 4), (4, 4)$. All points in the rectangle where $y > x^2$ are colored blue, whereas every other point is colored red. What fraction of the rectangle is colored blue?

A. $\frac{2}{9}$ B. $\frac{1}{3}$ C. $\frac{2}{3}$ D. $\frac{7}{9}$ E. NOTA

16. $\int_1^e x^{\ln x - 1} \ln x \, dx =$

A. $\frac{e-1}{2}$ B. $\frac{e}{2}$ C. $e - 1$ D. $e - \frac{1}{e}$ E. NOTA

17. $\int_{-2}^{3} \frac{1}{x^2 - 1} dx =$

A. $-\frac{1}{2}\ln 6$ B. $\frac{1}{2}\ln\frac{2}{3}$ C. $\frac{1}{2}\ln\frac{3}{2}$ D. $\frac{1}{2}\ln 6$ E. NOTA

18. $\int_{-2}^{2} \frac{x \sin(\cos x) + 2}{x^2 + 4} dx =$

A. $-\frac{\pi}{2}$ B. 0 C. $\frac{\pi}{2}$ D. π E. NOTA

19. The Lumobile travels along a straight line at 15 $\frac{m}{s}$ at time $0s$ and 40 $\frac{m}{s}$ at time $5s$. Which of the following statements is/are ALWAYS true, assuming the velocity function is differentiable?

I. The Lumobile has traveled at most $200m$.
II. The Lumobile accelerated at 5 $\frac{m}{s^2}$ sometime between $0s$ and $5s$.
III. The Lumobile has traveled at least $75m$.
IV. The Lumobile traveled at 20.19 $\frac{m}{s}$ sometime between $0s$ and $5s$.

A. II only B. IV only C. II and IV only D. I, II, III, IV E.NOTA

20. $\int_0^3 |x^2 - 4x + 3|\, dx$

A. $\frac{4}{3}$ B. $\frac{8}{3}$ C. $\frac{13}{3}$ D. $\frac{17}{3}$ E. NOTA

21.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{in + n^2} =$$

A. $\ln 2 - 1$ B. $1 - \ln\frac{2}{3}$ C. $1 - \ln\frac{3}{2}$ D. $\ln\frac{e}{2}$ E. NOTA

22. Evaluate:

$$\int_{\frac{\sqrt{3}}{2}}^{1} \frac{\sqrt{1 - x^2}}{x}\, dx$$

A. $\frac{1}{2}\ln 3 - \frac{1}{2}$ B. $\frac{1}{2}\ln 3 - \frac{\sqrt{3}}{2}$ C. $\frac{\sqrt{3}}{2} - \frac{1}{2}\ln 3$ D. $\ln 3 - \frac{1}{2}$ E. NOTA

23. Let f and g be defined such that $f'(x) = f(x)^2 + g(x)^2$ and $g'(x) = 2f(x)g(x) + 1$. Given $f(0) = \frac{1}{5}$ and $g(0) = \frac{4}{5}$, find the value of $f\left(\frac{\pi}{12}\right) + g\left(\frac{\pi}{12}\right)$.
(Hint: add the two given equations.)

A. 0 B. $\frac{\sqrt{3}}{3}$ C. $\sqrt{3}$ D. 1 E. NOTA

24. Jonathan loves dancing to TWICE songs and improves as he dances more. If he learns TWICE dances at a rate of x% per second, where x is the number of seconds that have elapsed since he started, how many seconds will it take for Jonathan to learn one full TWICE dance? (i.e. get to 100%)

A. $10\sqrt{2}$ B. 15 C. $15\sqrt{2}$ D. 20 E. NOTA

25. Let $f(x) = e^x \sin x$ and $g(x) = e^x \cos x$. Find the value of the integral:

$$\int_0^\pi \left(g(x) - \int_0^x \big(f(y) + g(y)\big)\, dy \right) dx$$

A. $-1 - e^\pi$ B. $1 - e^\pi$ C. 0 D. e^π E. NOTA

26. Let $f(x) = -2x^3 - x - 9$. Partition $[0, 2019]$ into 4 non-overlapping subintervals of equal length, then use this partition to approximate $\int_0^{2019} f(x)dx$ by L, R, S, and T, where
$L =$ the left-hand Riemann approximation, $R =$ the right-hand Riemann approximation
$S =$ the Simpson's Rule approximation, $T =$ the trapezoidal rule approximation
Which of the following is true (assuming that L, R, S, and T approximate the integral and not the area)?

A. $R > T > S > L$ B. $L > T > S > R$ C. $L > S > T > R$
D. $L > T > R > S$ E. NOTA

27. Let $f(x) = 1 + 2x + 3x^2 + \cdots + 2019x^{2018}$ and $F(x) = \int f(x)\, dx$. Evaluate $F(1)$.

A. 0 B. 2018 C. 2019 D. 2020 E. NOTA

28. Alice Ha keeps integrating every single term in $\sum_{n=1}^{\infty} \frac{x}{2^n}$, but can't quite seem to reach the end! Find the following integral: $\int \left(\sum_{n=1}^{\infty} \frac{x}{2^n} \right) dx$.

A. $\frac{x}{2} + C$ B. $x + C$ C. $\frac{x^2}{2} + C$ D. $x^2 + C$ E. NOTA

29. David dislikes numbers less than 25. Let the David utility function, $D(x)$, be described as the following piecewise function: $D(x) = \begin{cases} 1 & if\ x \geq 25 \\ 0 & if\ x < 25 \end{cases}$. Find $\int_0^{10} D(x^2)dx$.

A. 5 B. 25 C. $\frac{125}{3}$ D. $\frac{875}{3}$ E. NOTA

30. Evaluate:

$$\int_0^1 \frac{1 - x}{e^x + x}\, dx$$

A. $\ln\left(1 - \frac{1}{e}\right)$ B. 0 C. $\ln(\frac{e}{e-1})$ D. $\ln(e + 1) - 1$ E. NOTA