1. C

The antiderivative is x^3, so this is $(-1)^3 - (-2)^3 = -1 - (-8) = 7$

2. D

$$\int_{-1}^{7} \big(1 + f(x)\big)dx = x|_{-1}^{7} + \int_{-1}^{7} f(x)dx = 7 - (-1) + \int_{-1}^{3} f(x)dx + \int_{3}^{7} f(x)dx = 8 + 4 - (-1) = 13$$

3. C

The average of $f(x) = \frac{\int_0^4 f(x)dx}{4-0}$ and 2019 is constant. $\int_0^4 4e^x dx = 4(e^4 - 1)$. The answer is then one-fourth of this, plus the original $2019 = e^4 - 1 + 2019 = e^4 + 2018$.

4. A

This is equal to $2(\ln 2 + \ln 4 + \ln 6) = 2\ln\big(2(4)(6)\big) = 2\ln 48$

5. D

We write $f(x) = (x-a)(x-b) = x^2 - (a+b)x + ab$, then $F(x) = \frac{x^3}{3} - \frac{(a+b)x^2}{2} + abx$. Plugging in $x = a$ as a root, we have $F(a) = \frac{a^3}{3} - \frac{a^2(a+b)}{2} + a^2 b = 0$. Since $a \neq 0$, we simplify this to $\frac{a}{3} - \frac{a+b}{2} + b = 0$, then $a = 3b$ so $\frac{a}{b} = 3$.

6. B

We see the derivative of the denominator with respect to x is very nearly the numerator. And thus,

$$\int_1^2 \frac{2x-3}{x^2-2x+2}dx = \int_1^2 \frac{2x-2}{x^2-2x+2}dx - \int_1^2 \frac{1}{x^2-2x+2}dx = \ln(x^2-2x+2) - \int_1^2 \frac{dx}{(x-1)^2+1}$$

Clearly, the second integral is just $\arctan(x-1)$. Evaluating from 1 to 2, we obtain $\ln 2 - \tan^{-1} 1 - \ln 1 + \tan^{-1} 0 = \ln 2 - \frac{\pi}{4}$.

7. A

We attempt directly substituting $t \to \infty$ and quickly see that both numerator and denominator go to ∞, which is indeterminate. We then apply L'Hopital's Rule and take the derivative of both the numerator and denominator to obtain $\lim_{t\to\infty} \frac{t^t}{t^t(1+\ln t)}$, where we find the derivative of the denominator by setting $y = t^t$, $\ln y = t\ln t$, $\frac{y'}{y} = \ln t + 1$ and thus $y' = t^t(1 + \ln t)$, and clearly the limit is now zero.

8. B

Oh why summer integral? Integrate to get $\frac{x^2}{2}$ and plug in the bounds.

9. C

This is $\int_1^2 (3^x \ln 9)dx = 2\int_1^2 (3^x \ln 3)dx = 2[3^x]_1^2 = 2(3^2 - 3^1) = 2(9-3) = 12$

10. B

We have $\frac{dP}{dt} = \frac{k}{\sqrt{P}}$ for some constant k. Separating variables, we obtain $\int \sqrt{P}dP = \int kdt$, and $\frac{2}{3}P^{\frac{3}{2}} = kt + C$. Plugging in $P(0)$, we have $144 = C$ and since $P' = \frac{k}{\sqrt{P}}$, at $t = 0, 2 = \frac{k}{6}$ so $k = 12$. So $P(3)^{\frac{3}{2}} = \frac{3}{2}(12 * 3 + 144) = 270$, and $P(3) = \sqrt[3]{270 * 270} = 9\sqrt[3]{100}$.

11. C

This is $\pi\int_{-1}^{1}(1-x^2)^2\,dx = 2\pi\int_0^1(1-2x^2+x^4)\,dx = 2\pi\left[x-\frac{2}{3}x^3+\frac{1}{5}x^5\right]_0^1 = 2\pi\left(1-\frac{2}{3}+\frac{1}{5}\right) = \frac{16}{15}$

12. E

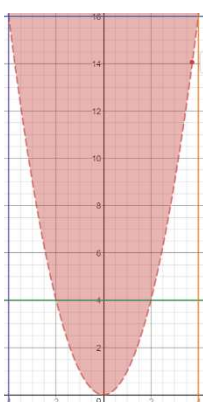
The first two nonzero terms of $\sin x = x - \frac{x^3}{6}$, so we take $\int_{\pi}^{2\pi} 1 - \frac{x^2}{6}dx = x - \frac{x^3}{18}\Big\{\begin{matrix}2\pi\\ \pi\end{matrix} = 2\pi - \frac{8\pi^3}{18} - \pi + \frac{\pi^3}{18} = \pi - \frac{7\pi^3}{18}$.

13. D

$x^2 = x + 2$ at $x = -1$ and $x = 2$. In this interval, $x + 2 > x^2$, so the area is $\int_{-1}^{2}(x+2-x^2)\,dx$.
The area is $\left[\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2} = 2+4-\frac{8}{3}-\left(\frac{1}{2}-2+\frac{1}{3}\right) = \frac{9}{2}$

14. C

This is a half circle with a 45 degree slice missing from it and is then just $\frac{3}{4} * \frac{1}{2} * 16\pi = 6\pi$.



15. D

(see graph to the left). We can clearly see that this shaded area (that is not blue
- but red since this is not the exact area we want) is just $\int_{-4}^{4} 16 - x^2 dx$ but the
- very bottom portion should not be included since this is below the rectangle.
- Therefore, we should subtract out the portion below $y = 4$, $\int_{-2}^{2} 4 - x^2 dx$. We
- then obtain our area as $\frac{256}{3} - \frac{32}{3} = \frac{224}{3}$. Finding the ratio, we have $\frac{\frac{224}{3}}{96} = \frac{7}{9}$.

16. A

We u-sub $u = \ln x$, $du = \frac{dx}{x}$ to obtain new integral $\int_0^1 ue^{u^2}du$, which by $v = u^2$, $dv = 2udu$, we have $\frac{1}{2}\int_0^1 e^v dv$, which is clearly just $\frac{e-1}{2}$.

17. E

We decompose this by partial fractions to obtain $\frac{1}{2}\int_{-2}^{3}\frac{1}{x-1} - \frac{1}{x+1}dx$, where if one did not already notice, they should notice that these bounds pass through $x = 1$ and -1, which are vertical asymptotes and thus the answer is E.

18. C

Clearly, this is ugly and we should not proceed to trig-sub. Rather we should notice the bounds and that if $f(x) = x\sin(\cos x)$, then $f(-x) = -x\sin(\cos(-x)) = -x\sin(\cos x) = -f(x)$, which implies $f(x)$ is odd. Therefore, the integral is equal to $\int_{-2}^{2} \frac{2}{x^2+4}dx = \tan^{-1}\frac{x}{2}\Big\{\begin{matrix}2\\-2\end{matrix} = \frac{\pi}{2}$.

19. C

I. This has no relevance on distance traveled. Imagine the Lumobile accelerates to 2019 $\frac{m}{s}$ in the first 0.01 s and then stays at that velocity until the last 0.01 s where it decelerates to 40 $\frac{m}{s}$.

II. This follows from the Mean Value theorem, since $\frac{40-15}{5-0} = 5\frac{m}{s^2}$.

III. Same reasoning as I. but replace 2019 $\frac{m}{s}$ with $-2019\frac{m}{s}$.

IV. This follows from Intermediate Value theorem.

20. B

Clearly, this factors into $(x-1)(x-3)$ and thus we can integrate $x^2 - 4x + 3$ from 0 to 3, which is 0 and then we must add back in twice the portion that was negative from 1 to 3, which gives us $\frac{4}{3}$. So our answer is just $0 + 2 * \frac{4}{3} = \frac{8}{3}$.

21. D

This is a Riemann-sum. We rewrite $\frac{i}{in+n^2} = (\frac{1}{n})(\frac{\frac{i}{n}}{\frac{i}{n}+1})$ such that $\frac{1}{n}$ is our dx ("width" of our rectangle) and $\frac{i}{n}$ is our x ("index" of our rectangle). Thus, our integral is $\int_0^1 \frac{x}{x+1}dx = \int_0^1 1 - \frac{1}{x+1}dx = x - \ln(x+1)\Big\{\begin{matrix}1\\0\end{matrix} = 1 - \ln 2 = \ln\frac{e}{2}$.

22. A

We can trig-sub this. $x = \sin u$, $dx = \cos u\, du$. We obtain $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{\cos^2 u}{\sin u}du$. We then substitute $v = \cos u$, $dv = -\sin u\, du$ to obtain $\int_{0.5}^{0} -\frac{v^2}{\sin^2 u}dv = \int_0^{0.5}\frac{v^2}{1-v^2}dv = \int_0^{0.5} -1 + \frac{1}{1-v^2}dv = \int_0^{0.5} -1 + \frac{0.5}{1-v} + \frac{0.5}{1+v}dv = -v - 0.5\ln(1-v) + 0.5\ln(1+v)\Big\{\begin{matrix}0.5\\0\end{matrix} = -0.5 - 0.5\ln 0.5 + 0.5\ln 1.5 = -0.5 + 0.5\ln 3$.

23. C

Let us follow the hint: $f'(x) + g'(x) = f(x)^2 + g(x)^2 + 2f(x)g(x) + 1 = \big(f(x)+g(x)\big)^2 + 1$. Then clearly we have, $\frac{f'(x)+g'(x)}{\left(f(x)+g(x)\right)^2+1} = 1$. Taking the integral with respect to x of both sides yields $\tan^{-1}\big(f(x)+g(x)\big) = x + C$ or $f(x) + g(x) = \tan(x + C)$. We now use our given values to find $C = \frac{\pi}{4}$ and so $f\left(\frac{\pi}{12}\right) + g\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$.

24. A

We find t such that $\int_0^t x\,dx = 100$ to find when he completes the dance. This happens when $\frac{t^2}{2} = 100$, so $t = 10\sqrt{2}$.

25. A

We notice that $f'(x) = f(x) + g(x)$ and that $g'(x) = g(x) - f(x)$. Therefore, $\int_0^{\pi}\left(g(x) - \int_0^x \big(f(y) + g(y)\big)\,dy\right)dx = \int_0^{\pi} g(x) - f(x) + f(0)dx = g(\pi) - g(0) = -1 - e^{\pi}$.

26. C

We notice on the interval that this is decreasing and concave down and that Simpson's rule is an exact approximation for polynomials of degree 3 or less. Since this is decreasing, the left-hand approximation is the largest and right-hand approximation is the smallest and the trapezoidal (which is the exact average of the two) is somewhere in between. However, since this function is concave down, it must be less than the actual value (drawing any concave down function, this is clear).

27. E

Don't forget the plus C! Any function $F(x) = C + x + x^2 + x^3 + \cdots + x^{2019}$, will satisfy this, but we can have any C and thus our answer for $F(1) = C + 2019$ is undetermined.

28. C

We can evaluate the sum within the integral first to help Alice Ha. $\sum_{n=1}^{\infty} \frac{x}{2^n}$ is an infinite geometric series with common ratio $1/2$, so this sum is equal to x. Then clearly we may integrate x to obtain $\frac{x^2}{2} + C$.

29. A

Since $D(x)$ is at most 1, we know the integral is less than 10. Since x^2 becomes ≥ 25 after 5, we can split the integral as $\int_0^5 0dx + \int_5^{10} 1\,dx$, achieving our answer of 5.

30. D

It is in good practice of integration to check the derivative of the denominator when performing integration on these fractional functions. In this case the derivative of $e^x + x$ is $e^x + 1$, which can be achieved by rewriting $\frac{1-x}{e^x+x}$ as $\frac{1-x}{e^x+x} + 1 - 1 = \frac{e^x+1}{e^x+x} - 1$. And thus it clearly integrates to $\ln(e^x + x) - x$, where plugging in the bounds achieves $\ln(e+1) - 1$.