#0 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \lim_{x \to 0} \left(\frac{x - x^3}{x^2 + 2x} \right) \qquad B = \lim_{x \to \infty} \left(1 - \frac{5}{x}\right)^{3x}$$

$$C = \lim_{x \to \infty} \left(\frac{3x^2 + x - 1}{2x^2 - x^3 + 7} \right) \qquad D = \lim_{x \to \infty} \left(\sqrt{x^2 + 2x} - \sqrt{x^2 - x} \right)$$

Find $\frac{AD}{B+C}$.

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Find $\frac{AD}{B+C}$.

#1 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \frac{d}{dx}\left[e^{-x}\cos(x^2)\right]\Big|_{x=0} \qquad B = \frac{d}{dx}\left[x(x^2+1)^2(x^3+2)^3\right]\Big|_{x=1}$$

$$C = \frac{d}{dx}\left[(x+1)^{x^3-1}\right]\Big|_{x=1} \qquad D = \frac{d}{dx}\left[\frac{\tan^2(x)}{x+\sin(x)+1}\right]\Bigg|_{x=0}$$

Find $A + B + C + D$.

#1 Mu Bowl
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Find $A + B + C + D$.

#2 Mu Bowl
MAΘ National Convention 2019

Let

$$f(x) = \frac{1 - x^2}{a}$$

$$g(x) = a(x^2 - 1)$$

for a positive real constant a.

Find the minimum possible area of the finite region bounded by $f(x)$ and $g(x)$.

#2 Mu Bowl
MAΘ National Convention 2019

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for a positive real constant a.

Find the minimum possible area of the finite region bounded by $f(x)$ and $g(x)$.

#3 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k+n}$$

$$B = \lim_{n\to\infty} \prod_{k=1}^{n} \left(1+\frac{k}{n}\right)^{\frac{2}{n}}$$

$$C = \lim_{x\to 0} \frac{4\arctan(1+x)-\pi}{x}$$

$$D = \lim_{x\to 0} \frac{\arctan(x)-x}{x^3}$$

Find $A+\ln(B)+C+D$.

#3 Mu Bowl
MAΘ National Convention 2019

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Find $A+\ln(B)+C+D$.

#4 Mu Bowl
MAΘ National Convention 2019

Let

$$f(x) = x^{x+x^{x+x^{x+x^{\cdot^{\cdot^{\cdot}}}}}}$$

$$g(x) = \sqrt{4x^2 + \sqrt{4x^2 + \sqrt{4x^2 + \cdots}}}$$

If

$$A = \frac{d}{dx}[f(x)]_{x=1} \qquad B = \frac{d}{dx}[g(x)]_{x=1}$$

$$C = \int_1^2 (\ln(f(x)) - f(x)\ln(x))\,dx \qquad D = \int_0^1 x\,g(x)\,dx$$

Find $A + 17B + 4C + 96D$.

#4 Mu Bowl
MAΘ National Convention 2019

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$$f(x) = x^{x+x^{x+x^{x+x^{\cdot^{\cdot^{\cdot}}}}}}$$

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Find $A + 17B + 4C + 96D$.

#5 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \int_1^2 \left(x^3 + \frac{1}{x^2}\right)\, dx$$

$$B = \int_0^{\sqrt{\pi/3}} x \sin(x^2)\, dx$$

$$C = \int_0^{\pi/4} (\cos(x) + \sin(x))^2\, dx$$

$$D = \int_1^2 \frac{1}{x^2 + 2x}\, dx$$

Find $\frac{A+C+e^{2D}}{B}$.

#5 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \int_1^2 \left(x^3 + \frac{1}{x^2}\right)\, dx$$

$$B = \int_0^{\sqrt{\pi/3}} x \sin(x^2)\, dx$$

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$$D = \int_1^2 \frac{1}{x^2 + 2x}\, dx$$

Find $\frac{A+C+e^{2D}}{B}$.

#6 Mu Bowl
MAΘ National Convention 2019

Consider the polynomial

$$p(x) = rx^3 + (1-r)x^2 - r^2x + (r+1)^3$$

where r is a real number changing at a constant rate of +2 units per second.

In units per second, let:

A be the rate of change of the sum of the roots when $r = 3$

B be the rate of change of the product of the roots when $r = 3$

Find $A - B$.

#6 Mu Bowl
MAΘ National Convention 2019

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$$p(x) = rx^3 + (1-r)x^2 - r^2x + (r+1)^3$$

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In units per second, let:

A be the rate of change of the sum of the roots when $r = 3$

B be the rate of change of the product of the roots when $r = 3$

Find $A - B$.

#7 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \int_{1}^{2019} \frac{x^{2019}}{x^{2019} + (2020 - x)^{2019}}\, dx \qquad B = \int_{0}^{\pi/2} \frac{1}{\cos(x) + \sin(x) + 2}\, dx$$

$$C = \int_{0}^{1} \frac{x - 1}{\ln(x)}\, dx$$

It turns out that $2A + B + C = pq + \sqrt{p}\arctan(\sqrt{p}) - \sqrt{p}\arctan\left(\frac{1}{\sqrt{p}}\right) + \ln(p)$ for a prime number p. Find q.

#7 Mu Bowl
MAΘ National Convention 2019

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$$A = \int_{1}^{2019} \frac{x^{2019}}{x^{2019} + (2020 - x)^{2019}}\, dx \qquad B = \int_{0}^{\pi/2} \frac{1}{\cos(x) + \sin(x) + 2}\, dx$$

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#8 Mu Bowl
MAΘ National Convention 2019

Let R denote the finite region bounded by the x-axis and the curve $y = 3 - 3x^2$.

Let A be the area of R.

Let B be the volume of the solid formed when R is rotated about the x-axis.

Let C be the volume of the solid formed when R is rotated about the line $x = 1$.

Let D be the volume of the solid formed when R is rotated about the line $y = x - 1$.

Find $\frac{C}{A} + \frac{D}{B}$.

#8 Mu Bowl
MAΘ National Convention 2019

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Find $\frac{C}{A} + \frac{D}{B}$.

#9 Mu Bowl
MAΘ National Convention 2019

Let L represent the line tangent to the curve $x^2y - xy^2 - x + y = -1$ at the point $(1,2)$.

Let M represent the line tangent to the curve $\begin{cases} x(t) = e^t - 2t \\ y(t) = t^2 - \ln(t+1) \end{cases}$ at $t = 0$.

If (A, B) is the point of intersection between L and M, find $A + B$.

#9 Mu Bowl
MAΘ National Convention 2019

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Let M represent the line tangent to the curve $\begin{cases} x(t) = e^t - 2t \\ y(t) = t^2 - \ln(t+1) \end{cases}$ at $t = 0$.

If (A, B) is the point of intersection between L and M, find $A + B$.

#10 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \int_{-1}^{0} e^{x} \arctan(x+1)\ dx$$

$$B = \int_{\pi/4}^{\pi/3} \sec(x)\tan^{2}(x)\ dx$$

$$C = \int_{\pi/4}^{\pi/3} \sec^{3}(x)\ dx$$

$$D = \int_{-1}^{0} \frac{e^{x}}{x^{2}+2x+2}\ dx$$

Find $A + B + C + D$.

#10 Mu Bowl
MAΘ National Convention 2019

Let

$$A = \int_{-1}^{0} e^{x} \arctan(x+1)\ dx$$

$$B = \int_{\pi/4}^{\pi/3} \sec(x)\tan^{2}(x)\ dx$$

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$$D = \int_{-1}^{0} \frac{e^{x}}{x^{2}+2x+2}\ dx$$

Find $A + B + C + D$.

#11 Mu Bowl
MAΘ National Convention 2019

Let $f(x) = 3x^4 - 2x^3 + x^2 - x + 2$.

If

$$A = f(2) \qquad B = f'(2) \qquad C = \frac{f''(2)}{2!}$$

$$D = \frac{f'''(2)}{3!} \qquad E = \frac{f^{(4)}(2)}{4!} \qquad F = \frac{f^{(5)}(2)}{5!}$$

Find $A + B + C + D + E + F$.

#11 Mu Bowl
MAΘ National Convention 2019

Let $f(x) = 3x^4 - 2x^3 + x^2 - x + 2$.

If

$$A = f(2) \qquad B = f'(2) \qquad C = \frac{f''(2)}{2!}$$

$$D = \frac{f'''(2)}{3!} \qquad E = \frac{f^{(4)}(2)}{4!} \qquad F = \frac{f^{(5)}(2)}{5!}$$

Find $A + B + C + D + E + F$.

#12 Mu Bowl
MAΘ National Convention 2019

Let $f(x) = 4x^3 - 2x + 2019$. Let R be the finite region bounded by $f(x)$, the x-axis, $x = 1$, and $x = 3$.

Let A be the value obtained when the area of R is approximated using a Left-handed Riemann Sum with 8 equal subintervals.

Let B be the value obtained when the area of R is approximated using a Right-handed Riemann Sum with 8 equal subintervals.

Let C be the value obtained when the area of R is approximated using the Trapezoidal Rule with 8 equal subintervals.

Let D be the value obtained when the area of R is approximated using Simpson's Rule with 8 equal subintervals.

Find $A + B - 2C + D$.

#12 Mu Bowl
MAΘ National Convention 2019

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Let D be the value obtained when the area of R is approximated using Simpson's Rule with 8 equal subintervals.

Find $A + B - 2C + D$.

#13 Mu Bowl
MAΘ National Convention 2019

Let $A = 2019$ if the statement

"There exists $c \in (1,3)$ such that the slope of the tangent line to $f(x) = \frac{1}{x-2}$ at $x = c$ is -1"

is true, or -2019 if it is false.

Let $B = 2020$ if the statement

"A function may only cross its oblique asymptote a finite number of times"

is true, or -2020 if it is false.

Let $C = 2021$ if the statement

"There exists a function that is continuous and differentiable everywhere, but has a second derivative nowhere"

is true, or -2021 if it is false.

Find $A + B + C$.

#13 Mu Bowl
MAΘ National Convention 2019

Let $A = 2019$ if the statement

"There exists $c \in (1,3)$ such that the slope of the tangent line to $f(x) = \frac{1}{x-2}$ at $x = c$ is -1"

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is true, or -2021 if it is false.

Find $A + B + C$.

#14 Mu Bowl
MAΘ National Convention 2019

Let

A be the area contained within the curve
$$\frac{(x-2)^2}{4}+\frac{(y+1)^2}{9}=1$$

B be the area contained within the curve
$$|x|+|y|=20$$

C be the area contained within the polar curve
$$r^2=3\sin(2\theta)$$

D be the area contained within the polar curve
$$r=1-\cos(\theta)$$

Find $A+B+C+D$.

#14 Mu Bowl
MAΘ National Convention 2019

Let

A be the area contained within the curve
$$\frac{(x-2)^2}{4}+\frac{(y+1)^2}{9}=1$$

B be the area contained within the curve
$$|x|+|y|=20$$

C be the area contained within the polar curve
$$r^2=3\sin(2\theta)$$

D be the area contained within the polar curve
$$r=1-\cos(\theta)$$

Find $A+B+C+D$.