Let

\[ A = \lim_{x \to 0} \left( \frac{x - x^3}{x^2 + 2x} \right) \]

\[ B = \lim_{x \to \infty} \left( 1 - \frac{5}{x} \right)^{3x} \]

\[ C = \lim_{x \to \infty} \left( \frac{3x^2 + x - 1}{2x^2 - x^3 + 7} \right) \]

\[ D = \lim_{x \to \infty} \left( \sqrt{x^2 + 2x} - \sqrt{x^2 - x} \right) \]

Find \( \frac{AD}{B+C} \).
Let

\[ A = \frac{d}{dx} [e^{-x} \cos(x^2)] \bigg|_{x=0} \]
\[ B = \frac{d}{dx} [x(x^2 + 1)^2(x^3 + 2)^3] \bigg|_{x=1} \]
\[ C = \frac{d}{dx} [(x + 1)^{x^3-1}] \bigg|_{x=1} \]
\[ D = \frac{d}{dx} \left[ \tan^2(x) \right] \bigg|_{x=0} \]

Find \( A + B + C + D \).
Let

\[ f(x) = \frac{1 - x^2}{a} \]

\[ g(x) = a(x^2 - 1) \]

for a positive real constant \( a \).

Find the minimum possible area of the finite region bounded by \( f(x) \) and \( g(x) \).
Let

\[ A = \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{k + n} \]

\[ B = \lim_{n \to \infty} \prod_{k=1}^{n} \left(1 + \frac{k}{n}\right) \]

\[ C = \lim_{x \to 0} \frac{4 \arctan(1 + x) - \pi}{x} \]

\[ D = \lim_{x \to 0} \frac{\arctan(x) - x}{x^3} \]

Find \( A + \ln(B) + C + D \).
Let
\[ f(x) = x^{x+x^x+x^{x^x}} \]
\[ g(x) = \sqrt{4x^2 + \sqrt{4x^2 + \sqrt{4x^2 + \cdots}}} \]

If
\[ A = \frac{d}{dx}[f(x)]_{x=1} \quad B = \frac{d}{dx}[g(x)]_{x=1} \]
\[ C = \int_1^2 (\ln(f(x)) - f(x) \ln(x)) \, dx \quad D = \int_0^1 x \, g(x) \, dx \]

Find \( A + 17B + 4C + 96D \).
Let

\[ A = \int_{1}^{2} \left( x^3 + \frac{1}{x^2} \right) \, dx \]

\[ B = \int_{0}^{\pi/3} x \sin(x^2) \, dx \]

\[ C = \int_{0}^{\pi/4} (\cos(x) + \sin(x))^2 \, dx \]

\[ D = \int_{1}^{2} \frac{1}{x^2 + 2x} \, dx \]

Find \( \frac{A+C+e^{2D}}{B} \).
Consider the polynomial

\[ p(x) = rx^3 + (1 - r)x^2 - r^2x + (r + 1)^3 \]

where \( r \) is a real number changing at a constant rate of +2 units per second.

In units per second, let:

- \( A \) be the rate of change of the sum of the roots when \( r = 3 \)
- \( B \) be the rate of change of the product of the roots when \( r = 3 \)

Find \( A - B \).
Let
\[ A = \int_{1}^{2019} \frac{x^{2019}}{x^{2019} + (2020-x)^{2019}} \, dx \]
\[ B = \int_{0}^{\pi/2} \frac{1}{\cos(x) + \sin(x) + 2} \, dx \]
\[ C = \int_{0}^{1} \frac{x - 1}{\ln(x)} \, dx \]

It turns out that \(2A + B + C = pq + \sqrt{p} \arctan(\sqrt{p}) - \sqrt{p} \arctan(\frac{1}{\sqrt{p}}) + \ln(p)\) for a prime number \(p\). Find \(q\).
Let $R$ denote the finite region bounded by the $x$-axis and the curve $y = 3 - 3x^2$. Let $A$ be the area of $R$.

Let $B$ be the volume of the solid formed when $R$ is rotated about the $x$-axis.

Let $C$ be the volume of the solid formed when $R$ is rotated about the line $x = 1$.

Let $D$ be the volume of the solid formed when $R$ is rotated about the line $y = x - 1$.

Find $\frac{C}{A} + \frac{D}{B}$.
Let $L$ represent the line tangent to the curve $x^2y - xy^2 - x + y = -1$ at the point (1,2).

Let $M$ represent the line tangent to the curve $\begin{cases} x(t) = e^t - 2t \\ y(t) = t^2 - \ln(t + 1) \end{cases}$ at $t = 0$.

If $(A,B)$ is the point of intersection between $L$ and $M$, find $A + B$. 
Let

\[ A = \int_{-1}^{0} e^x \arctan(x + 1) \, dx \]
\[ B = \int_{\pi/4}^{\pi/3} \sec(x) \tan^2(x) \, dx \]
\[ C = \int_{\pi/4}^{\pi/3} \sec^3(x) \, dx \]
\[ D = \int_{-1}^{0} \frac{e^x}{x^2 + 2x + 2} \, dx \]

Find \( A + B + C + D \).
Let $f(x) = 3x^4 - 2x^3 + x^2 - x + 2$.

If

\[
A = f(2) \qquad B = f'(2) \qquad C = \frac{f''(2)}{2!}
\]

\[
D = \frac{f'''(2)}{3!} \qquad E = \frac{f^{(4)}(2)}{4!} \qquad F = \frac{f^{(5)}(2)}{5!}
\]

Find $A + B + C + D + E + F$. 

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\]

Find $A + B + C + D + E + F$. 

Let $f(x) = 4x^3 - 2x + 2019$. Let $R$ be the finite region bounded by $f(x)$, the $x$-axis, $x = 1$, and $x = 3$.

Let $A$ be the value obtained when the area of $R$ is approximated using a Left-handed Riemann Sum with 8 equal subintervals.

Let $B$ be the value obtained when the area of $R$ is approximated using a Right-handed Riemann Sum with 8 equal subintervals.

Let $C$ be the value obtained when the area of $R$ is approximated using the Trapezoidal Rule with 8 equal subintervals.

Let $D$ be the value obtained when the area of $R$ is approximated using Simpson's Rule with 8 equal subintervals.

Find $A + B - 2C + D$. 

Let \( A = 2019 \) if the statement

“There exists \( c \in (1,3) \) such that the slope of the tangent line to \( f(x) = \frac{1}{x^2} \) at \( x = c \) is \(-1\)”
is true, or \(-2019\) if it is false.

Let \( B = 2020 \) if the statement

“A function may only cross its oblique asymptote a finite number of times”
is true, or \(-2020\) if it is false.

Let \( C = 2021 \) if the statement

“There exists a function that is continuous and differentiable everywhere, but has a second derivative nowhere”
is true, or \(-2021\) if it is false.

Find \( A + B + C \).
Let

\[ A \text{ be the area contained within the curve } \frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1 \]

\[ B \text{ be the area contained within the curve } |x| + |y| = 20 \]

\[ C \text{ be the area contained within the polar curve } r^2 = 3\sin(2\theta) \]

\[ D \text{ be the area contained within the polar curve } r = 1 - \cos(\theta) \]

Find \( A + B + C + D \).