For all questions, the answer choice "(E) NOTA" means none of the above answers is correct. $\mathbb{Z}_n$ represents the ring of integers under the operations of addition and multiplication modulo n. Good luck and have fun!

1. Given that $5|(x+4y)$, which of the following expressions is necessarily divisible by 5?
(A) $3x+y$ (B) $7x+6y$ (C) $6x+9y$ (D) $2x+5y$ (E) NOTA

2. What is 17^{-1} under $\mathbb{Z}_{47}$?
(A) 28 (B) 13 (C) 36 (D) 7 (E) NOTA

3. What is the smallest integer that can be written as a sum of two positive cubes in two distinct ways?
(A) 3645 (B) 87539319 (C) 7453 (D) 1332 (E) NOTA

4. Find the sum of all AB_{10} such that $AB_{10} = BA_7$, where A and B are digits of the two-digit numbers.
(A) 69 (B) 104 (C) 42 (D) 37 (E) NOTA

5. Ben is trying to figure out how many Pokémon he has caught in Pokémon Sun. If he puts them into boxes containing 16 Pokémon each, he has a box with only 5 Pokémon in it; if he puts them into boxes of 9, he has a box with only 4 Pokémon in it; and if he puts them into boxes of 5, he has a box with only 3 Pokémon in it. Given that there are 802 Pokémon available for capture in Sun and Ben does not catch duplicate Pokémon, how many Pokémon has he caught?
(A) 101 (B) 796 (C) 229 (D) 373 (E) NOTA

6. Ellen plays a game where she rolls 6 fair, six-sided dice, then sums the numbers on the top faces (six numbers in total). If she gets a multiple of 6 for her sum, then she wins! What is the probability that Ellen wins this game?
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{25}{36}$ (E) NOTA

7. How many positive integers less than 60 are relatively prime to 60?
(A) 24 (B) 36 (C) 16 (D) 48 (E) NOTA

8. Let a and b be non-negative integers. Let c be the largest integer that cannot be represented in the form $6a + 7b$. What is the sum of the digits of c?
(A) 10 (B) 11 (C) 12 (D) 13 (E) NOTA

9. What is the sum (in $\mathbb{Z}_{35}$) of the zeros of the expression $x^2 + 2x + 32$ under $\mathbb{Z}_{35}$?
(A) 31 (B) 7 (C) 12 (D) 0 (E) NOTA

10. How many positive integers less than 1500 are divisible by 7 or 11, but not by both?
(A) 312 (B) 363 (C) 331 (D) 350 (E) NOTA

11. What is the third smallest positive integer that has exactly 9 factors?
(A) 100 (B) 196 (C) 225 (D) 128 (E) NOTA

12. Find the number of zeros at the end of the expansion of 2018!.
(A) 360 (B) 2011 (C) 502 (D) 403 (E) NOTA

13. What is the product of $\gcd(84, 126)$ and $\text{lcm}(84, 126)$?
(A) 10863 (B) 5292 (C) 21168 (D) 31752 (E) NOTA

14. For how many positive integer pairs (m, n) is $2m + 5n = 99$?
(A) 8 (B) 10 (C) 6 (D) 4 (E) NOTA

15. What is the sum of the positive integral factors of 3288?
(A) 8119 (B) 8280 (C) 8357 (D) 8068 (E) NOTA

16. Let a, b, c be positive integers which satisfy the equation $a^2 + b^2 = c^2$. What is the largest possible value of k such that $k|abc$ for all such a, b, c?

(A) 4 (B) 12 (C) 15 (D) 60 (E) NOTA

17. For how many integer pairs (m, n) is $8m^3 + n^2 = 2019$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

For questions 18-21:

Define **Gaussian integers** to be a ring $\mathbb{Z}[i] = \{a + bi | a, b \in \mathbb{Z}\}$, where $i^2 = -1$. We define the norm of a Gaussian integer $\alpha = a + bi$ to be $N(\alpha) = a^2 + b^2$. For these problems, assume $\alpha, \beta, \gamma \in \mathbb{Z}[i]$.

18. What is the norm of $(5 + 3i)(6 - 2i)(2 + i)$?

(A) 96 (B) 192 (C) 6800 (D) 3400 (E) NOTA

19. We say that $\alpha|\beta$ if there exists γ such that $\gamma\alpha = \beta$. If $(1 - 5i)|\alpha$, which of the following integers divides $N(\alpha)$?

(A) 7 (B) 5 (C) 13 (D) 11 (E) NOTA

20. Define a *unit* in $\mathbb{Z}[i]$ to be α such that there exists β with the property $\alpha\beta = \beta\alpha = 1$. A Gaussian integer γ is a *Gaussian prime* if there do not exist α, β such that $\gamma = \alpha\beta$ and neither of α, β is a unit. Which of the following is a Gaussian prime?

(A) 2 (B) 41 (C) $4 + i$ (D) $1 + 5i$ (E) NOTA

21. Which of the following properties is/are true under $\mathbb{Z}[i]$?

(I) $N(\alpha\beta) = N(\alpha)N(\beta)$

(II) The only units of $\mathbb{Z}[i]$ are 1 and -1

(III) If $N(\alpha)|N(\beta)$ under $\mathbb{Z}$, then $\alpha|\beta$ under $\mathbb{Z}[i]$

(IV) Primes $p = 4k + 3$ for $k \in \mathbb{Z}$ are also primes under $\mathbb{Z}[i]$

(A) I, III, IV (B) I (C) II, IV (D) III, IV (E) NOTA

22. What is the sum of all positive integers n for which $(n - 4)|(n^3 + 7n^2 - 13n + 19)$?

(A) 8 (B) 187 (C) 45 (D) 73 (E) NOTA

23. What is the remainder when 3^{391} is divided by 91?

(A) 9 (B) 3 (C) 27 (D) 81 (E) NOTA

24. The cubic equation $n^3 - 40n^2 + c_1 n + c_2$ has prime roots p_1, p_2, p_3. What is the sum of all possible values for $|c_2|$?

(A) 265 (B) 434 (C) 722 (D) 138 (E) NOTA

25. A sequence a_n is given by $a_0 = 1, a_1 = 1, a_n = 3a_{n-1} - a_{n-2}$ for $n \geq 2$. What is $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$?

(A) $\frac{\sqrt{6}+\sqrt{2}}{2}$ (B) $\frac{5}{2}$ (C) $\frac{3+\sqrt{5}}{2}$ (D) $\sqrt{7}$ (E) NOTA

26. Which of the following values for x satisfies the equation $27^5 + 84^5 + 110^5 + x^5 = 144^5$?

(A) 133 (B) 126 (C) 143 (D) 130 (E) 139

27. How many distinct prime factors does $4^5 + 5^4$ have?

(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA

28. What is the sum of all $x + y$ where (x, y) are non-negative integer solutions to the equation $x^3 = 3^y - 1$?

(A) 4 (B) 0 (C) 30 (D) 17 (E) NOTA

29. What is the sum of all $a < 50$ such that positive integers (a, b) satisfy the equations $a - b = p$ and $ab = k^2$ for some integer k and prime p?

(A) 75 (B) 89 (C) 114 (D) 29 (E) NOTA

30. There exist positive integers $m, n > 4$ such that m, n are solutions to the equation $(n-2)^2 - (m-2)^2 = 2mn$. What is the least possible value of $m + n$?

(A) 48 (B) 60 (C) 78 (D) 84 (E) NOTA