

For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good luck, have fun!

**PART I: Mr. Jensen's Math Class**

Mr. Jensen has planned a fun class activity for today! To begin, solve the first five equations.

1. Solve for  $A$ :  $\log_2 A + \log_4 A^2 + \log_8 A^3 = 9$ .

- A. 2                      B. 4                      C. 8                      D. 16                      E. NOTA

2. Solve for  $B$ :  $2(4^B) - 7(2^B) - 1 = 3$ .

- A. 1/4                      B. 1/2                      C. 2                      D. 4                      E. NOTA

3. Compute the sum of solutions to the following equation:  $|1 - |x - |2x + 1|| - 2 = 5$ . For question 6, let  $C$  be the absolute value of the answer to this problem. For this problem, bubble the actual sum of the solutions.

- A. 7                      B. 4                      C. -3                      D. -5                      E. NOTA

4. Let  $D$  be the number of integral solutions between  $-10$  and  $10$ , inclusive, to the following inequality:  $\frac{(2x-3)(x+2)}{(x+2)(x-3)} \leq \frac{2}{x-3}$ .

- A. 4                      B. 8                      C. 13                      D. 20                      E. NOTA

5. Let  $M$ ,  $N$ , and  $P$  be the numerators of the partial fraction decomposition below. Compute  $E = |M + N + P|$ .

$$\frac{11x + 5}{(x + 1)^2(x - 2)} = \frac{M}{x - 2} + \frac{N}{(x + 1)^2} + \frac{P}{x + 1}$$

- A. 2                      B. 4                      C. 6                      D. 8                      E. NOTA

6. Take  $A, B, C, D,$  and  $E$  as in the previous questions. Mr. Jensen has a wheel with sectors labeled  $A, B, C, D,$  and  $E$ , where the probability of the wheel landing on  $A$  is  $\frac{A}{A+B+C+D+E}$ , and likewise for the other four sectors. If you spin the wheel twice (assuming the first spin is independent of the second), in which of the following ranges is the probability that you land on  $A$  and  $E$  in some order?
- A.  $(0, 0.1]$       B.  $(0.1, 0.2]$       C.  $(0.2, 0.3]$       D.  $(0.3, 0.4]$       E. NOTA

### **PART 2: All's Fair in Love and War**

Daniel is trying to win Catherine's heart. Catherine wants to ensure that Daniel is a man of mathematical skill, so she puts him through some assessments.

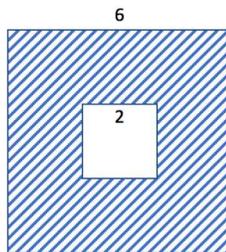
7. Let's start off easy! Catherine wants to make sure that she and Daniel will have a great couple name. One option is to form some permutation of CATHIEL. The other option is to form some permutation of DANERINE. If these are the only two options, how many possible couple names are there?
- A. 12600      B. 15120      C. 25200      D. 45360      E. NOTA
8. Catherine gives Daniel eight equally-sized basketballs and one softball that need to go in a box whose length and width are both 32 inches. He puts four basketballs in the box first, such that their centers form a square. He then balances the softball on top of the four basketballs, with the center of the softball directly above the center of the square formed by the centers of the basketballs. Then he puts the other four basketballs on top, again with centers forming a square. If the radius of each basketball is eight inches and the radius of the softball is six inches, what is the minimum height of a box that completely contains the nine balls?
- A.  $2\sqrt{17}$       B.  $4\sqrt{17}$       C.  $8 + 2\sqrt{17}$       D.  $16 + 4\sqrt{17}$       E. NOTA
9. Daniel is trying to impress Catherine with his polyhedron-guessing capabilities. Catherine has a polyhedron in mind, and she tells him that her polyhedron has 24 vertices and 36 edges. Daniel would like to win Catherine's heart by guessing the correct number of faces. What number should he guess?
- A. 16      B. 14      C. 12      D. 10      E. NOTA

10. Impressed by Daniel's polyhedron-guessing skills, Catherine tests his mathematical abilities while jet-skiing. They go to a straight river (with no current) that is  $M$  meters across, and start skiing towards each other at exactly the same time from directly opposite points. They first meet 50 meters from the side of the river that Daniel started on. They keep skiing and turn around at the banks of the river without losing time. They meet for the second time 20 meters from the opposite bank, and immediately stop. Catherine tells him the above information and asks him to tell her how wide the river is. Help him out! What is  $M$ ?

A. 100                      B. 130                      C. 160                      D. Need more info                      E. NOTA

11. Catherine is thinking of a polynomial,  $f(x)$ . When she divides it by  $x - 3$ , it leaves a remainder of 5. When she divides it by  $x + 2$ , it leaves a remainder of  $-3$ . She asks Daniel to find the remainder,  $g(x)$ , when  $f(x)$  is divided by  $x^2 - x - 6$ ! Daniel asks you for a hint – what is  $g(8)$ ?

A.  $-3$                       B.  $9/5$                       C. 5                      D. 13                      E. NOTA



12. Daniel has passed all of Catherine's tests so far! As a final test, Catherine assembles a 6-by-6-by-6 cube made of unit cubes and removes a 2-by-2-by-1 block from the center of each of the six faces, as shown above for one face. She paints every visible face of the remaining object purple, her favorite color. She then disassembles the object and picks a unit cube at random, with all cubes equally probable. She then tosses the cube into the air. If it lands purple side up, she will go on a date with Daniel. What is the probability that she will go on a date with Daniel?

A.  $11/54$                       B.  $11/48$                       C.  $19/48$                       D.  $19/24$                       E. NOTA

**PART 3: Field Trip**

You and your classmates take a trip to Ace's Amazing Animals, a local animal shelter, to help Ace and his staff take care of the animals.

13. In the Hotelling Room of the animal shelter, there are only pigs and chickens. When you and nine of your classmates walk into the room, the number of legs increases by five percent and the number of arms increases by ten percent. How many pigs are there in the room? Assume that pigs have four legs and zero arms, and that chickens have two arms and two legs.
- A. 45                      B. 50                      C. 90                      D. 100                      E. NOTA
14. It's time to feed Cournot the Cat her daily Magic Milk Mixture, consisting of a combination of Bertrand milk, which is 12% sugar, and Stackelberg milk, which is 20% sugar. Cournot the Cat prefers a 15% sugar mixture for her Magic Milk. Ace gives you eight fluid ounces of Stackelberg milk, and wants you to make eight fluid ounces of Magic Milk Mixture for Cournot the Cat. How much Stackelberg milk should you remove and replace with Bertrand milk to reach Cournot the Cat's optimal sugar level?
- A. 5                      B. 4                      C. 3                      D. 2                      E. NOTA
15. Allais and Ellsberg are two rabbits who are racing each other. They set a starting point and an ending point ninety inches apart. Allais hops at five inches per second. Allais thinks that Ellsberg is slower, so she gallantly gives Ellsberg a three-second head start. They race, and Allais discovers that Ellsberg reaches the finish line when Allais has five inches to go. How fast does Ellsberg hop (in inches per second), if she hops at a constant speed?
- A. 4.5                      B.  $45/11$                       C.  $45/8$                       D.  $45/7$                       E. NOTA
16. Brian Wheaton the Parrot is flying around his rectangular cage. The cage has a rectangular base of length 10 feet and width 8 feet, with a height of 6 feet. The cage is sitting flat on the ground. Brian Wheaton the Parrot, however, has special powers that allow him to fly 36 inches outside of any point on the cage, in addition to anywhere inside the cage. How many cubic feet of space (inside or outside the cage) does Brian Wheaton have access to?
- A.  $888 + 135\pi$                       B.  $1368 + 252\pi$                       C.  $1608 + 135\pi$                       D.  $1608 + 252\pi$                       E. NOTA

17. You play catch with Shephard the Sheep on a Euclidean plane, where Shephard the Sheep is standing at the point  $(3,0)$ . Suppose Shephard can only run along the  $x$ -axis. You throw the ball along the parabola  $y = -\frac{1}{10}(x - 2)^2 + 10$ , starting at the  $y$ -intercept and traveling in the positive  $x$  direction. Shephard the Sheep must run to catch the ball when it reaches the  $x$ -axis, and can run at a rate of 3 units per second. How many seconds will it take for Shephard to reach the landing point of the ball from his current position?
- A. 3                      B. 4                      C. 9                      D. 12                      E. NOTA
18. Ace is expanding his animal shelter! You enlist two of your friends, Hicks and Marshall, to help you paint a room. You can paint fifteen square feet an hour, while Hicks and Marshall can each paint ten square feet an hour. Hicks starts painting first for thirty minutes, after which you and Marshall join in. How long will it take, in hours, from the time Hicks starts painting, for the three of you to finish painting 250 square feet?
- A. 7                      B.  $50/7$                       C. 7.5                      D.  $107/14$                       E. NOTA

#### **PART 4: Game Time**

You and your classmates decide to rebel against Mr. Jensen and play a few popular classroom games.

19. Joe Blitzstein decides he doesn't want to participate in the festivities and goes to swing on the swing set. He starts in a vertical, stationary position, then swings back and forth, trying to build height. Suppose the length of the swing is nine feet, and that his own size is negligible. He starts by swinging forward, traveling  $\frac{\pi}{2}$  feet along the arc, then backwards  $\frac{3\pi}{2}$  feet along the arc, then forwards  $\frac{5\pi}{2}$  feet along the arc, and so on. He jumps off the swing when the swing reaches  $60^\circ$  from vertical for the first time. What is the total distance that Joe travels on the swing?
- A.  $21\pi/2$                       B.  $15\pi$                       C.  $18\pi$                       D.  $21\pi$                       E. NOTA
20. You and your good friend, Leontief, take a box of chalk from the classroom closet and start drawing outside on the concrete. Leontief draws a large circle, and challenges you to create as many intersections as possible by drawing ten straight line segments of any finite length. Assume that an intersection is counted each time a line crosses the circle or another line. What is the maximal number of intersections?
- A. 55                      B. 60                      C. 65                      D. 70                      E. NOTA

21. You and your desk partner, Giffen, head outside to build sand castles in the sand box. You can build one sand castle in twenty minutes, while Giffen can build one sand castle in forty minutes. The classroom dog, Pareto, destroys sand castles at a rate of two sand castles per hour. How long, in minutes, will it take you and Giffen to build three sand castles, if you two start working and Pareto starts destroying at the same time? (For the sake of the problem, sand castles are not discrete objects. In other words, a castle that was built but  $x\%$  of which has been destroyed by Pareto, when combined with a castle that is  $x\%$  into its construction, still count as one.)
- A. 120/13      B. 24      C. 360/13      D. 72      E. NOTA
22. Walras and Cara start playing a game where they stand one hundred yards apart and simultaneously start running towards each other until they collide. Walras runs at 60 feet per second and Cara runs at 40 feet per second. Bernoulli the Bee is currently at Cara's location. When Walras and Cara start running, Bernoulli will fly to Walras, then turn around (without losing any time) and fly to Cara, repeating this pattern until Bernoulli is crushed between the two runners. If Bernoulli flies at seventy yards per second, how many yards will Bernoulli fly from the time the running begins until his untimely death?
- A. 70      B. 210      C. 350      D. 1050      E. NOTA
23. Ed, Greg, and John are eating candy. In each round, the cube of the number of pieces Ed eats is directly proportional to the number of pieces Greg eats and inversely proportional to the square of the number that John eats. If, in the first round, Ed eats four pieces, John eats six pieces, and Greg eats eight pieces, how many pieces will Ed eat in the second round if Greg eats six pieces and John eats eight pieces? Fractional pieces of candy are allowed.
- A. 3      B. 4      C. 24      D. 4096/27      E. NOTA
24. Ed then orders two pizzas. A pizza can be small, medium, or large, and there are three possible toppings. A pizza can be any of the three sizes, with any combination of the toppings (no toppings is also allowed). How many different combinations of pizzas can Ed order?
- A. 276      B. 300      C. 324      D. 576      E. NOTA

**PART 5: Casino Party**

You're in Las Vegas – it's time to go to a casino!

25. Mr. Lu shuffles a standard deck of 52 cards and starts dealing them one by one. What is the probability that exactly one King, one Queen, and one Jack (in any order) are dealt before the first Ace?
- A.  $3/16$       B.  $1/4$       C.  $7/16$       D.  $15/32$       E. NOTA
26. Mr. Frazer plays a series of three games, entitled  $A_1$ ,  $A_2$ , and  $A_3$ . He has a probability of 0.6 of winning the first game,  $A_1$ . If he wins game  $A_i$ , he wins game  $A_{i+1}$  with probability 0.8. However, if he loses game  $A_i$ , he gets discouraged and wins game  $A_{i+1}$  with probability 0.4. Given that Mr. Frazer wins  $A_1$ , what's the probability that he wins  $A_3$ ?
- A. 0.08      B. 0.68      C. 0.72      D. 0.76      E. NOTA
27. In poker, a full house (three of one kind, two of another kind) is more desirable than a flush (five of a suit). Let  $X$  be the number of possible full house hands, and  $Y$  be the number of possible flush hands in a standard 52-card deck. What is  $X/Y$ ? (Include royal flushes, which consist of a 10, Jack, Queen, King, and Ace of a suit, as well as straight flushes, which consist of five consecutive cards of the same suit, when counting flushes.)
- A.  $8/11$       B.  $26/33$       C.  $33/26$       D.  $11/8$       E. NOTA
28. Chris Rycroft has a deck of 4 cards, consisting of the numbers 1 through 4. You choose 3 at random and with replacement, and you win if the numbers on the three cards you choose form the side lengths of a nondegenerate triangle. What is the probability that you win?
- A.  $7/16$       B.  $15/32$       C.  $17/32$       D.  $5/8$       E. NOTA

29. Honty Mall invites you to play his game! He has three doors, numbered Door 1, Door 2, and Door 3. For each door, there is a goat behind the door with probability  $\frac{2}{3}$ , and a car behind the door with probability  $\frac{1}{3}$ . Assume that the contents behind each door are independent. You pick a door, and Honty then opens Door 2. You can then stay with your current choice of door or switch to the unopened door. Your strategy is to pick Door 1, and to switch to the unopened door, Door 3, with probability  $\frac{1}{2}$ . Given Honty opens Door 2 to reveal a goat, what is the probability that you win a car?
- A.  $\frac{2}{9}$       B.  $\frac{1}{3}$       C.  $\frac{4}{9}$       D.  $\frac{5}{9}$       E. NOTA
30. The rapper B. Tshishi is coincidentally at the same casino! He wants impress the casino's guests, so he claims to know the number of zeros at the end of  $1011!$ , when expanded completely. He confides to you that he needs your help, however – what is the correct number of zeros?
- A. 202      B. 250      C. 251      D. 310      E. NOTA