PART I: Mr. Jensen's Math Class

1. **ANS: 8 (C)**. Using change-of-base, we see that the second and third terms on the left side reduce to $\log_2 A$. So, we have that $3\log_2 A = 9$, or $\log_2 A = 3$, so $A = 8$.

2. **ANS: 2 (C)**. Letting $y = 2^B$, the equation becomes $2y^2 - 7y - 4 = (2y+1)(y-4) = 0$. Since B must be positive, we have that $B = 2$.

3. **ANS: 4 (B)**. Moving the 2 to the other side gives $\big|1 - |x - |2x+1||\big| = 7$, which means that $|x - |2x+1|| = 8$ or -5, which isn't possible since absolute values must be positive. So, we either have that $x - |2x+1| = 8$ or $x - |2x+1| = -8$. The first gives solutions -9 and $\frac{7}{3}$, both of which are invalid. The second gives solutions 7 and -3, both of which are valid. Thus, the sum of solutions is 4.

4. **ANS: 0 (E)**. First, we'll move everything to one side and get a common denominator:
$$\frac{(2x-3)(x+2) - 2(x+2)}{(x+2)(x-3)} = \frac{2x^2 - x - 10}{(x+2)(x-3)} \leq 0$$

Simplifying,
$$\frac{(2x-5)(x+2)}{(x+2)(x-3)} \leq 0$$

This gives us a solution interval of $\frac{5}{2} \leq x < 3$, so there are no integral solutions.

5. **ANS: 2 (A)**. First, we'll get a common denominator:

$$\frac{M(x+1)^2 + N(x-2) + P(x+1)(x-2)}{(x+1)^2(x-2)} = \frac{11x+5}{(x+1)^2(x-2)}$$

Matching coefficients, we have that $M + P = 0$, $2M + N - P = 11$, and $M - 2N - 2P = 5$. Solving, we obtain $M = 3$, $P = -3$, and $N = 2$, so their sum is 2.

6. **ANS: (0.1, 0.2) (B)**. From the past five answers, the probability of landing on A is $\frac{8}{16} = \frac{1}{2}$ and the probability of landing on E is $\frac{2}{16} = \frac{1}{8}$. So, the probability of landing on A and E in some order is $2 \cdot \frac{1}{2} \cdot \frac{1}{8} = \frac{1}{8} = 0.125$.

PART 2: All's Fair in Love and War

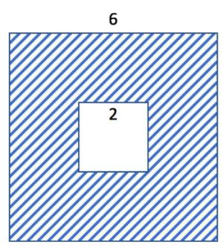
7. **ANS: 15120 (B)**. For permutations of CATHIEL, there are no repeated letters, so there are $7! = 5040$ possibilities. For permutations of DANERINE, there are two E's and two N's, so we have $\frac{8!}{2^2} = 10080$ possibilities, giving a total of 15120 possibilities.

8. **ANS: $16 + 4\sqrt{17}$ (D)**. We need to calculate the vertical distance from the center of the softball to the center of the square formed by the centers of the four basketballs. Consider the right triangle formed by these two points and the center of one basketball. The hypotenuse has length 14 inches since it is the sum of the radii of the two types of balls. One leg has length $8\sqrt{2}$ inches since it is half the diagonal of the square formed by the centers of the four basketballs. So, the height is $2\sqrt{17}$ by the Pythagorean Theorem. So, the height of the box is twice this height, plus twice the radius of the basketball at the bottom and top, which is just $16 + 4\sqrt{17}$.

9. **ANS: 14 (B)**. For any polyhedron, we must have that the number of faces plus the number of vertices must equal the number of edges plus two. So, we see that $F + 24 = 38$, or $F = 14$.

10. **ANS: 130 (B)**. Note that distance equals rate times time. So, we have that $\frac{M-50}{c} = \frac{50}{d}$ and $\frac{50+M-20}{c} = \frac{M-50+20}{d}$. Rearranging, we have that $\frac{M-50}{50} = \frac{M+30}{M-30}$. Cross-multiplying, we obtain that $M^2 - 80M + 1500 = 50M - 1500$. Thus, $M = 130$.

11. **ANS: 13 (D)**. 1From the information in the question, we know that $f(x) = (x-3)p(x) + 5$ and that $f(x) = (x+2)q(x) - 3$. So, let $f(x) = (x+2)(x-3)r(x) + g(x)$, where $g(x) = ax + b$. So, we know that $f(3) = 5$ and $f(-2) = -3$ from the first two equations. Plugging into the third, we obtain $f(3) = 5 = 3a + b$ and $f(-2) = -3 = -2a + b$. Solving this system, $a = \frac{8}{5}$ and $b = \frac{1}{5}$. So, $g(8) = \frac{64}{5} + \frac{1}{5} = 13$.



12. **ANS: 11/48 (B)**. There are eight cubes with three sides painted (corners). There are 48 cubes on the edges of the original cube plus 48 cubes on the edges of the removed sections, each with two painted sides. There are 48 cubes with just one face exposed (centers of squares). Finally, there are 40 cubes that are completely hidden. There are a total of 192 possible cubes to choose from, so the total probability is $\frac{8}{192} \times \frac{3}{6} + \frac{96}{192} \times \frac{2}{6} + \frac{48}{192} \times \frac{1}{6} = \frac{11}{48}$.

PART 3: Field Trip

13. **ANS: 50 (B)**. Let P be the number of pigs and C be the number of chickens. The number of legs is then $4P + 2C$ and the number of arms is then $2C$. The number of legs increases by five percent when twenty legs are added, so $4P + 2C = 400$. Similarly, $2C = 200$. So, we have that $C = 100$ and $P = 50$.

14. **ANS: 5 (A)**. Let x be the amount to remove and replace. We'll construct an equation representing the amount of sugar: $0.2(8-x)+0.12x=0.15(8)=1.2$. Solving, we obtain $x=5$.

15. **ANS: 4.5 (A)**. Let e represents how fast Ellsberg hops, in inches per second. So, using the fact that time equals distance over rate, $\frac{90}{e}=\frac{85}{5}+3$. Solving, we have that $e=\frac{90}{20}=4.5$.

16. **ANS: $\mathbf{1368+153\pi}$ (E)**. Brian clearly has access to the inside of the cage, which has volume 480 cubic feet. He also has access to the points that are within three feet of the walls (measured perpendicular to the walls), which results in rectangular prisms protruding from the cage in all sides except the bottom. This amounts to $3(10)(8)+2(3)(10)(6)+2(3)(8)(6)=888$ cubic feet. For the edges, there are four identical vertical quarter-cylinders, amounting to one whole cylinder of space Brian can access, with volume $\pi(3)^2(6)=54\pi$. On the top edges, there are again four vertical quarter-cylinders with radius 3, but two have height 4 and two have height six. This amounts to a volume of $(0.5)\pi(3)^2(10)+(0.5)\pi(3)^2(8)=81\pi$. Finally, around the top four corners, there are four eighth-spheres of additional access, giving $\left(\frac{4}{3}\right)(0.5)\pi(3)^3=18\pi$ more cubic feet. So, in total, there are $1368+153\pi$ cubic feet of flying space.

17. **ANS: 3 (A)**. We just need to find the x-intercept of the parabola, which can be done by setting $y=0$. So, we have that $(x-2)^2=100$, or $x=12$. So, Shephard needs to run nine units, which will take Shephard 3 seconds.

18. **ANS: 7.5 (C)**. In the thirty minutes that Hicks paints alone, five square feet are painted. So, there are 245 square feet left to be painted. The three of you combined can paint 35 square feet an hour, so this comes out to seven more hours. Thus, it will take 7.5 hours from the time Hicks starts painting to finish 250 square feet.

PART 4: Game Time

19. **ANS: $\mathbf{18\pi}$ (C)**. Note that Joe travels forward and back $\frac{\pi}{2}$ to center position, then backwards and forward π to center position, then $\frac{3\pi}{2}$, and so on. Sixty degrees represents a length of $\frac{1}{6}\cdot 18\pi=3\pi$ from center position, so we need to calculate $2\cdot\frac{\pi}{2}+2\cdot\pi+\cdots+2\cdot\frac{5\pi}{2}+3\pi=18\pi$.

20. **ANS: 65 (C)**. We have $\binom{10}{2}=45$ line-line intersections, as well as 20 line-circle intersections (2 per line). So, this makes 65 total intersections.

21. **ANS: 72 (D)**. Your rate is $\frac{1}{20}$ per minute, and Giffen's rate is $\frac{1}{40}$ per minute. Pareto's rate of destruction is $\frac{1}{30}$ per minute. So, let t be the time it takes to build a sand castle. We have that $\left(\frac{1}{20}+\frac{1}{40}-\frac{1}{30}\right)t=1$, or $t=24$. They need to build three sand castles, so our answer is 72.

22. **ANS: 210 (B)**. First, we'll find the time elapsed until they crash, which we'll call t. Since they are 100 yards, or 300 feet apart, we have that $60t + 40t = 300$, or $t = 3$. Since Bernoulli flies continuously during this time, he will fly for $3(70) = 210$ yards.

23. **ANS: 3 (A)**. Call the number of pieces Ed eats e, and similarly assign j and g for John and Greg, respectively. Then, we know that $\frac{e^3 j^2}{g}$ is a constant. Thus, $\frac{(4^3)(6^2)}{8} = \frac{e^3(8)^2}{6}$. Solving we get $e = 3$.

24. **ANS: 300 (B)**. There three possible sizes, and eight possible combinations of toppings (2^3), which comes out to 24 possible pizzas. If the two pizzas are the same, there are 24 possible combinations. If the two pizzas are different, there are $\binom{24}{2} = 276$ possible combinations, for a total of 300.

PART 5: Casino Party

25. **ANS: 16/455 (E)**. Let's just consider the 16 Kings, Queens, Jacks, and Aces, since the order of the other cards doesn't matter. The probability we want is $6 \cdot \frac{4}{16} \cdot \frac{4}{15} \cdot \frac{4}{14} \cdot \frac{4}{13} = \frac{16}{455}$, since the King, Queen, and Jack can be reordered in $3! = 6$ ways.

26. **ANS: 0.72 (C)**. Given the result of the first game, Mr. Frazer can win the second and third, or he can lose the second and win the third. The probability of the former happening is $(0.8)(0.8) = 0.64$. The probability of the latter happening is $(0.2)(0.4) = 0.08$. This gives us a total probability of 0.72.

27. **ANS: 8/11 (A)**. The number of full house hands is $13\binom{4}{3}12\binom{4}{2} = 13(12)(24) = X$. The number of flush hands is $4\binom{13}{5} = Y$. We then obtain $\frac{(13)(12)(24)}{(13)(12)(11)(3)} = \frac{8}{11}$.

28. **ANS: 17/32 (C)**. We'll casework by the largest card picked, noting that the total number of possibilities is 64.

    a. There is exactly one triplet whose largest card is 1: (1, 1, 1).
    b. For triplets whose largest card is 2, we can have (1, 2, 2), of which there are three, and (2, 2, 2), of which there is only 1, giving us a total of 4.
    c. For triplets whose largest card is 3, we can have (1, 3, 3), (2, 2, 3), (2, 3, 3), and (3, 3, 3), giving a total of $3 + 3 + 3 + 1 = 10$.
    d. For triplets whose largest card is 4, we can have (1, 4, 4), (2, 3, 4), (2, 4, 4), (3, 3, 4), (3, 4, 4), and (4, 4, 4), giving a total of $3 + 6 + 3 + 3 + 3 + 1 = 19$.

    So, we have a total of $1 + 4 + 10 + 19 = 34$, or a probability of $\frac{34}{64} = \frac{17}{32}$.

29. **ANS: 1/3 (B)**. We are conditioning on the fact that there is a goat behind door 2. So, if all three doors contain goats, with probability $\frac{4}{9}$, you will not win the car. If there are goats behind doors 1 and 2, and a car behind door 3, with probability $\frac{2}{9}$, you win the car with probability $\frac{1}{2}$. If there are goats behind doors 2 and 3, and a car behind door 1, with probability $\frac{2}{9}$, you win the car with probability $\frac{1}{2}$. Finally, if there are cars behind doors 1 and 3, and a goat behind door 2, with probability $\frac{1}{9}$, you win the car with probability 1. This gives a total probability of

$$\frac{4}{9}\cdot 0+\frac{2}{9}\cdot\frac{1}{2}+\frac{2}{9}\cdot\frac{1}{2}+\frac{1}{9}\cdot 1=\frac{1}{3}$$

30. **ANS: 251 (C)**. We know that $1011! = 1\cdot 2\cdot \ldots \cdot 1010\cdot 1011$. Zeros appear at the end of a number when a 2 is multiplied with a 5. So, we just need to count the number of 2's and 5's in this number's prime factorization. However, note that there is an abundance of 2's relative to 5's, so we'll just count the number of 5's. There are $\left\lfloor\frac{1011}{5}\right\rfloor = 202$ multiples of 5, $\left\lfloor\frac{1011}{25}\right\rfloor = 40$ multiples of 25, $\left\lfloor\frac{1011}{125}\right\rfloor = 8$ multiples of 125, and $\left\lfloor\frac{1011}{625}\right\rfloor = 1$ multiple of 625. So, when we count the total number of 5's, we compute the sum $202+40+8+1=251$ zeros. Note that for the number 625, each of its four 5's are counted once in the numbers 202, 40, 8, and 1.