1. B. $\frac{8!}{2!2!}$ = 10080

2. C. The probability of drawing a king is $\frac{4}{52}$ and the probability of a heart is $\frac{13}{52}$. You must subtract out the intersection of the king of hearts, which gives $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{4}{13}$.

3. C. Fill in a chart with the given information, also noting that $\frac{1}{5} * 20 = 6$ males with stripes.

	Females	Males	Total
Stripes	2	4	6
No Stripes	8	6	14
Total	10	10	20

P(female with no stripes) = $\frac{8}{20} = \frac{2}{5}$.

4. C. If Cindy and Annie are both already on the officer team, we can choose 2 of the 8 remaining people to be the other officers. ${}_8C_2$ = 28. If neither are on the team, we can choose 4 of the other 8 people to be officers. ${}_8C_4$ = 70. 28+70 = 98.

5. A. If there are n people at a party, the first person shakes hands with the other $n - 1$ people, the second person then shakes hands with the other $n - 2$ people, and so on until the $(n - 1)th$ person shakes his hand with the nth person. Thus, the number of handshakes is $(n - 1) + (n - 2) \ldots + 3 + 2 + 1 = \frac{(n-1)(n)}{2}$ = ${}_nC_2$. Setting this equal to 861 and solving gives 42 as the answer.

6. B. The constant term is ${}_5C_2(4x^3)^2\left(\frac{-2}{x^2}\right)^3 = -1280.$

7. B. The simplest way to solve this is to list out the pairings that result in a multiple of four: (1,4), (1,8), (2,4), (2,6), (2,8), (3,4), (3,8), (4,5), (4,6), (4,7), (4,8), (4,9), (5,8), (6,8), (7,8), (8,9) = 16 out of 36 possible pairings = $\frac{4}{9}$.

8. A. The best approach is to use geometric probability. Draw a 6x6 square from the origin (0,0) to (6,6). Let the x-axis be Steffi's arrival time and the y-axis be Ali's arrival time. Draw two 45-degree lines from (1,0) to (6,5) and (0,2) to (4,6). The area between the lines represents the chances of meeting. The probability of them meeting is equal to this area divided by the total area of the square. Thus, the probability of them meeting is equal to $\frac{36-4*4*0.5-5*5*0.5}{36} = \frac{31}{72}$.

9. B. There are $({}_{15}C_3)$ different ways that the three can be assigned to roles. For them to all be on the same team, they must be given 3 of the 8 town roles (${}_8C_3$), 3 of the 4 mafia roles (${}_4C_3$), or all 3 of the neutral roles (${}_3C_3$). (${}_8C_3$ + ${}_4C_3$ + ${}_3C_3$) / ${}_{15}C_3$ = $\frac{61}{455}$.

10. C. For Kanye to win, Lil Wayne must get a question wrong before him. This can occur on the second turn if Kanye gets the first question right and Lil Wayne misses the second, which has a

$\frac{2}{5} * \frac{1}{3} = \frac{2}{15}$ chance of occurring. However, if Lil Wayne gets the second question right, the soonest that Kanye can win is on the fourth turn if he gets the third question right and Lil Wayne misses the fourth, which has a $\frac{2}{5} * \frac{2}{3} * \frac{2}{5} * \frac{1}{3} = \frac{8}{225}$. The pattern continues, and you can see that this is an infinite geometric series with a first term of $\frac{2}{15}$ and a common ratio of $\frac{4}{15}$. $\frac{2}{15} / \left(1 - \frac{4}{15}\right) = \frac{2}{11}$.

11. E. A set with n elements has 2^n subsets. $2^6 = 64$.

12. C. It takes 5 steps to get from (0,0) to (2,3) and you can choose any two of them to be steps in the x direction. This gives you ${}_5C_2$=10 ways to get to (2,3). It takes 7 steps to get from (2,3) to (5,7), with 3 of them being steps in the x direction, so ${}_7C_3$=35 ways to get there. Multiplying gives the total number of ways, 350.

13. D. The only way for them to match is if they both chose a red shirt and shorts. The probability of this is $\left(\frac{3}{9} * \frac{3}{4}\right) * \left(\frac{4}{9} * \frac{4}{7}\right) = \frac{4}{63}$.

14. D. There is a total of $1 + 2 + 3 + 4 + 5 + 6 = 21$ dots. If the dot is removed from an even face (which has a $\frac{2+4+6}{21} = \frac{4}{7}$ chance of happening), there are 4 odd faces, giving a probability of $\frac{4}{7} * \frac{4}{6} = \frac{8}{21}$. If the dot is removed from an odd face ($\frac{1+3+5}{21} = \frac{3}{7}$ chance), there are 2 odd faces, giving a probability of $\frac{3}{7} * \frac{2}{6} = \frac{1}{7}$. $\frac{8}{21} + \frac{1}{7} = \frac{11}{21}$.

15. C. I: There are 5 ways to get a sum of 6 when rolling 2 die (1 and 5, 2 and 4, 3 and 3, 4 and 2, 5 and 1) out of 36 $(6 * 6)$ possible combinations, so $\frac{5}{36}$.

II: There are four suits to choose from and 13 cards in each suite to pick out 5, so the probability is ${}_4C_1 * {}_{13}C_5$ divided by the total number of possible hands of 5, ${}_{52}C_5$. $\frac{33}{16660}$.

III: Only perfect squares have an odd number of factors. There are 10 perfect squares under 100 including 100, so $\frac{10}{100} = \frac{1}{10}$.

The correct order is I, III, II.

16. D. $\frac{1-P(Arjun\ wins)}{P(Arjun\ wins)} = \frac{11}{7}$ → $7 - 7P(Arjun\ wins) = 11P(Arjun\ wins)$ → $7 = 18P(Arjun\ wins)$ → $P(Arjun\ wins) = \frac{7}{18}$.

17. E. Chris can put on all his footwear in 16! ways, but the probability that he puts it on the in the right order on any leg is ½. The probability that he puts them on correctly on all legs is $\frac{1}{2^8}$, giving $\frac{16!}{2^8}$ ways to put them on correctly.

18. A. The prime factorizations of 420 and 2860 are $2^2 * 3 * 5 * 7$ and $2^2 * 5 * 11 * 13$, respectively. Any positive integral divisors of 420 that are also divisors of 2860 must come from $2^2 * 5$ since that is the part of the factorization shared by both. Therefore, there are a total of $(2+1)(1+1) = 6$ such divisors.

19. D. Let p be the probability that Andy does not run a stop sign. $p(1-p) = \frac{7}{64}$. Solving the equation gives $p = \frac{7}{8}$. The probability that he doesn't run the first 3 signs he comes across but runs the fourth is $\frac{7}{8} * \frac{7}{8} * \frac{7}{8} * \frac{1}{8} = \frac{343}{4096}$.

20. B. There are 3! ways to arrange Helena, Kira, and Alice in the top three and 5! ways to arrange the other 5 competitors, giving a total of $3! * 5! = 720$ possible arrangements.

21. A. A rectangle can be made by the intersection of any two vertical lines and any two horizontal lines on the grid, so ${}_7C_2 * {}_7C_2 = 441$.

22. A. The probability of drawing different colors jelly beans is $P(red) * P(not\ red) + P(blue) * P(not\ blue) + P(green)P(not\ green)$. $\frac{8}{24} * \frac{16}{23} + \frac{12}{24} * \frac{12}{23} + \frac{4}{24} * \frac{20}{23} = \frac{44}{69}$.

23. D. Let p be the desired probability. If the first roll is a 6, we are done and the sum is even. If it is a 2 or a 4, the sum of the rest of the terms must be even. If it is a 1, 3, or 5, the sum of the rest must be odd. Thus: $p = \frac{1}{6} + \frac{1}{3} * p + \frac{1}{2} * (1-p)$ which simplifies to $p = \frac{4}{7}$.

24. C. If each room must get at least 1 egg, there are only 6 $(10-4)$ eggs to be freely distributed among the 4 rooms. We can use the "stars and bars" technique in which we imagine these 6 eggs as "stars" and three dividers or "bars" splitting the stars. The stars to the left of the first bar are hidden in the first room, those between the first and second bars are hidden in the second room, and so on. This becomes an arrangement of 9 items in which the 6 stars and 3 bars are identical, giving $\frac{9!}{6!3!} = 84$ as the answer.

25. A. In order for Bryan to win on the fifth match, both Bryan and Ben must win two times in the first four matches in any order. Bryan must win the fifth game. Thus, the probability is ${}_4C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2 \left(\frac{1}{3}\right)$.

26. B. $1 - .25 = .75$ people are not infected and $1 - .8 = .2$ of infected people test negative, so the percentage of people who test negative is $(.25)(.2) + (.75)(.4) = .35 = 35\%$.

27. E. The probability of landing in the inner circle is $\frac{2^2\pi}{7^2\pi} = \frac{4}{49}$. The probability of landing in the middle ring is $\frac{5^2\pi - 2^2\pi}{7^2\pi} = \frac{21}{49}$. The probability of landing in the outer ring is $\frac{7^2\pi - 5^2\pi}{7^2\pi} = \frac{24}{49}$. The expected earnings is $\left(\frac{4}{49}\right)(10-3) + \left(\frac{21}{49}\right)(3-3) + \left(\frac{24}{49}\right)(1-3) = \frac{4}{7} + 0 - \frac{48}{49} = -\frac{20}{49}$.

28. B. The only way Justin can make a net profit (win more than the 6 dollars it costs him to play twice) is if he lands in the inner circle both times or once in the inner circle and once in the middle ring. The probability of this is $\left(\frac{4}{49}\right)\left(\frac{4}{49}\right)+\left(\frac{4}{49}\right)\left(\frac{21}{49}\right)+\left(\frac{21}{49}\right)\left(\frac{4}{49}\right)=\frac{184}{2401}$.

29. B. By putting the values into a Venn diagram, there are $24-15=9$ who play PAD and Clash Royale only and $20-15=5$ who play PAD and Pokémon Go. This means that $44-(15+9+5)=15$ people play PAD only. Let a = Pokémon Go only, b = Pokémon Go and Clash Royale only, and c = Clash Royale only. $a+b=36-(15+5)=16, b+c=42-(15+9)=18$, and $a+b+c=75-(45+3)=28$. Solving this system gives $a=10, b=6, c=12$. The number that less than two games is $10+15+12+3=40$.

30. D. There are 42 that play Clash Royale, and using the information from above, 15+6=21 of those play Pokémon Go. $\frac{21}{42}=\frac{1}{2}$.