1. C	11. E	21. A
2. C	12. B	22. B
3. A	13. A	23. B
4. A	14. D	24. B
5. B	15. A	25. E one point
6. D	16. B	26. A
7. C	17. D	27. E 15/14
8. A	18. B	28. D
9. E 0	19. C	29. E $k > 8$
10. D	20. B	30. C

1. The focal width is the reciprocal of the coefficient of the y^2 term if the x-coefficient is 1. Multiply by 1/8 and the coefficient is 7/12.

2. All parabolas have an eccentricity of 1.

3. When $x=0$, $y^2+4y=4 \to y^2+4y+4=8$ $\to (y+2)^2=8 \to y=-2\pm 2\sqrt{2}$. The distance between the points is $4\sqrt{2}$.

4. Distance between center and line: $D=\dfrac{|12(2)+9(1)+14|}{\sqrt{12^2+9^2}}=\dfrac{47}{15}$. Subtract 3, the length of the radius: $\dfrac{47-45}{15}=\dfrac{2}{15}$.

5. $xy-2y-5x+7=0 \to xy-2y=5x-7$ $y(x-2)=5x-7 \to y=\dfrac{5x-7}{x-2}$. After synthetic division, we have $y=\dfrac{3}{x-2}+5$. $x=2$, $y=5$.

6. Area $=ab\pi$ and focal width $=\dfrac{2b^2}{a}$. Substituting $b=\frac{18}{\pi}$ we get $\frac{2}{a}\left(\frac{324}{a^2}\right)=3 \to a^3=216 \to a=6$ The sum of the focal radii is $2a$, so 12.

7. The x-intercepts are 0 and 1, so the vertex will be at $x=\dfrac{1}{2}$. At $x=\dfrac{1}{2}$, $y=-\dfrac{1}{2}$. The rectangle has dimensions $1\times\dfrac{1}{2}$, so the area is $\dfrac{1}{2}$.

8. $m=\dfrac{7-2}{3-(-3)}=\dfrac{5}{6}\to m_{\perp}=-\dfrac{6}{5}$. $y-7=-\dfrac{6}{5}(x-3)\to 5y-35=-6x+18$ $6x+5y-53=0$.

9. Let $y=a(x-1)(x-2)$. We know that (0, 6) is on the graph, so $6=a(0-1)(0-2)\to a=3$. $y=3(x-1)(x-2)=3x^2-9x+6$. The sum is 0.

10. Let the rectangle have dimensions $2r\times x$. The length of the track is $2\pi r+2x=1320\to$ $\pi r+x=660$. Rectangle area $=2r(660-\pi r)$ $\to -2\pi r^2+1320r \to r=-\dfrac{1320}{2(-2\pi)}=\dfrac{330}{\pi}$; $x=330$. Rectangle area $=2\left(\dfrac{330}{\pi}\right)(330)$.

11. The center is (4, −3) and the b-value is 4, so the conjugate axis endpoints are (4, 1) and (4, −7). The distance between the endpoints is 8. For the parabola, $4a=8\to a=2$. The vertex has to be (6, −3) or (2, −3). The possible equations are $-8(x-6)=(y+3)^2$ and $8(x-2)=(y+3)^2$.

12. Confocal conics have the same focus. The two that are not confocal are $\dfrac{y^2}{4}-\dfrac{x^2}{8}=1$ and $\dfrac{x^2}{6}+\dfrac{y^2}{6}=1$.

13. The 42 million miles is between the focus and the vertex. For convenience, let $c > a$, giving $c-a=42$. The focal width, $\frac{2b^2}{a}$, will be 224, so $\frac{b^2}{a}=112$. For hyperbolas, $a^2+b^2=c^2$, so $\frac{c^2-a^2}{a}=112=\frac{c^2-(c-42)^2}{c-42}$.
$112(c-42)=84c-1764 \to 28c=2940 \to$
$c=105 \to a=63 \to 63+42=105$.

14. We want a circle with center (c, c) and radius c: $(x-c)^2+(y-c)^2=c^2$. Since the circle passes through (4, 4), we have
$(4-c)^2+(4-c)^2=c^2$. $2\left(c^2-8c+16\right)=c^2 \to$
$c^2-16c+32=0$. $c=\frac{16\pm\sqrt{256-128}}{2}=8\pm4\sqrt{2}$.
Circumference= $2\pi r=2\pi\left(8-4\sqrt{2}\right)=$
$16\pi-8\pi\sqrt{2}$.

15. Let a and b be the two roots. Their product is $3072=3\bullet2^{10}$. $a-b=244$. There are two possible choices for a and b: 12 and 256 and –12 and –256. The absolute value of the sum is 268.

16. If $z=x+yi$, then the given equation is the sum of the distances of (x, yi) to (3, 0) and (–5, 0). This is the definition of an ellipse, where the given points are the foci and 14 is the sum of the focal radii, $2a$. The center is (–1, 0), so $c=4$. Eccentricity $=\frac{c}{a}=\frac{4}{7}$.

17. Let point L be (x, y). The line that contains L is $y=-\sqrt{3}x \to y^2=3x^2$. $9x^2+4y^2=36 \to$
$9x^2+4\left(3x^2\right)=36 \to 7x^2=12$. $x^2=\frac{12}{7}$, $y^2=\frac{36}{7}$.

18. An equation for the parabola is $y=a(x-10)^2+70$, since the vertex is (10, 70). The initial point is (0, 50), so
$50=a(0-10)^2+70 \to -20=100a \to$
$a=-\frac{1}{5}$. $0=-\frac{1}{5}(x-10)^2+70 \to$
$-350=-(x-10)^2 \to \sqrt{350}=x-10 \to$
$x=10+5\sqrt{14}$.

19. Due to symmetry, the center of R is $(r, 0)$. We need the distance from $(r, 0)$ to (–2, 0) and (4, 2) or (4, –2):
$(r+2)^2+(0-0)^2=(r-4)^2+(0-2)^2$.
$r^2+4r+4=r^2-8r+16+4 \to 12r=16 \to r=\frac{4}{3}$.

20. By sketching the graph we can see that the hyperbola opens horizontally. The slopes of the asymptotes are $\pm\frac{3}{4}$. Using the asymptote equations as a system of equations, we find that they intersect at (3, –1). We now know that $c=5$. Since $b=\frac{3}{4}$ and $a^2+b^2=c^2$, we can find the equations of the directrices.
$x=3\pm\frac{a^2}{c}=3\pm\frac{16}{5}=\frac{31}{5}, -\frac{1}{5}$.

21. From the given information, we have $\frac{(x-1)^2}{a^2}+\frac{(y-3)^2}{b^2}=1$. We can substitute the given values to form a system of equations:
$$\begin{cases}\frac{16}{a^2}+\frac{9}{b^2}=1 \ \ (4)\\ \frac{36}{a^2}+\frac{4}{b^2}=1 \ \ (-9)\end{cases} \to -\frac{260}{a^2}=-5 \to a^2=52.$$
$\frac{16}{52}+\frac{9}{b^2}=1 \to b^2=13$. Now we have
$h=-1, k=-3, a^2=52, b^2=13 \to$
$\frac{52}{13}+\frac{-3}{-1}=7$.

22. $\begin{cases} x^2+3xy=28 \ (2) \\ 4y^2+xy=8 \ (7) \end{cases} \to \begin{cases} 2x^2+6xy=56 \\ 28y^2+7xy=56 \end{cases}$

Subtract to get $2x^2-xy-28y^2=0$, which factors into $(2x+7y)(x-4y)=0$. We can substitute either of these factors into the system as one of the equations.

$\begin{cases} xy+4y^2=8 \\ 2x+7y=0 \end{cases}$ $\begin{cases} xy+4y^2=8 \\ x-4y=0 \end{cases}$

$\left(-\frac{7}{2}y\right)y+4y^2=8$ $(4y)y+4y^2=8$

$y^2=16$ $y^2=1$

$y=\pm4,\ x=\mp14$ $y=\pm1,\ x=\pm4$

$(14,-4)(-14,4)$ $(4,1),(-4,-1)$

$$\text{Area}=\pm\frac{1}{2}\begin{vmatrix} 14 & -4 \\ -4 & -1 \\ -14 & 4 \\ 4 & 1 \\ 14 & -4 \end{vmatrix}.$$

$$=\pm\frac{1}{2}[(-14-16-14-16)-(16+14+16+14)]$$
$$=60$$

23. Add the sides to get $x^2+2xy+y^2=25$. This factors into $(x+y)^2=25$. We can solve this use each solution with the other equation.

$\begin{cases} x+y=5 \\ xy=5 \end{cases}$ $\begin{cases} x+y=-5 \\ xy=5 \end{cases}$

$x(5-x)=5$ $x(-5-x)=5$

$x^2-5x+5=0$ $x^2+5x+5=0$

$x=\frac{5\pm\sqrt{5}}{2}$ $x=\frac{-5\pm\sqrt{5}}{2}$

$y=\frac{5\mp\sqrt{5}}{2}$ $y=\frac{-5\mp\sqrt{5}}{2}$

The shortest distance is between the points On the left or the points on the right. $D=\sqrt{10}$.

24. $4x^2-12xy+9y^2+20x-30y+25=0$ factors into $(2x-3y)^2+10(2x-3y)+25=0 \to$ $(2x-3y+5)^2=0$, which is one line.

25. We will rewrite $x^2+3xy+3y^2-x+1=0$ and use the quadratic formula: $x^2+(3y-1)x+(3y^2+1)=0$.

$$x=\frac{1-3y\pm\sqrt{9y^2-6y+1-12y^2-4}}{2}\to$$

$$x=\frac{1-3y\pm\sqrt{-3(y+1)^2}}{2},$$ which is only defined at the point $(2,-1)$.

26. Create a system of equations with the given information:

$$\begin{cases} \frac{1}{2}a+v_0+s_0=52 \\ 2a+2v_0+s_0=52 \\ \frac{9}{2}a+3v_0+s_0=20 \end{cases} \to \begin{cases} a+2v_0+2s_0=104 \\ 2a+2v_0+s_0=52 \\ 9a+6v_0+2s_0=40 \end{cases}$$

Add −2 times the first equation to the second equation and −9 times the first equation to the third equation.

$$\begin{cases} a+ & 2v_0+ & 2s_0= & 104 \\ & -2v_0- & 3s_0= & -156 \\ & -12v_0- & 16s_0= & -896 \end{cases}$$

Add −6 times the second equation to the third equation.

$$\begin{cases} a+ & 2v_0+ & 2s_0= & 104 \\ & -2v_0- & 3s_0= & -156 \\ & & 2s_0= & 40 \end{cases}$$

$s_0=20,\ v_0=48,\ a=-32$. (When measured in feet, a will always be −32.)

27. The circumcenter is the intersection of the perpendicular bisectors of the sides. We find the perpendicular bisectors of $\overline{AB}$ and $\overline{BC}$ and then find their intersection. If the midpoints of each line are M and N, respectively, then $M\left(\frac{3}{2},1\right)$ and $N\left(\frac{5}{2},-\frac{3}{2}\right)$. The slopes are $-\frac{2}{5}$ and 1, respectively. The two lines are $\frac{5}{2}\left(x-\frac{3}{2}\right)=y-1 \rightarrow y=\frac{5}{2}x-\frac{11}{4}$ and $-\left(x-\frac{5}{2}\right)=y+\frac{3}{2} \rightarrow y=-x+1$. Now find the intersections: $\frac{5}{2}x-\frac{11}{4}=-x+1 \rightarrow x=\frac{15}{14}$.

28. The volume of an ellipsoid can be found in the same manner as the area of an ellipse—they're analogous to circles. The volume of a circle is found by $\frac{4}{3}\pi r^3$. For the ellipsoid, the radii are the semi-major and semi-minor axes. Since we are rotating around the major axis, we will use the semi-minor axis length twice.
$V=\frac{4}{3}\pi(7)\left(\sqrt{33}\right)^2=308\pi.$

29. We need to complete the square and get a negative value on the right side. This automatically eliminates choices A and B.
$x^2-4x+4+y^2+8y+16=-k-12+20$
$(x-2)^2+(y+4)^2=8-k$
$8-k<0 \rightarrow k>8.$

30. Statement I is just the Pythagorean theorem. The inscribed angle is a right angle.

Statement II is false. The distance from the center to a directrix is $\frac{a^2}{c}$, and the distance from the center to the corresponding focus is c. $\frac{a^2}{c}-c \rightarrow \frac{a^2-c^2}{c}$. For an ellipse, $a^2-b^2=c^2$, so the correct statement is $p=\frac{b^2}{c}$.

Statement III is true. We know that $p=\frac{b^2}{c}$. The length of the major axis is $2a$, which we can write as $\frac{2\left(\frac{b^2}{a}\right)}{\left(\frac{b^2}{a^2}\right)} \rightarrow \frac{2\left(\frac{b^2}{a}\right)}{\left(\frac{a^2-c^2}{a^2}\right)} \rightarrow \frac{2\left(\frac{b^2}{c}\bullet\frac{c}{a}\right)}{\left(1-\frac{c^2}{a^2}\right)}=\frac{2pe}{1-e^2}$.

Statement IV is also true.
$p=c-\frac{a^2}{c} \rightarrow \frac{c^2-a^2}{c} \rightarrow \frac{b^2}{c}$. The length of the transverse axis is $2a$. This can be rewritten as
$2a=\frac{2\left(\frac{b^2}{a}\right)}{\left(\frac{b^2}{a^2}\right)} \rightarrow \frac{2\left(\frac{b^2}{a}\right)}{\left(\frac{c^2-a^2}{a^2}\right)} \rightarrow \frac{2\left(\frac{c}{a}\bullet\frac{b^2}{c}\right)}{e^2-1} \rightarrow \frac{2ep}{e^2-1}.$

In statement V, the radical axis of two circles is the locus of points at which tangents drawn to both circles have the same length. It is always a straight line perpendicular to the line connecting the centers. When the circles are unequal in size, the radical axis is closer to the circumference of the larger circle; since these circles are congruent, it goes through the midpoint of the line connecting the centers.