

“NOTA” stands for “None Of These Answers.”

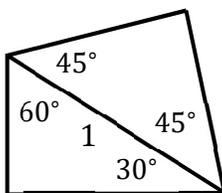
Diagrams are not necessarily to scale. Good luck and have fun!

- In an isosceles right triangle with a hypotenuse of length 2, what is the sum of the lengths of the medians?
  - $\sqrt{10}$
  - $1 + \sqrt{10}$
  - $\sqrt{2} + \sqrt{5}$
  - $\sqrt{2} + 2\sqrt{5}$
  - NOTA
- $\triangle AHS$  has side lengths  $AH = 5$ ,  $HS = 5$ ,  $SA = 8$ . Let the medians be  $AX$ ,  $HY$ , and  $SZ$ , which intersect at  $P$ . What is the value of the ratio  $\frac{AP}{PX}$ ?
  - $\frac{5}{4}$
  - $\frac{8}{5}$
  - 2
  - $\frac{5}{2}$
  - NOTA
- What is the circumcenter of the triangle defined by the points  $(-1,0)$ ,  $(1,0)$ ,  $(0,\sqrt{3})$ ?
  - $(0, \frac{\sqrt{3}}{3})$
  - $(0, \frac{\sqrt{3}}{2})$
  - $(0, \frac{2\sqrt{3}}{3})$
  - $(0, -\frac{\sqrt{3}}{3})$
  - NOTA
- One of Paloma’s Dungeons and Dragons dice is an icosahedron with edge length 5mm. Another is an octahedron with edge length 10mm. They have 20 and 8 equilateral triangular faces, respectively. What is the ratio of the surface area of the icosahedron to the surface area of the octahedron?
  - $\frac{5}{2}$
  - $\frac{5}{4}$
  - $\frac{5}{6}$
  - $\frac{5}{8}$
  - NOTA
- A point is chosen inside an equilateral triangle with side length 1. What is the expected value of the sum of the distances from the point to each side of the triangle?
  - $\frac{\sqrt{3}}{3}$
  - $\frac{\sqrt{3}}{2}$
  - $\frac{1}{2}$
  - 1
  - NOTA
- A point  $P$  is randomly chosen inside equilateral triangle  $YOU$ , using the uniform distribution. Find the probability that the closest vertex to  $P$  is  $Y$  and the closest side to  $P$  is  $\overline{YO}$ .
  - $\frac{1}{2}$
  - $\frac{1}{3}$
  - $\frac{1}{6}$
  - $\frac{1}{9}$
  - NOTA
- Obtuse triangle  $WOT$  has side lengths  $WO = 8$ ,  $OT = 11$ . What is the sum of all possible integral lengths for  $WT$ ?
  - 165
  - 102
  - 63
  - 28
  - NOTA
- The lengths of the sides of a triangle are 6, 9, and  $y$ . Find all restrictions on  $y$ .
  - $y > 0$
  - $y < 15$
  - $3 < y < 15$
  - $0 < y < 15$
  - NOTA

9. In triangle  $XYZ$ , the measure of angle  $X$  is 4 times as large as the measure of angle  $Y$ . If the measure of the exterior angle at  $Z$  is 130 degrees, what is the number of degrees in the measure of angle  $X$ ?
- A. 10                      B. 26                      C. 40                      D. 104                      E. NOTA
10. Which of these could not be the side lengths of a right triangle?
- A.  $2\sqrt{6}, 4\sqrt{3}, 6\sqrt{2}$     B. 8, 15, 17              C.  $0.\bar{3}, 0.\bar{4}, 0.\bar{5}$               D.  $\sqrt[4]{2}, \sqrt[4]{3}, \sqrt{2} + \sqrt{3}$     E. NOTA
11. In  $\triangle ABC$ ,  $AB = 25, AC = 24, BC = 7$ . Which triangle must be congruent to  $\triangle ABC$ ?
- A.  $\triangle DEF$  where  $m\angle D = 90^\circ, DE = 25, DF = 7$   
 B.  $\triangle GHI$  where  $m\angle I = 90^\circ, GH = 25, GI = 24$   
 C.  $\triangle JKL$  where  $m\angle K = \sin^{-1}\frac{7}{25}, JK = 25, KL = 7$   
 D.  $\triangle MNO$  where  $m\angle M = \tan^{-1}\frac{7}{24}, NO = 7, MO = 24$   
 E. NOTA
12. Consider the parabola  $y = x^2 + 2x + 1$ . What is the area of the triangle formed by the parabola's vertex and the endpoints of the parabola's latus rectum?
- A.  $\frac{1}{2}$                       B.  $\frac{1}{3}$                       C.  $\frac{1}{4}$                       D.  $\frac{1}{6}$                       E. NOTA
13. Richard is glamping at the top of a cliff, 150ft above the ocean. He spots a bird on the water, straight ahead but at a  $30^\circ$  angle of depression from his eye level. What is the distance from Richard's eyes to the bird, in feet?
- A.  $150\sqrt{3}$               B. 150                      C.  $300\sqrt{3}$               D. 300                      E. NOTA
14. Luna draws the Deathly Hallows symbol for Harry, as shown to the right. The Cloak of Invisibility is an equilateral triangle, the Resurrection Stone is its incircle, and its altitude is the Elder Wand. If the altitude is 15 inches long, like the real Elder Wand, what is the total length she draws, in inches?
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- A.  $15 + 30\sqrt{3} + 10\pi$     B.  $15 + 15\sqrt{3} + 10\pi$     C.  $15 + 30\sqrt{3} + 5\pi$     D.  $30\sqrt{3} + 10\pi$     E. NOTA
15.  $\triangle ZHU$  is equilateral with side length 13, and  $\triangle MAO$  has vertices on  $\triangle ZHU$  such that  $M$  is on  $\overline{ZH}$ ,  $A$  is on  $\overline{HU}$ ,  $O$  is on  $\overline{UZ}$ , and  $ZM = HA = UO = 5$ . What is the area of  $\triangle MAO$ ?
- A.  $\frac{89}{4} + 10\sqrt{3}$               B.  $\frac{89}{4} - 10\sqrt{3}$               C.  $\frac{49\sqrt{3}}{4}$                       D.  $\frac{129\sqrt{3}}{4}$                       E. NOTA

16. A line passes through the incenter of an equilateral triangle and is parallel to one of the sides. This divides the triangle into two regions. What is the ratio of the area of the smaller region to the area of the larger region?
- A.  $\frac{4}{5}$       B.  $\frac{\sqrt{3}}{2}$       C.  $\frac{3}{4}$       D.  $\frac{2}{3}$       E. NOTA
17. Let  $\theta$  be the angle between the longest and second-longest sides of a triangle inscribed in a circle with radius  $r$ , where one side is a diameter of the circle. What is the triangle's area?
- A.  $2r^2 \sin \theta \tan \theta$       B.  $2r^2 \cos \theta \tan \theta$       C.  $2r^2 \tan^2 \theta$       D.  $2r^2 \sin^2 \theta$       E. NOTA
18. The median to the hypotenuse of a right triangle divides the triangle into two triangles that are both:
- A. Acute      B. Isosceles      C. Scalene      D. Obtuse      E. NOTA

For questions 19-20, use the quadrilateral below, the shown diagonal has length 1.



19. What is the quadrilateral's perimeter?
- A.  $\frac{1+2\sqrt{2}+\sqrt{3}}{2}$       B.  $\frac{3+2\sqrt{2}+\sqrt{3}}{2}$       C.  $\frac{1+\sqrt{2}+\sqrt{3}}{2}$       D.  $\frac{3+\sqrt{2}+\sqrt{3}}{2}$       E. NOTA
20. What is the quadrilateral's area?
- A.  $\frac{1+\sqrt{3}}{4}$       B.  $\frac{1+2\sqrt{3}}{8}$       C.  $\frac{1+2\sqrt{3}}{4}$       D.  $\frac{2+\sqrt{3}}{4}$       E. NOTA
21. Of the following triangles with the given side lengths, for which triangle does the orthocenter lie on one of the sides?
- A. 7, 7, 7      B. 6, 9, 11      C. 8, 8, 13      D. 5, 12, 13      E. NOTA
22. An acute scalene triangle has angle measures that form an arithmetic sequence. Which of these is a possible measure for the largest angle of the triangle?
- A.  $31^\circ$       B.  $59^\circ$       C.  $68^\circ$       D.  $90^\circ$       E. NOTA

23. An isosceles triangle and a square have the same base of length  $b$ . The isosceles triangle has height  $h$  where  $h > b$ . What is the area of the region (shaded in the diagram to the right) outside the triangle but inside the square?



- A.  $\frac{b^2}{2h}$       B.  $\frac{b^3}{h}$       C.  $\frac{b^4}{2h^2}$       D.  $\frac{b^3}{2h}$       E. NOTA

For questions 24-25, use the information below.

In  $\triangle GEO$ ,  $M$  is on  $\overline{GO}$  such that  $\overline{EM}$  is the angle bisector of  $\angle GEO$ . Also,  $GE = \sqrt{2}$  and  $EO = 2^{1/3}$ .

24. What is  $\frac{OM}{MG}$ ?
- A.  $\left(\frac{1}{2}\right)^{1/6}$       B.  $2^{1/6}$       C.  $2^{1/3}$       D.  $4^{1/3}$       E. NOTA
25. What is the ratio of the area of  $\triangle OEM$  to the area of  $\triangle GEM$ ?
- A.  $2^{1/6}$       B.  $2^{1/3}$       C.  $2^{1/2}$       D. 2      E. NOTA
26. Which of these is true of a triangle with side lengths 13, 14, 15?
- A. Its area is 168      B. None of its altitudes has integer length      C. It is obtuse      D. Its circumcenter lies outside the triangle      E. NOTA
27. In  $\triangle ILY$ , the sides have lengths  $a, b, c$ , where  $a < b < c$ , the angles measure  $105^\circ, 45^\circ, 30^\circ$ , and the area is 2019. What is the value of the product  $bc$ ?
- A.  $\frac{8076\sqrt{3}}{3}$       B. 8076      C.  $4038\sqrt{2}$       D. 4038      E. NOTA

For questions 28-29, use the information below.

Eugene has a triangular house with wall lengths 100, 130, and 130 feet. His house is covered with Wi-Fi routers, but they are so weak that he only gets Wi-Fi signal when he is within 10 feet of a wall.

28. If Eugene only wants to walk around outside the house, what is the total area (in square feet) that he can roam and still get Wi-Fi signal?
- A. 3600      B.  $3600 + 50\pi$       C.  $3600 + 75\pi$       D.  $3600 + 100\pi$       E. NOTA
29. What is the house's area, in square feet?
- A. 6000      B. 5600      C. 4800      D. 3000      E. NOTA

30. Super Radz, always the encouraging coach, sends you this star for completing the test. As an astute geometer, you immediately notice the star is formed by extending the sides of a regular pentagon (which has five interior angles measuring  $108^\circ$  each) until they meet at five points. Assuming the star is constructed this way, what is the angle formed when the extended sides meet at those points?



- A.  $36^\circ$                       B.  $48^\circ$                       C.  $60^\circ$                       D.  $72^\circ$                       E. NOTA