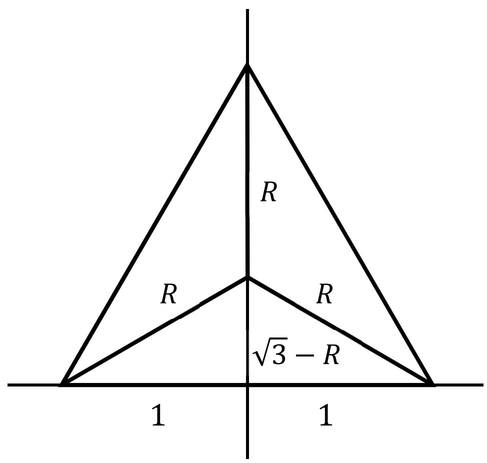
Answers

1.	B	11.	B	21.	D
2.	C	12.	E	22.	C
3.	A	13.	D	23.	D
4.	D	14.	A	24.	A
5.	B	15.	C	25.	E
6.	C	16.	A	26.	E
7.	B	17.	E	27.	B
8.	C	18.	B	28.	D
9.	D	19.	A	29.	A
10.	D	20.	E	30.	A

Solutions

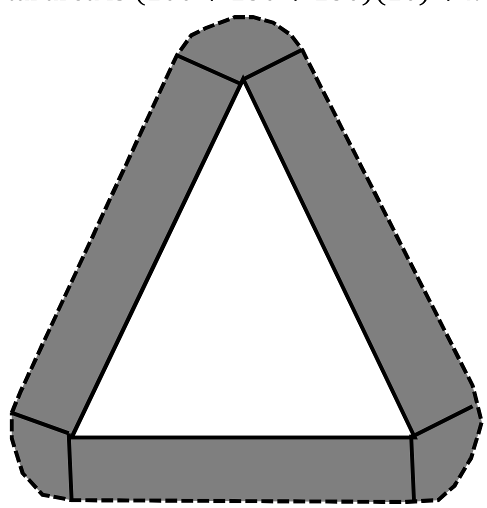
1. B The median to the hypotenuse is half the length of hypotenuse, 1. Two medians are the hypotenuses of right triangles with legs $\frac{\sqrt{2}}{2}$ and $\sqrt{2}$, which are $\frac{\sqrt{10}}{2}$ each. The total is $1+\sqrt{10}$.
2. C For the two parts that the centroid splits each median into, the ratio of the longer part of the median to the shorter part is 2 in any triangle.
3. A Plotting the points and noting the distances, we see that these are the coordinates of an equilateral triangle of side length 2 with a height along the y-axis. Splitting the triangle into three isosceles triangles, where the circumradius is R, we can use the right triangle with sides 1, $\sqrt{3}-R$, and R to find that $R=\frac{2\sqrt{3}}{3}$. Therefore, the circumcenter is at $\left(0,\frac{\sqrt{3}}{3}\right)$.



4. D The icosahedron has surface area $20\left(\frac{5^2\sqrt{3}}{4}\right)$ and the octahedron has surface area $8\left(\frac{10^2\sqrt{3}}{4}\right)$, so the ratio is $\frac{20(5^2)}{8(10^2)}=\frac{5}{8}$.
5. B The sum of the distances from a point inside an equilateral triangle is always equal to the height. The expected value weights each possible value of this sum by the probability of it occurring, which is just $\frac{\sqrt{3}}{2}$ with probability 1, so the expected value is $\frac{\sqrt{3}}{2}$.
6. C The probability is the favorable area (the area where the condition is fulfilled if a point is chosen there) over the total area. The left diagram shows the set of all points whose closest vertex is B (which covers $\frac{1}{3}$ of the total area), the center diagram shows the set of all points whose closest side is BI (which covers $\frac{1}{3}$ of the total area), and the right diagram shows the intersection of these sets, which covers $\frac{1}{6}$ of the total area.
7. B Let the third side length be x. From the other two side lengths we know that $11-8<x<11+8$. From the obtuse condition we know that either $8^2+x^2<11^2$ or $8^2+11^2<x^2$ (the last arrangement $11^2+x^2<8^2$ is impossible). These two imply $x^2<57, x^2>185$ respectively. Combining all the conditions and assuming x is an integer we get: $4\le x\le 18, x\le 7, x\ge 14$. So $4+5+6+7+14+15+16+17+18=102$.
8. C Triangle inequality
9. D 5A=130 A=26, 4(26)=104

10. $\boxed{D}$ Choice D is the only one that is not a right triangle: $\sqrt{2}+\sqrt{3} \neq \left(\sqrt{2}+\sqrt{3}\right)^2$. (Choice A is a right triangle: $24 + 48 = 72$. Choice B is a Pythagorean triple. Choice C is a right triangle, as it is a multiple of a 3-4-5 Pythagorean triple $(\frac{3}{9},\frac{4}{9},\frac{5}{9})$.)
11. $\boxed{B}$ Choice B is the only one congruent by HL. (Choice A does not have the right leg lengths. Choice C does not have the right angle. Choice D would be using SSA where the corresponding shorter side is opposite the corresponding angle, which does not guarantee congruence.)
12. $\boxed{E}$ This factors to the vertex form $y = 1(x+1)^2 + 0$, so the distance p from the vertex to the focus or directrix is given by $\frac{1}{4p} = 1$ or $p = \frac{1}{4}$. The latus rectum has length $4p$ and is perpendicular to the segment from the focus to the vertex of length p. These are the base and height of the triangle respectively, so the area is $\frac{1}{2}(4p)(p) = 2p^2 = \frac{1}{8}$.
13. $\boxed{D}$ We can form a 30-60-90 triangle with Richard's eyes (the 60 degree angle), the bird (the 30 degree angle), and the base of the cliff Richard is standing on (the 90 degree angle). The distance from Richard's eyes to the bird is the hypotenuse, which has length $(150)(2) = 300$.
14. $\boxed{A}$ The altitude of the triangle is 15, so the side of the triangle is $10\sqrt{3}$. The inradius is 5, using the formula $r = \frac{A}{s}$. The total length drawn is the altitude, three sides, and incircle circumference: $15 + 30\sqrt{3} + 10\pi$.
15. $\boxed{C}$ There are three triangles formed where one side is 5, one side is 8, and the angle between them is 60°. Using the law of cosines, the side opposite the angle is $\sqrt{5^2 + 8^2 - 2(5)(8)\cos 60^\circ} = 7$. Each side of ΔMAO is 7, so its area is $\frac{49\sqrt{3}}{4}$.
16. $\boxed{A}$ Using properties of 30-60-90 triangles, we find that the smaller region is a similar equilateral triangle with height $\frac{2}{3}$ that of the original triangle, so its area is $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ that of the original triangle. The other region must have area $\frac{5}{9}$, so the ratio of areas is $\frac{4}{5}$.
17. $\boxed{E}$ If one side is a diameter of the circle, that side must be the longest, and the triangle must be a right triangle with the diameter as the hypotenuse. Thus the legs have length $2r\sin\theta$ and $2r\cos\theta$ and the area of the triangle is $2r^2\sin\theta\cos\theta$.
18. $\boxed{B}$
19. $\boxed{A}$ From the 30-60-90 and 45-45-90 triangles formed, the sides on the quadrilateral's boundary are $\frac{1}{2},\frac{\sqrt{3}}{2},\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}$. The sum is $\frac{1+\sqrt{3}+2\sqrt{2}}{2}$.
20. $\boxed{E}$ The area is $\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{2+\sqrt{3}}{8}$.
21. $\boxed{D}$ This occurs in a right triangle. The only answer choice with side lengths of a right triangle is D.
22. $\boxed{C}$ Let the angles be $a-d, a, a+d$ for positive numbers a, d. Then since they sum to 180, $3a = 180$ and the middle angle must be $a = 60^\circ$. Since it is acute, choice A is impossible. Since the largest angle is $a + d > 60^\circ$, the only remaining possibility is 68°.
23. $\boxed{D}$ Drop an altitude from the apex of the isosceles triangle to the top side of the square. Then two pairs of similar triangles are formed. If we let the shaded triangle have base x, the shaded triangle has height b, and the unshaded triangle has height $h - b$ and base $\frac{1}{2}b - x$. Solving for x gives $x = \frac{b^2}{2h}$ so the area of one shaded triangle is $\frac{b^3}{4h}$ for a total of $\frac{b^3}{2h}$.

24. [A] We apply the Angle Bisector Theorem to get that $\frac{2^{\frac{1}{3}}}{2^{\frac{1}{2}}} = 2^{-\frac{1}{6}} = \left(\frac{1}{2}\right)^{\frac{1}{6}}$.
25. [E] If we consider the bases of both triangles to be along GO, then the heights are the same, and the ratio of the areas is just the ratio of the bases. This is just OM/MG from the previous problem, or $2^{-\frac{1}{6}}$.
26. [E] Using Heron's formula, or noticing that this triangle can be formed by sticking together two right triangles with sides 5, 12, 13 and 9, 12, 15, we can see that choices A and B are false. Since $a^2 + b^2 > c^2$ for all arrangements of sides a, b, c, choice C is false. Choice D is equivalent to choice C, so it is also false.
27. [B] We know that the area is equal to $2019 = \frac{1}{2}bc \sin A$. We also know that a is the shortest side, so A is the smallest angle, the 30 degree angle. Therefore, $2019 = \frac{1}{2}bc\left(\frac{1}{2}\right)$ so $bc = 8076$.
28. [D] The area Eugene can roam is shown below. It is the area of three rectangles with widths 100, 130, and 130 and height 10, plus three circular sectors of radius 10 at each of the vertices. The circular sectors' degree measures are unknown. But we know they must add to 360° because if the triangles' angles are a, b, c, then the sum of the circular sectors' angles (which are each 360, minus 90 degrees for each of the two rectangles, minus the interior angle, for a total of 180 minus the interior angle) is $(180 - a) + (180 - b) + (180 - c) = 540 - (a + b + c) = 360$. This means that we can treat the three sectors together as a circle with radius 10. Thus the total area is $(100 + 130 + 130)(10) + \pi(10)^2 = 3600 + 100\pi$.



29. [A] Use Heron's formula or use the fact that the triangle is isosceles to drop an altitude that is also a perpendicular bisector to the side of length 100; from there, we find that the area is 6000.
30. [A] The point is the vertex angle of an isosceles triangle with base angles $180 - 108 = 72°$ each, so each point has an angle of $180 - 2(72) = 36°$.