Choose answer choice E. NOTA if "None of the Above" answers are correct. Good luck!
If the word "Ellipse" appears in an answer choice, it is referring to a *non-circular* ellipse.

1. Which term best describes this conic, given that it is non-degenerate?

$$2x^2 + 4xy - 3y^2 + 4x + 7y = 14$$

A. Circle B. Hyperbola C. Parabola D. Ellipse E. NOTA

2. Polar graphs A and B are defined as $r^2 = 8\cos(2\theta)$ and $r^2 = 9\sin(2\theta)$ respectively. What is the ratio of the area bounded by graph A to the ratio of the area bounded by graph B?

A. $\frac{8}{9}$ B. $\frac{2\sqrt{2}}{3}$ C. $\frac{16\sqrt{2}}{27}$ D. $\frac{3\sqrt{2}}{4}$ E. NOTA

3. What is the smallest angle by the which the coordinate axes can be rotated counterclockwise to eliminate the xy term from the equation $4x^2 + \sqrt{3}xy + y^2 + 3x - 5y = 8$?

A. $\frac{\pi}{24}$ B. $\frac{\pi}{12}$ C. $\frac{\pi}{6}$ D. $\frac{\pi}{3}$ E. NOTA

4. Which of the following accurately describes the graph of $|z + i| - |z - i| = 12$ in the complex plane?

A. Hyperbola B. Ellipse C. Line D. Parabola E. NOTA

5. Find the shortest distance between the two planes $3x + 4y - z = 3$ and $3x + 4y - z = 4$.

A. $\frac{\sqrt{26}}{26}$ B. $\frac{1}{5}$ C. $\frac{2\sqrt{26}}{13}$ D. 1 E. NOTA

6. Let the foci of the ellipse $4x^2 + 9y^2 = 1$ be located at points B and C. Point A is a point on the ellipse chosen such the measure of angle BAC is 45 degrees. What is the perimeter of triangle ABC?

A. $\frac{\sqrt{5}+3}{3}$ B. $\frac{\sqrt{5}+6}{6}$ C. $\frac{\sqrt{5}+3}{6}$ D. $\frac{2\sqrt{5}+3}{6}$ E. NOTA

7. What is the eccentricity of the ellipse $7x^2 + 4y^2 + 28x - 24y + 36 = 0$?

A. $\frac{\sqrt{21}}{7}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{\sqrt{231}}{7}$ D. $\frac{3}{7}$ E. NOTA

8. What is the volume of the tetrahedron with vertices at $(0, 0, 0), (-5, 6, 7), (4, 2, 0)$, and $(-6, 9, 21)$?

A. 63 B. 126 C. 189 D. 378 E. NOTA

9. What is the tangent of the angle between the two vectors $\langle 3, 5, -7\rangle$ and $\langle 2, -1, 4\rangle$?

A. $-\frac{9\sqrt{1023}}{16}$ B. $-\frac{27}{\sqrt{1743}}$ C. $-\frac{\sqrt{6}}{3}$ D. $-\frac{13\sqrt{6}}{27}$ E. NOTA

10. At what point do the asymptotes of the hyperbola $5x^2 - y^2 - 10x - 6y - 29 = 0$ intersect?

A $(3, 1)$ B $(-3, 1)$ C $(1, -3)$ D $(1, 3)$ E. NOTA

11. Points A, B, C, and D are chosen on circle O independently at random. What is the probability that ABCD is a convex quadrilateral?

A. $\frac{1}{24}$ B. $\frac{1}{12}$ C. $\frac{1}{6}$ D. $\frac{1}{3}$ E. NOTA

12. What is length of the longer axis (major, minor, transverse or conjugate) in the parametrically defined conic $x = \frac{3^t + 3^{-t}}{2}$ and $y = 3^t - 3^{-t}$?

A. 2 B. $2\sqrt{2}$ C. 4 D. $4\sqrt{2}$ E. NOTA

13. The point $(3, -5)$ is rotated 90 degrees clockwise about the origin. What is the resultant point?

A (3,5) B (5,3) C $(-5, -3)$ D $(5, -3)$ E. NOTA

14. Let A equal the minimum number of points such that if you are given any set of A distinct points, there is at most one ellipse passing through all of them. Let B equal the minimum number of points such that if you are given any set of B distinct points, there is at most one parabola with a vertical directrix passing through all of them. What is $A + B$?

A. 7 B. 8 C. 9 D. 10 E. NOTA

15. Which of the following is a directrix of the conic $r = \frac{3}{1-\sin(\theta)}$? Give your answer in Cartesian coordinates.

A. $y = 6$ B. $y = 3$ C. $y = -3$ D. $y = -6$ E. NOTA

16. How many perfect squares greater than 1 and less than 1000 have the property that there is no rose curve with that number of petals?

A. 30 B. 15 C. 8 D. 1 E. NOTA

17. What is the area of a triangle with side lengths $\sqrt{2}, \sqrt{17}$, and $\sqrt{13}$?

A. $\frac{5}{2}$ B. 2 C. 444 D. 5 E. NOTA

18. What is the circumcenter of the triangle defined by the points $(6, 0), (8, -4)$, and $(-1, -1)$?

A. $(3, -4)$ B. $(3, 4)$ C. $(-3, -4)$ D. $(-3, 4)$ E. NOTA

19. For some real-valued function $f(x)$, all complex numbers z that satisfy $Im(z) = f(Re(z))$ also satisfy $|z - 1| = |Re(z) + 1|$. Which of the following could be the definition of $f(x)$?

A. $4x^2$ B. $2\sqrt{x}$ C. $\frac{x^2}{4}$ D. $\frac{\sqrt{x}}{2}$ E. NOTA

Questions 20 and 21 pertain to the line defined parametrically as $x = 3t, y = 5t - 2$ and $z = -7t + 1$.

20. For what value of t is the distance between the line and the origin minimized, given that the line does not pass through the origin?

A $-\frac{1}{5}$ B $-\frac{17}{15}$ C $\frac{17}{15}$ D $\frac{17}{83}$ E. NOTA

21. A second line is defined by the parametric equations $x = 2s + 1, y = 5s + 3$, and $z = as$. For what value of a will the lines be coplanar?

A. -1 B. -4 C. 1 D. 4 E. NOTA

22. Find the volume bounded by $x^2 + y^2 - z^2 = 0$ and $|z| = 60$.

A. 72000π B. 144000π C. 216000π D. 432000π E. NOTA

23. The conic $4x^2 - 13xy + 3y^2 + 2y - 8x - 7 = 0$ is rotated about the origin by some angle. The rotated graph can be written as $Ax^2 + Cy^2 + Dx + Ey - 14 = 0$. What is the value of $A + C + AC$?

A. $-\frac{93}{4}$ B. $-\frac{107}{2}$ C. -107 D. -128 E. NOTA

24. Which of the following best describes the following polar graph?

$$r = \left(\frac{1}{2}\sin(\theta)\cos(2\theta) - \frac{\sqrt{3}}{2}\sin(\theta)\sin(2\theta)\right) + \left(\frac{\sqrt{3}}{2}\cos(\theta)\cos(2\theta) + \frac{1}{2}\cos(\theta)\sin(2\theta)\right)$$

A. Rose curve B. Circle C. Lemniscate D. Cardioid E. NOTA

25. What is the eccentricity of the hyperbola $xy + 5x - 3y - 18 = 0$?

A. $\frac{\sqrt{2}}{2}$ B. $\sqrt{2}$ C. $\sqrt{3}$ D. 2 E. NOTA

26. What is the area on the complex plane bounded by all solutions to $z^{12} = 1$ that are not solutions to $z^6 = 1$?

A. $\frac{3\sqrt{3}}{4}$ B. $\frac{3\sqrt{3}}{2}$ C. $\frac{9\sqrt{3}}{4}$ D. $\frac{9\sqrt{3}}{2}$ E. NOTA

27. Find the furthest possible distance between a point on the circle $x^2 + y^2 = 5$ and a point on the circle $x^2 + y^2 - 4x + 22y + 80 = 0$.

A. $5\sqrt{5}$ B. $8\sqrt{5}$ C. $9\sqrt{5}$ D. $13\sqrt{5}$ E. NOTA

28. Three points of a parallelogram are $(6, -3), (-5, 4)$, and $(1, 2)$. What is the sum of all possible x-coordinates of the fourth point?

A. 2 B. -2 C. 4 D. -4 E. NOTA

29. What is the sum of the elements of the matrix $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{2256}$?

A. $\sqrt{3}$ B. 0 C. 1 D. 2 E. NOTA

30. In a certain conic, the distance between a point and one of the directrices is 17. The distance between the same point and the focus that corresponds to the directrix from before is 453. Which of these best describes the conic?

A. Ellipse B. Circle C. Hyperbola D. Parabola E. NOTA