NOTA denotes "None of These Answers." Good luck!

1. Find the length of the major axis of an ellipse with equation:
$$160y + 27x^2 + 212 - 162x + 16y^2 = 1.$$
A. $6\sqrt{3}$ B. 8 C. 16 D. 27 E. NOTA

2. A conic section has foci at $(1,5)$ and $(1,-23)$. If the eccentricity is $\frac{7}{3}$, find the length of a latus rectum.
A. $\frac{80}{3}$ B. $\frac{160}{3}$ C. $\frac{9\sqrt{10}}{5}$ D. 6 E. NOTA

3. Find the area of the ellipse with equation $r = \frac{12}{2+sin\theta}$.
A. $128\pi\sqrt{3}$ B. 128π C. $32\pi\sqrt{3}$ D. 32π E. NOTA

4. A triangle is formed with the three cube roots of 27 on the complex plane. Find the area of this triangle.
A. 9 B. $9\sqrt{3}$ C. $\frac{9\sqrt{3}}{4}$ D. $\frac{27\sqrt{3}}{4}$ E. NOTA

5. Find one of the asymptotes of the hyperbola with foci $(-13,2)$ and $(7,2)$ and a vertex at $(1,2)$.
A. $y = x\sqrt{21} - 6$ B. $y = x\sqrt{21} - 3\sqrt{21}$
C. $y = x\sqrt{21} + 6$ D. $y = x\sqrt{21} + 3\sqrt{21}$ E. NOTA

6. A parabola has equation $-12y = x^2 - 6x - 51$. If a triangle is formed using the latus rectum of the parabola and a point on the directrix, find the area of the triangle.
A. 36 B. $\frac{1}{576}$ C. $\frac{2}{3}$ D. $\frac{3}{32}$ E. NOTA

7. A rectangle is formed with the latus rectums of a hyperbola being two opposite sides. If the hyperbola has equation $\frac{(y-5)^2}{64} - \frac{(x+2)^2}{225} = 1$, find the area of the rectangle.

A. 1020 B. $\frac{3825}{2}$ C. $\frac{4352}{15}$ D. 544 E. NOTA

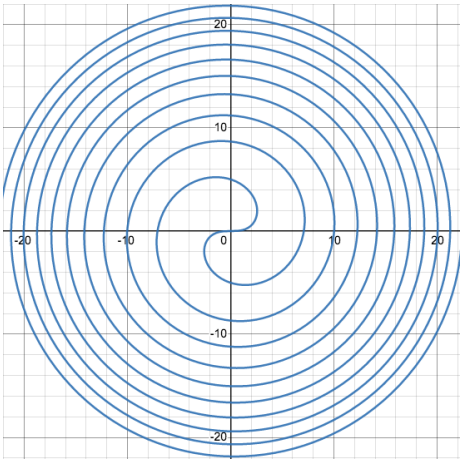
8. On a piece of paper, Hailey draws a coordinate grid and then two distinct points on the grid before passing the paper to Carol. Carol then draws the locus of points that is equally distant from both points. Which of the following correctly describes what Carol drew?

A. Circle B. Hyperbola C. Line D. Parabola E. NOTA

9. The ellipse $\frac{x^2}{16} + \frac{y^2}{6} = 1$ has foci A, B and a point P on the ellipse. Given APB is a right triangle, find the sum of all possible values for the square of the area APB.

A. 56.5 B. 57.5 C. 58.5 D. 59.5 E. NOTA

10. What is the name and equation of the graph below? $(a > 0)$



A. Fermat's spiral; $r = \pm a\sqrt{\theta}$

B. Fermat's spiral; $r = \pm \frac{a}{\sqrt{\theta}}$

C. Archimedean spiral; $r = \pm a\sqrt{\theta}$

D. Archimedean spiral; $r = \pm \frac{a}{\theta}$

E. NOTA

11. Complex numbers $E = 2\sqrt{2} + 2i\sqrt{2}$ and $X = -3\sqrt{3} + 3i$ are graphed as points on the Argand plane. Find the square of the distance between E and X.

A. $4(13 + 3\sqrt{6} - 3\sqrt{2})$ B. $4(13 + 3\sqrt{6} + 3\sqrt{2})$
C. $4(13 - 3\sqrt{6} + 3\sqrt{2})$ D. $54 + 12\sqrt{6} - 12\sqrt{2}$ E. NOTA

12. Find an equation of the plane containing points $(3,6,-1)$, $(-1,2,7)$, and $(8,5,0)$.

A. $x - 10y + 6z = 63$ B. $x + 11y + 6z = 63$
C. $2x + 13y - 6z = 72$ D. $x + 7y + 9z = 36$ E. NOTA

13. The maximum y-coordinate of the cardioid-esque graph $r = (1 + \cos(\theta))^2$ is equal to $\frac{p\sqrt{q}}{r}$ in simplest form. Find $p + q + r$. (Hint: maximize y^2 instead of y directly.)

A. 54 B. 57 C. 60 D. 63 E. NOTA

14. A tetrahedron has vertices at $(-1,3,2)$, $(6,-4,-3)$, $(7,5,-2)$, and $(9,-7,12)$. Find the volume of the tetrahedron.

A. 1200 B. 600 C. 400 D. 200 E. NOTA

15. Identify the conic with equation $369 + 96x + 9y^2 + 16x^2 - 54y = 0$.

A. Ellipse B. Hyperbola C. Line D. Point E. NOTA

16. Vector $\vec{\boldsymbol{u}}$ is found by projecting $\vec{\boldsymbol{a}} = \langle 5, 3, 2 \rangle$ onto $\vec{\boldsymbol{b}} = \langle 1, 2, 1 \rangle$. What is $\vec{\boldsymbol{u}}$?

A. $\left\langle \frac{13\sqrt{6}}{36}, \frac{13\sqrt{6}}{18}, \frac{13\sqrt{6}}{36} \right\rangle$ B. $\left\langle \frac{13\sqrt{6}}{6}, \frac{13\sqrt{6}}{3}, \frac{13\sqrt{6}}{6} \right\rangle$
C. $\left\langle \frac{13}{36}, \frac{13}{18}, \frac{13}{36} \right\rangle$ D. $\left\langle \frac{13}{6}, \frac{13}{3}, \frac{13}{6} \right\rangle$ E. NOTA

17. Consider point P with positive x-coordinate on the graph $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Let $A = (0, q), B = (-5,0)$. Given the minimum of $AP + BP$ is equal to 15, compute q^2.

A. 11 B. 13 C. 22 D. 24 E. NOTA

For questions 18 and 19, use the following equation for nondegenerate conic A

$$7x^2 + 4xy + 8y^2 - 36x + 16y - 75 = 0$$

18. Which of the following describes the graph of Conic A?

A. Parabola B. Hyperbola C. Ellipse D. Circle E. NOTA

19. Let θ be the acute angle the axes need to be rotate counterclockwise by to eliminate the xy term in conic A's equation. Find $\tan\theta$.

A. $\frac{\sqrt{17}-1}{4}$ B. $\frac{1+\sqrt{17}}{4}$ C. $\frac{\sqrt{17}-4}{16}$ D. $\frac{\sqrt{17}+4}{16}$ E. NOTA

20. Ellipse E has equation $\frac{x^2}{81} + \frac{y^2}{72} = 1$, and foci F_1, F_2 with F_1being the focus with the smaller x-coordinate. Circle C has equation $x^2 + (y-4)^2 = 9$. Let P be a point on E in the first quadrant, and the line passing through P and F_1 intersects C at two distinct points A, B. Let A be the point closer to F_1. Then, the possible values of $AP + PF_2$ fall in the range $(r, s]$. Compute $r + s$.

A. 24 B. 26 C. 28 D. 30 E. NOTA

21. Find the area enclosed by the graph of the parametric equations: $\begin{matrix} x = 14\sin(t)\cos(t) \\ y = 12\sin^2(t) + 10 \end{matrix}$.

A. 1764π B. 1534π C. 42π D. $\pi\sqrt{42}$ E. NOTA

22. Let line $m = (3,5,-1) + \langle 2,-1,4\rangle t$ and line $n = (2,1,1) + \langle 3,-2,7\rangle r$. What is the distance between line m and n?

A. $\sqrt{6}$ B. $\frac{7\sqrt{6}}{6}$ C. $\frac{20\sqrt{67}}{67}$ D. $\frac{5\sqrt{6}}{6}$ E. NOTA

23. If an ellipse is in the form of $A(x-h)^2 + B(x-h)(y-k) + C(y-k)^2 = 1$, then what is its area?

A. $\frac{2\pi i}{\sqrt{B^2-4AC}}$ B. $\frac{2\pi}{i\sqrt{B^2-4AC}}$ C. $\frac{i\sqrt{B^2-4AC}}{2\pi}$ D. $\frac{\sqrt{B^2-4AC}}{2\pi i}$ E. NOTA

24. Point J has coordinates of (14, 8). If J was rotated clockwise by $\frac{5\pi}{4}$ about the origin, find the new coordinates of J.

A. $(-11\sqrt{2}, 3\sqrt{2})$ B. $(-3\sqrt{2}, -11\sqrt{2})$
C. $(3\sqrt{2}, 11\sqrt{2})$ D. $(11\sqrt{2}, -3\sqrt{2})$ E. NOTA

25. Which of the following does not always uniquely define a plane?

A. Two intersecting lines B. Two parallel lines
C. A line and a point not on the line D. Three points E. NOTA

26. Find the area of the region enclosed by the graph of $|13x + 23| + |7y - 8| = 91$.

A. 486 B. 364 C. 243 D. 182 E. NOTA

27. Find the area of intersection between $(x-1)^2 + (y+2)^2 + (z-4)^2 \leq 64$ and $x + 2y + 2z = 14$.

A. 55π B. 56π C. 57π D. 58π E. NOTA

28. A parabolic archway has height of 2024 feet at its center and a width of 2024 feet. What is the height, in feet, above a coin 253 feet away from one of the edges? Assume that the height of the coin is neglectable and the arch starts on the ground.

A. 506 B. $\frac{1771}{2}$ C. 1518 D. $\frac{3795}{2}$ E. NOTA

29. Find the equation of the hyperbola that shares latus recta with the ellipse described by the equation $\frac{(x-1)^2}{112}+\frac{(y-5)^2}{256}=1$.

A. $\frac{(y-5)^2}{81}-\frac{(x-1)^2}{63}=1$ B. $\frac{(x-1)^2}{81}-\frac{(y-5)^2}{63}=1$

C. $\frac{(y-5)^2}{80}-\frac{(x-1)^2}{64}=1$ D. $\frac{(x-1)^2}{80}-\frac{(y-5)^2}{64}=1$ E. NOTA

30. Congratulations on reaching the end of this test! For this last question, find the number that cannot be the number of petals in a rose curve from the following.

A. 18 B. 20 C. 21 D. 24 E. NOTA