1. B We find the discriminant, $B^2 - 4AC$, where A is the x^2 term, B is the xy term and C is the y^2 term to be $4^2 - 4(2)(-3) = 40$, which is greater than 0, meaning this conic is a hyperbola
2. A These lemniscates are similar. The differences between them are the angle of rotation, which will not affect the area, and the scaling factor in front of the trigonometric function. Since area is directly proportional to r^2, the ratio of the areas is simply the ratio of the coefficients on the trigonometric functions, which is $\frac{8}{9}$
3. B $\cot(2\theta) = \frac{A-C}{B} = \frac{4-1}{\sqrt{3}} = \sqrt{3}$, meaning $2\theta = \frac{\pi}{6}$, so $\theta = \frac{\pi}{12}$.
4. E At first glance, this may look like a hyperbola with foci at $-i$ and i, but the difference in the distances from these points must be 12. This is impossible, via the Triangle Inequality; the distance is at most 2, which is achieved by a point coincident with either focus.
5. A We will simply pick a point on the first plane, say $(1, 0, 0)$, and find the distance from the this point to the second plane using the point-to-plane distance formula. This gives us $\frac{|3+0+0-4|}{\sqrt{3^2+4^2+1^2}} = \frac{\sqrt{26}}{26}$.
6. A The location of the point is irrelevant if it is on the ellipse. The side of the triangle connecting the two foci will have length $2c$ where $c^2 = a^2 - b^2$ and $a^2 = \frac{1}{4}, b^2 = \frac{1}{9}$. Thus $2c = \frac{\sqrt{5}}{3}$. The sum of the other two side lengths is simply $2a = 1$ since it is the sum of the distances from a point on the ellipse to both foci, so the perimeter is $\frac{\sqrt{5}}{3} + 1 = \frac{\sqrt{5}+3}{3}$.
7. A This ellipse can be written in the form $\frac{(x+2)^2}{4} + \frac{(y-3)^2}{7} = 1$. Eccentricity is $\frac{c}{a}$ where $c^2 = a^2 - b^2$. $a^2 = 7, b^2 = 4$, so $c^2 = 3$. Thus $\frac{c}{a} = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$.
8. A We first find the vectors that make up the tetrahedron, which are $< -5, 6, 7 >$, $< 4, 2, 0 >$, and $< -6, 9, 21 >$. We then find the determinant of the matrix with these three vectors as rows, and take the absolute value of it to get 378. Finally we divide by 6 to get 63.
9. D The magnitude of the cross product of two vectors is $|A||B|\sin(\theta)$ while the dot product is $|A||B|\cos(\theta)$. Therefore, dividing the magnitude of the cross product (which is $< 13, -26, -13 > \rightarrow 13\sqrt{6}$) by the dot product ($6 - 5 - 28 = -27$) will give us the tangent of an angle, which is $-\frac{13\sqrt{6}}{27}$.
10. C The intersection of the asymptotes is simply the center, to find the center, we need to put the conic into the form $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ where the center is (h, k). Completing the squares in x and y and dividing both sides of the equation by 25, we get $\frac{(x-1)^2}{5} - \frac{(y+3)^2}{25} = 1$, so the center is $(1, -3)$.
11. D After we pick four random points on a circle, we can assume a certain one is A without loss of generality. In order for ABCD to form a valid quadrilateral, point A must be the point that is not adjacent to point D, meaning that once we choose A,

there is one choice out of 3 possible points for D that makes a valid quadrilateral. Thus the probability we seek is $\frac{1}{3}$.

12. C Multiplying x by 2, squaring both equations, and subtracting the equations gives us $x^2 - \frac{y^2}{4} = 1$. The larger axis is the conjugate axis, which has length $2\sqrt{4} = 4$.

13. C We can treat the point $(3, -5)$ as the complex number $3 - 5i$. Then a clockwise rotation about the origin by 90 degrees is simply a multiplication by $-i$. Thus the complex number representation of the resultant point is $(3 - 5i)(-i) = -5 - 3i$, which is $(-5, -3)$ when represented as a Cartesian point. Alternatively, we could have used a rotation matrix to get the same result.

14. B Given any 5 points there is 1 unique conic section that passes through all of them, so neither A nor B can be greater than 5. A must be 5 because 2 ellipses can have 4 intersection points between them, meaning there could be 2 ellipses passing through all points if we are only given 4 points. B is 3 because a parabola with a vertical directrix can be written in the form $x = ay^2 + by + c$, and 3 points are all that are needed to determine the coefficients. Thus, the answer is $5 + 3 = 8$.

15. C $r - r\sin\theta = 3$, so $r = 3 + y$ and $x^2 + y^2 = y^2 + 6y + 9$. Simplifying, $6y = x^2 - 9$, or $y = \frac{x^2}{6} - \frac{3}{2}$. This is a parabola with vertex $\left(0, -\frac{3}{2}\right)$ and a focus on the origin, so its directrix is $y = -3$.

16. E In order for any number to have this property, it must have a remainder of 2 when divided by 4. There are no perfect squares with a remainder of 2 when divided by 4, so the answer is 0.

17. A We can find coordinates on the Cartesian plane that match up to these side lengths and use the Shoelace theorem to find the area. One set of coordinates that works is $(0,0), (1,1)$, and $(-3,2)$. Applying the Shoelace theorem gives an area of $\frac{5}{2}$.

18. A We can find the circumcenter by taking two pairs of points, find the perpendicular bisectors, and finding the intersection point between those perpendicular bisectors. The perpendicular bisector of $(6,0)$ and $(8,-4)$ is $x - 2y = 11$ and the perpendicular bisector of $(6,0)$ and $(-1,-1)$ is $7x + y = 17$, which gives us an intersection point of $(3,-4)$.

19. B On the Cartesian plane, the given complex graph is the same as the parabola with focus $(1,0)$ and directrix $x = -1$. This parabola can be written as $x = \frac{y^2}{4}$. All points on the graph of $y = 2\sqrt{x}$ also lie on $x = \frac{y^2}{4}$, So $2\sqrt{x}$ is our answer.

20. D In terms of t the distance between a point on the line and the origin is $\sqrt{(3t)^2 + (5t-2)^2 + (1-7t)^2} = \sqrt{83t^2 - 34t + 5}$, and the quadratic under the radical is minimized at $t = \frac{17}{83}$.

21. B In order for lines to be coplanar, they must either be parallel or intersecting, and since they are not parallel, they must be intersecting. We solve for the values of t and s by setting the values of x and y equal, which gives us the two equations $3t = 2s + 1$ and $5t - 2 = 5s + 3$. Solving for s and t we get $t = -1$ and $s = -2$. Setting the z values equal, we get $-7 * (-1) + 1 = -2a \rightarrow\ a = -4$.

22. B This region is made up of two cones, both with height 60 and radius 60. Thus the volume is $2 * \frac{1}{3} * \pi * 60^2 * 60 = 144000\pi$.

23. C When the constant term stays the same in a rotation, the values of $A + C$ and $B^2 - 4AC$ stay the same. However, the constant term here is doubled, meaning A and C are both doubled, so $A + C$ is doubled and $B^2 - 4AC$ is quadrupled. $A + C$ is 7 for the original conic, so it becomes 14. $B^2 - 4AC$ is 121 in the original conic, so it becomes 484, meaning $AC = -121$ since $B = 0$. Thus, the answer is $-121 + 14 = -107$.

24. A Factoring $\sin(\theta)$ out of the first two terms and $\cos(\theta)$ out of the last two terms, we get $r = \sin(\theta)\left(\frac{1}{2}\cos(2\theta) - \frac{\sqrt{3}}{2}\sin(2\theta)\right) + \cos(\theta)\left(\frac{\sqrt{3}}{2}\cos(2\theta) + \frac{1}{2}\sin(2\theta)\right)$. Recognizing angle addition, this becomes $r = \sin(\theta)\cos\left(2\theta + \frac{\pi}{3}\right) + \cos(\theta)\sin(2\theta + \frac{\pi}{3})$ which is the same as $r = \sin(3\theta + \frac{\pi}{3})$ by recognizing angle addition again. This figure is a rose curve (the $\frac{\pi}{3}$ only rotates the figure).

25. B This hyperbola can be written as $(x - 3)(y + 5) = 3$. The center does not affect the eccentricity, so this is the same as finding the eccentricity of $xy = 3$. The eccentricity of any hyperbola in the form $xy = k$ is $\sqrt{2}$ because when rotated, hyperbolas in this form can always be written as $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$, which has $c^2 = a^2 + a^2 \rightarrow c = a\sqrt{2} \rightarrow \frac{c}{a} = \sqrt{2}$.

26. B This is the area of a hexagon with radius 1, which also has a side length of 1. Thus, the area is simply $\frac{3\sqrt{3}}{2}$ (the roots are $cis\left(\frac{\pi}{6}\right), cis\left(\frac{3\pi}{6}\right), cis\left(\frac{5\pi}{6}\right) \dots, cis(\frac{11\pi}{6})$).

27. C The furthest distance between the two circles will be the distance between their centers plus the sum of their radii. The center and radius of the first circle is clear, so to get the center and radius of the second circle, we complete the squares in x and y on the second circle to get $(x - 2)^2 + (y + 11)^2 = 45$. The two centers are $(0,0)$ and $(2, -11)$, which have a distance of $\sqrt{2^2 + 11^2} = 5\sqrt{5}$ between them. The radii of the circles are $\sqrt{5}$ and $3\sqrt{5}$ respectively, giving us a total distance of $5\sqrt{5} + 4\sqrt{5} = 9\sqrt{5}$.

28. A $6 - (-5) = 11$ and $-3 - 4 = -7$, so two possibilities are $(1 + 11, 2 - 7) = (12, -5)$ and $(1 - 11,\ 2 + 7) = (-10, 9)$. $6 - 1 = 5$ and $-3 - 2 = -5$, so two more possibilities are $(-5 + 5, 4 - 5) = (0, -1)$ and $(-5 - 5, 4 + 5) = (-10, 9)$, but we already counted $(-10, 9)$, so we do not count it twice. Finally, $-5 - 1 = -6$ and $4 - 2 = 2$, so two more possibilities are $(6 - 6, -3 + 2) = (0, -1)$ and $(6 + 6, -3 - 2) = (12, -5)$, but we already counted both, so we do not count them twice. The possible x-coordinates are $-10 + 12 + 0 = 2$.

29. D This is a rotation matrix with angle $\frac{\pi}{6}$ raised to a power that is a multiple of 12, meaning that the resultant matrix will also be a rotation matrix with an angle coterminal to 2π, meaning it will be the identity matrix, which has the sum of its elements equal to 2.

30. C The eccentricity is the ratio of the distance between a point and focus to the distance between the same point and the corresponding directrix. Here, the eccentricity is $\frac{453}{17}$, which is greater than one, so the figure is a hyperbola.